

***Mechanical and Energy Engineering***

**Simulation of Pose to Pose Moving of the Mobile Robot with Specified GPS Points**

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**ABSTRACT**

The applications of mobile robots in rescue scenarios, surviving to search, and exploration for outdoor navigation have received increasing attention due to their promising prospects. In this paper, a simulation of a differential wheeled mobile robot was presented, implementing a Global Positioning System (GPS) data points to specified starting points, final destination, and total error. In this work, a simple kinematic controller for polar coordinate trajectory tracking is developed. The tracking between two points, pose to pose, was specified by using the GPS data points. After that, the geodesy (GEO) formulation was used to convert the geodesy coordinate to Euclidean or polar coordinate. The Haversine equation obtained the distance between the two points. The system performance and stability of the tracking controller are proved using the Lyapunov theorem of the stability. A python script was used in this work as a simulator. Computer simulation with pose to pose trajectory strategy conform to the simplicity of the proposed controller.

**Keywords:** Haversine, Kinematic controller, Trajectory tracking, Polar coordinates.

**محاكاة حركة الروبوت النقل من نقطة الى نقطة المحددة بواسطة الـ GPS**

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**الخلاصة**

ان تطبيقات الروبوتات النقالة - تكنولوجيا الأتمتة في عمليات الانقاذ و البحث عن الحياة و الاستكشاف في الملاحة الخارجية اكتسبت اهتماما متزايدا بسبب افاقها الواعدة. في هذا البحث تم تقديم نمذجة للروبوت نقل ذو عجلات تفاضلية التي يستخدم بيانات النظام العالمي لتحديد المواقع GPS لتحديد نقاط البداية و النهاية والخطأ الكلي.

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في هذا البحث تم تطوير نظام سيطرة كينماتيكي لتتبع المسار في الاحداثيات القطبية. تم تعيين مسافة المسار بين نقطتين البداية والنهاية بواسطة بيانات ال GPS و بعدها تم استخدام المعادلات الجيوديسيا لتحويل النظام الجيوديسيا الى الاحداثيات الاقليدية او القطبية . المسافة بين نقطتين تم ايجادها بواسطة معادلة Haversine . تم اثبات استقرارية المسار الخاطيء للنظام باستخدام تحليل لياونوف للاستقرارية. استخدم البايثون كبرنامج للنمذجة. نتائج النمذجة الحاسوبية لتتبع المسار من نقطة الى نقطة بينت سهولة استخدام المسيطر المقترح.

الكلمات الرئيسية: هافرساين , السيطرة الكينماتيكية , تتبع المسار , الاحداثيات القطبية .

## 1. INTRODUCTION

The goal is to develop a kinematic controller of an autonomous mobile robot that can navigate from converting the geodesy coordinate to the polar coordinate system by haversine formulation. The wheeled mobile robot (WMR) is used in a wide range of applications that are dangerous for humans.

The rescue applications in dangerous scenarios depending on computer science and robotic technology; the mobile robot is used widely in rescue and military applications (Yang et al. 2019; Al-Araji and Ibraheem 2019). Mobile robot study must contain several topics, for instance, path planning (Yousif and Ali 2019; Jawad and Hadi 2019), localization, navigation, and tracking control. In general, mobile robot navigation is divided into two categories, local (Ibraheem 2010; Choi, Lee, and Won 2011) and global (Borenstein and Feng 1996; Qi and Moore 2002; Yang et al. 2019) navigation. The local navigation is used for indoor applications. On the other hand, the global navigation is used for outdoor navigation. The local navigation uses dead reckoning, but global navigation uses GPS devices (Qi and Moore 2002). For indoor navigation, in some cases, dead reckoning is used, which means obtaining the data from inertial sensors like accelerometer (Park et al. 1996). The dead reckoning includes some source of errors as white noise and accumulative errors. Another way for indoor navigation uses a camera (Cunha et al. 2011). The used camera is to acquire marks and information about the environment.

In outdoor navigation, (Urmson et al. 2008) used an autonomous DARPA vehicle named (Boss), and the vehicle used a GPS for global navigation and laser for obstacle avoidance.

The work (Yang et al. 2019) presented a series manner to solve the GPS problem by using a novel approach in a large scale environment. They used a 3D-laser scanner to solve poor signal localization problem in GPS-denied environments.

The tracking control of the mobile robot is another field of study in the last years. The tracking control topic is subdivided into two topic path tracking and trajectory tracking. The path tracking means the accurate following of the predefined path with time-independent. The Cartesian path tracking works were done by (Das, Kar, and Chaudhury 2006; Martins and Brandão 2018) while polar path tracking was done by (Lee et al. 2000; Jin and Tack 2011) as kinematic models.

(Muñoz, Muñoz-Panduro, and Ramos 2018) work proposed a collision-free trajectory using Artificial Potential Fields and sensed depth data from the surrounding environment. They used a closed loop controller based on polar coordinates, which is led by a potential field.

In general, control systems that have been proposed for various differential WMR ( Wheeled Mobile Robot ) can be divided into two classes, a dynamic controller (Martins and Brandão 2018; Al-Araji and Ibraheem 2019) and kinematic controller (Lee et al. 2000). The dynamic controller uses the torque of the dynamic model as controller term. In kinematic controller uses the kinematic model and the velocity as controller term. The work (Cornejo et al. 2018) focused



on the kinematic controller based on polar coordinates and compared with the kinematic LQR controller by using LabVIEW. Their comparison found that the LQRT is better than the polar controller and the  $y_{error}$ ,  $x_{error}$  in LQR better by 0.52% and 7.52 % .

The kinematic controller must produce a smooth signal of control. This work modified a polar kinematic controller that compensates the velocity before reach the limited value.

## 2. KINEMATIC MODEL

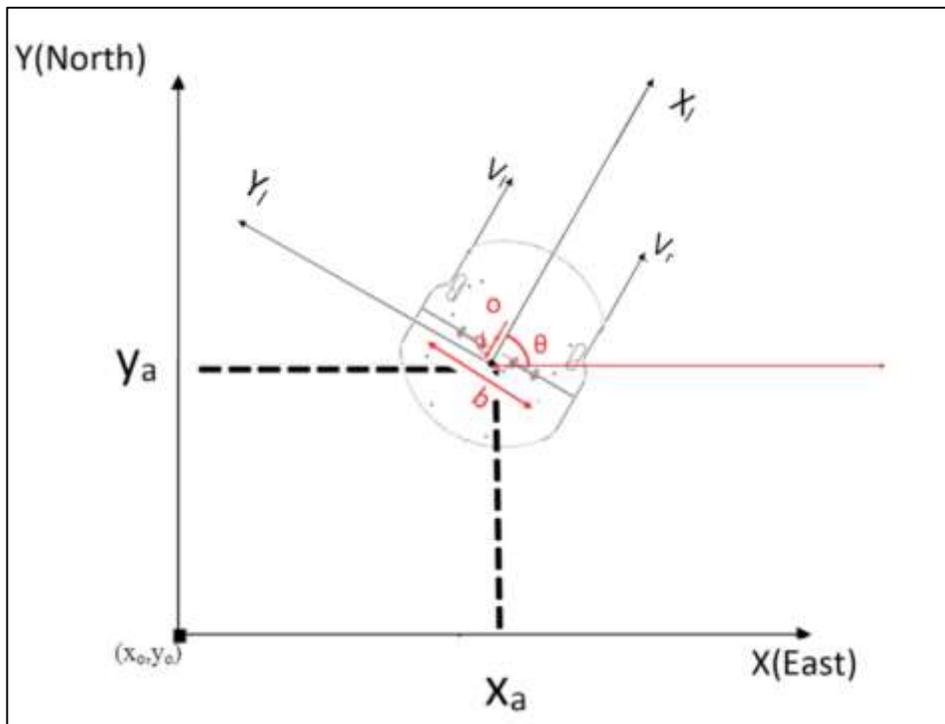
Consider the mobile robot in **Fig. 1** using generalized coordinate vector  $q = [X, Y, \theta]$  where  $Y$  is north of earth direction and  $X$  east of earth direction of the robot posture shown its all configuration space.

The linear and angular velocity of the frame at point  $(x_a, y_a)$  are:

$$v(t) = \frac{V_r(t) + V_l(t)}{2} \tag{1}$$

$$w(t) = \frac{(w_R - w_L)r}{b} \tag{2}$$

Where  $v(t)$  is the speed of the robot in the forward direction and  $w(t)$  is the angular velocity of the robot frame.  $V_r, V_l$  are the right and left wheels velocity as shows in **Fig. 1**,  $w_R, w_L$  are the angular of the left and right wheels velocities around their axes,  $b$  is the distance between the two wheels.  $r$  is the radius of the WMR wheels.



**Figure 1.** Coordinate System Mobile Robot.



The wheels driving the robot make it non-holonomic and imposes the pure rolling constraint on the expressed in Eq (1).

$$\dot{x}_a(t) = v(t) \cos\theta(t) \tag{3}$$

$$\dot{y}_a(t) = v(t) \sin\theta(t) \tag{4}$$

$$\dot{\theta}(t) = w(t) \tag{5}$$

$$\begin{bmatrix} \dot{x}_a(t) \\ \dot{y}_a(t) \\ \dot{\theta}_a(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ \dot{\theta}(t) \end{bmatrix} \tag{6}$$

Where  $\dot{x}_a$  is the velocity of the point (a) in  $x - axis$  direction and the  $\dot{y}_a$  is the velocity of point (a) in  $y - axis$  direction, and  $\theta$  is the orientation of WMR.

The velocity of the WMR state was represented with respect to the goal frame in the polar coordinate. The kinematics of the differential mobile robot model, as shown in **Fig. 2**, will be as follow (**Cornejo et al. 2018**):

$$\dot{\rho} = -v \cos(\theta - \gamma) = -v \cos A \tag{7}$$

$$\dot{\theta} = v \frac{\sin A}{\rho} \tag{8}$$

$$\dot{A} = -\omega + v \frac{\sin A}{\rho} \tag{9}$$

Where  $A = \theta - \gamma$  is the angle between the  $Xr$  robot chassis reference coordinate and the line of  $\rho$ ,  $\theta$  is the angle between the latitude and the bearing angle,  $\rho$  is the distance between two GPS points as shown in **Fig. 2**. Rewriting the above equation as follows:

$$\begin{bmatrix} \dot{\rho} \\ \dot{A} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\cos A & 0 \\ \frac{\sin A}{\rho} & -1 \\ -\frac{\sin A}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{10}$$

$\rho$  is defined as the distance between the two GPS points, and it is given by the Haversine equation (**De Smith, Goodchild, and Longley 2007**). Haversine equation assumes that the earth is a perfect sphere. This assumption has an error of about 0.5% in a large scale of 100 m.

$$\rho = R \cdot c \tag{11}$$

Where  $R$  is mean earth radius  $R = 6371 \text{ km}$ , and  $c$

$$c = 2 * \text{atan2}(\sqrt{\epsilon}, \sqrt{1-\epsilon}) \tag{12}$$

And  $\epsilon$  is defined with the following relation

$$\epsilon = \sin^2 \left( \frac{\Delta lat}{2} \right) + \cos(lat1) * \cos(lat2) * \sin^2 \left( \frac{\Delta log}{2} \right) \tag{13}$$

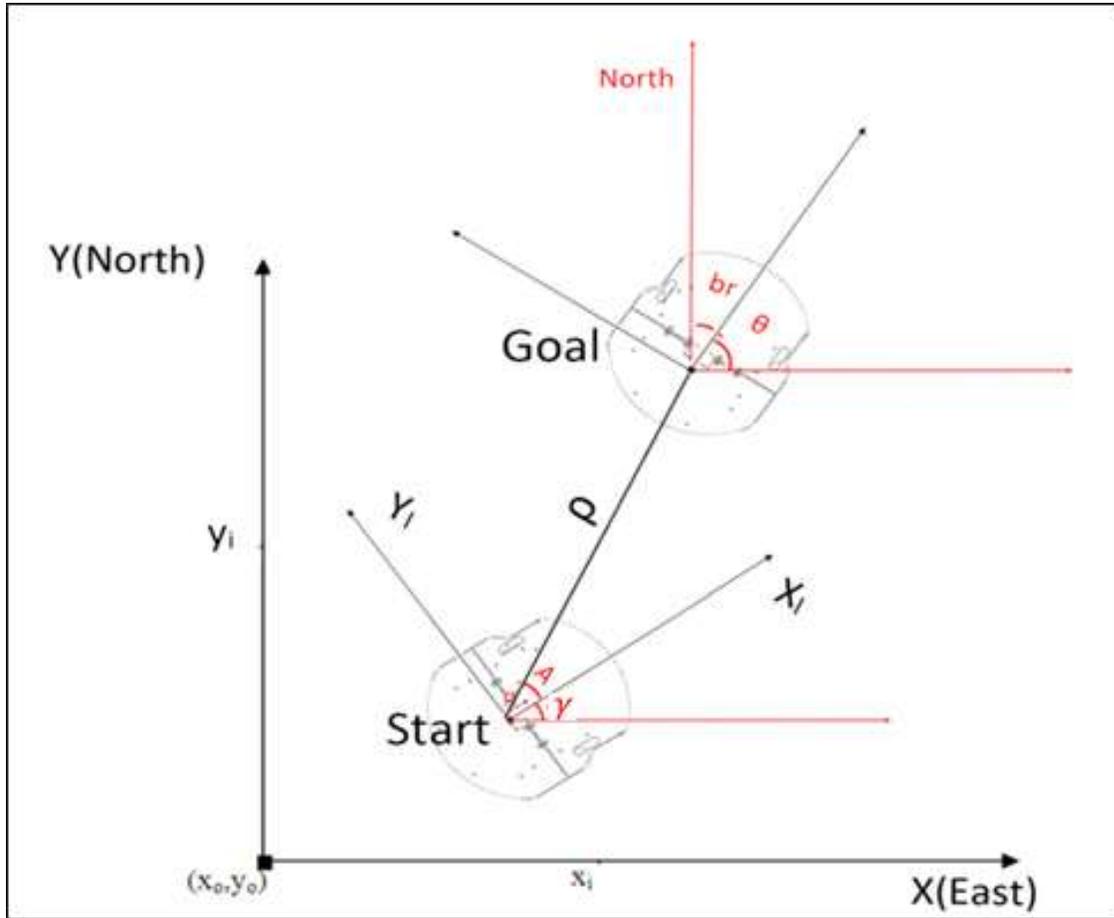


Figure 2. Mobile robot pose to pose configuration.

Recalling Eq. ( 6) to find  $\theta$  from the bearing angle which is obtained as follows:

$$\theta = 90 - br \tag{14}$$

Where  $br$  is the bearing and  $br$  is defined as :

$$br = atan2(\zeta_x, \zeta_y) \tag{15}$$

Where  $\zeta_x$  and  $\zeta_y$  is defined as follows:

$$\zeta_x = \cos(lat2) * \sin(\Delta long) \tag{16}$$

$$\zeta_y = \cos(lat1) * \sin(lat2) - \sin(lat1) * \cos(lat2) * \cos(\Delta long) \tag{17}$$

$lat$  are latitude and  $long$  longitude points, where all value of  $long$  and  $lat$  are in radius and obtained from GPS raw data.



### 3. CONTROLLER DESIGN BASED ON LYAPUNOV STABILITY

WMR is a non-linear system. To find a stable controller for WMR, the controller must base on the Lyapunov stability theory. Lyapunov equation Candidate as the following equation to find the stable controller.

$$= \frac{1}{2L} \rho^2 + \frac{1}{2} (A^2 + h\theta^2), h, L > 0 \tag{18}$$

Where the V means the error energy in both distance and direction of the wheeled mobile robot.

To find the stability of the WMR,  $\dot{V}$  must satisfy the below inequality:

$$\dot{V} = \frac{\rho \dot{\rho}}{L} + (A\dot{A} + h\theta\dot{\theta}) < 0 \tag{19}$$

By differentiating Eq.(18) and putting the Eq. (7) in Eq. (19)

$$\dot{V} = \frac{\rho}{L} (-v \cos A) + A \left( -\omega + \frac{v \sin A}{\rho} \right) + \theta h \left( \frac{v \sin A}{\rho} \right) < 0 \tag{20}$$

$$\dot{V} = \left[ \frac{\rho}{L} (-v \cos A) \right] + \left[ A(-\omega) + \frac{v \sin A}{\rho} (h\theta + A) \right] < 0 \tag{21}$$

If the Lyapunov equation is to be stable, the first part  $[\rho(-v \cos A)]$  must be negative, and for that, the control law for  $v$  was assumed as follows:

$$v = v_{max} (1 - \exp(-k_\rho \rho)) \cos A \tag{22}$$

The first part of the Lyapunov equation become  $\left[ -\frac{|\rho|}{L} (v_{max} (1 - \exp(-k_\rho \rho)) \cos^2 A) \right]$  and the square of  $\rho$ ,  $\cos A$  make any  $\rho$  and  $A$  stable. For the second part of Lyapunov  $\left[ \frac{A}{L} (-\omega) + \frac{v \sin A}{\rho * L} (h\theta + A) \right] < 0$ , so the  $\omega$  must be as follows

$$\omega > (A + h\theta) k_\alpha \cos A \sin A \tag{23}$$

So we proposed the following control law for  $\omega$  as following

$$\omega = k_\alpha A + \frac{(v_{max}(1-\exp(-k_\rho \rho)) \cos A \sin A)}{\rho_i} \frac{\cos A \sin A}{A} (A + h\theta) \tag{24}$$

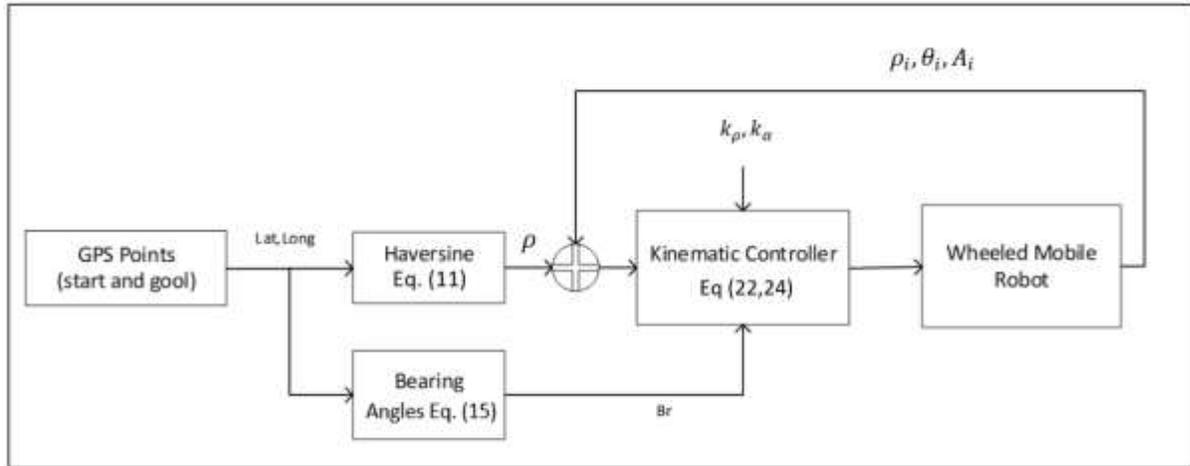
where the  $L, h, k_\alpha$  and  $k_\rho$  are designer positive constant parameters.  $v_{max}$  is the limit speed of the WMR. Substituting Eq. (24) in Eq. (21) leads to

$$\dot{V} = \left[ \frac{\rho}{L} (-v_{max}(1 - \exp(-k_\rho \rho)) \cos^2 A) \right] + \left[ A \left( -k_\alpha A - \frac{(v_{max}(1-\exp(-k_\rho \rho)) \cos A \sin A)}{\rho_i} \frac{\cos A \sin A}{A} (A + h\theta) \right) + (v_{max}(1 - \exp(-k_\rho \rho)) \cos A) \frac{\sin A}{\rho} (h\theta + A) \right] < 0 \tag{25}$$

$$\dot{V} = \left[ \frac{|\rho|}{L} (-v_{max}(1 - \exp(-k_\rho \rho)) \cos^2 A) \right] - k_A A^2 - k_A A - \frac{1-\exp(-k_\rho \rho)}{\rho} \frac{\cos A \sin A}{A} v_{max} h\theta \leq 0 \tag{26}$$



The proposed control law achieves stability, as indicated in the simulation results. This type of control is kinematic controlling, so in these types. Another inside control loop is needed that is faster ten times of the main loop for the wheels in any real experiment. The block diagram of the presented work structure is given in **Fig. 3**.



**Figure 3.** Block diagram of the mobile robot controller.

#### 4. SIMULATION RESULTS

The proposed controller Eq (22) and Eq (24) were verified with computer simulation by using Python scripts. For simulation, two types of reference paths have been selected. The first path is done with a circular path, and equations are given as:

$$x_r = 4 \cos\left(\frac{2\pi}{T}t\right) - 4, y_r = 4 \sin\left(\frac{2\pi}{T}t\right), \theta_r = \frac{2\pi}{T}t + \frac{\pi}{2}$$

Where T =40 sec is the cyclic time.

The initial error is 1 m in the x-direction and 2 m in the y-direction, as illustrated in **Fig. 4**. The initial reference starts at  $[x_r(0), y_r(0), \theta_r(0)]^T = [0, 0, \pi/2]^T$  and the initial robot posture of WMR is chosen as  $[x(0), y(0), \theta(0)]^T = [1, 2, 0]^T$ .

The control parameter was chosen as  $k_\rho = 1.9, k_\alpha = 1$  and  $h = 0.05$ , and for the kinematic controller, the dynamic parameter is not required. The tracking performance is shown in **Fig. 4**, where the tracking path is indicated by the red line. The trajectory error is decreased rapidly, as shown in **Fig. 5**, and the total error converges to 0.1 m at the end. The performance of the proposed kinematic control law is compared to the kinematic work of the (Cornejo et al. 2018) for the circular case as above trajectory case. For the proposed work, the parameters take as a previous case as for (Cornejo et al. 2018) is taken. The performance comparison is denoted in **Fig. 6** and illustrates the performance tracking of the proposed work. To see the performance of the proposed work, index  $Error = \sqrt{x_e^2 + y_e^2}$  is chosen, where  $x_e$  is the error in x direction and  $y_e$  is the error in y direction. **Fig. 7** gives a comparison between the proposed controller and (Cornejo et al. 2018). It is clear to observe that the proposed controller is better than the (Cornejo et al. 2018) controller.

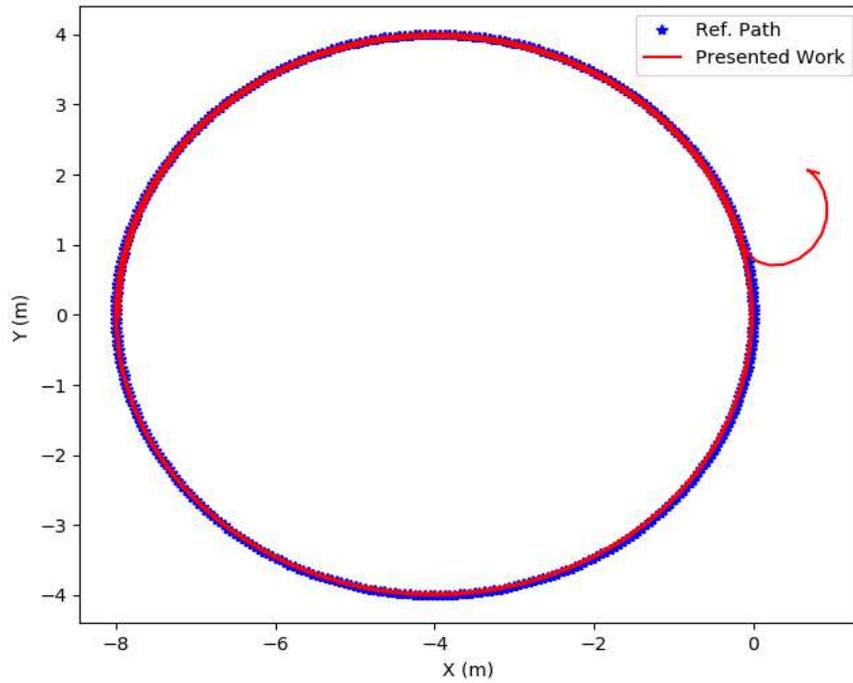


Figure 4. Block diagram of the mobile robot controller.

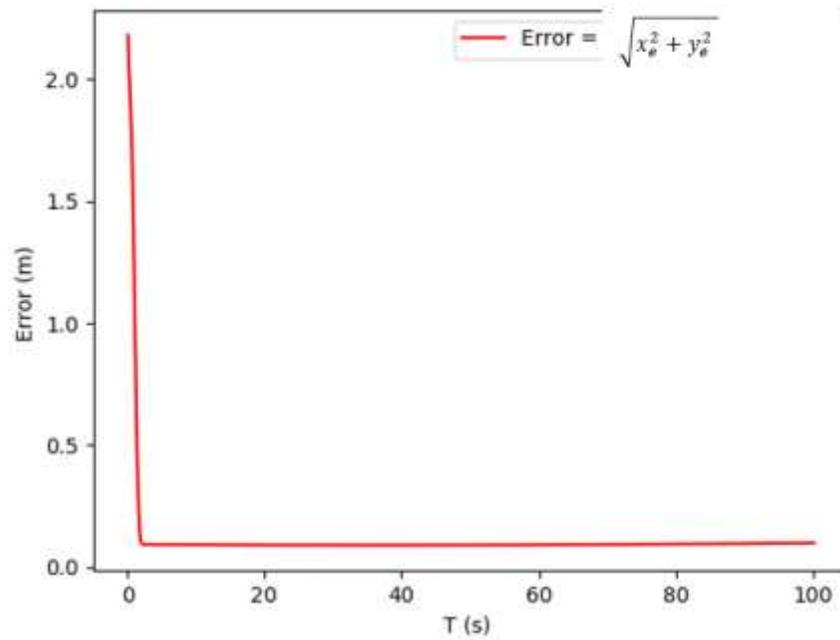


Figure 5. Block diagram of the mobile robot controller.

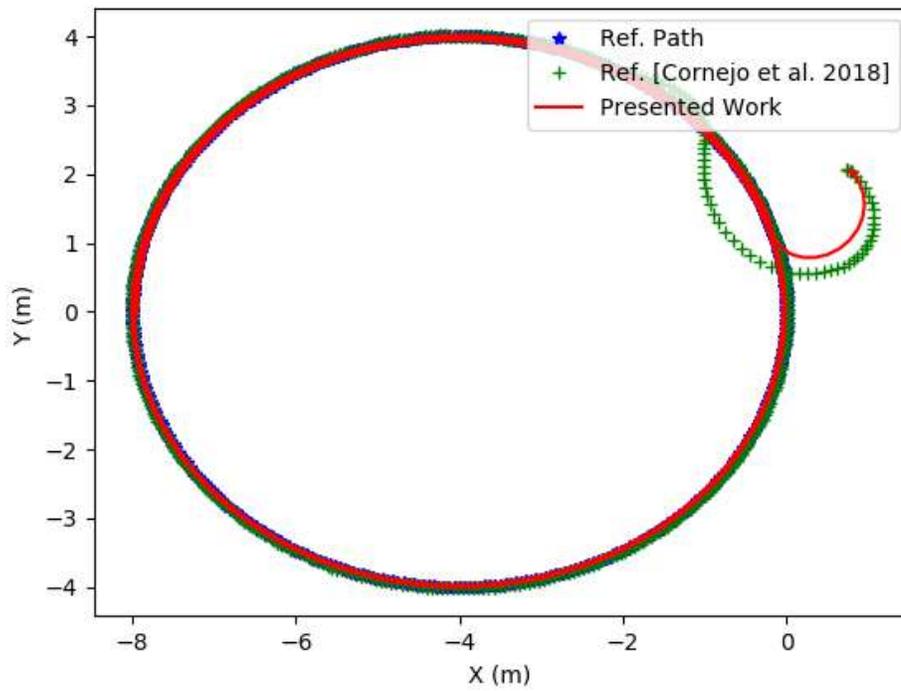


Figure 6. Block diagram of the mobile robot controller.

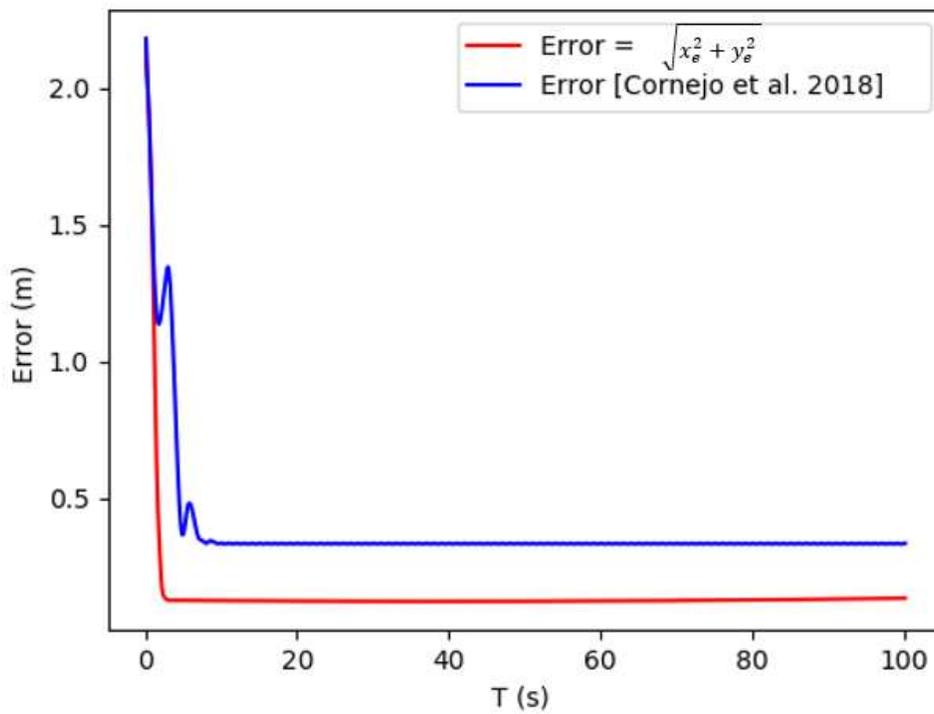


Figure 7. Block diagram of the mobile robot controller.



The second case is a simulation of the outdoor pose to pose tracking controller. For the present pose to pose work, two points from low-cost GPS Nema 8 data have been extracted. The two points are used in the presented work are GPS-point-1= (44.373160,33.273572) and GPS-point-2 = (44.373264, 33.274087).

The data were analyzed and transformed from GEO coordinate to Cartesian coordinate with Haversine formula Eq. (11) and (15). The straight distance between these two points is 58.07 meters, and the bearing angle calculated by Eq. (15) is 9.4 degrees. The starting point is at the GPS-point1 with an initial angle is  $2\pi/5$ , and the goal is the GPS-point2 with angle  $-2\pi/6$ . **Fig. 8, a-d** shows the simulation of the WMR from the starting point to the final point. The error at starting is 57 meters in the x-direction, as indicated in **Fig. 11**, and the error at y- direction is 8 meter, as indicated in **Fig. 12**. **Fig. 11** and **Fig 12** shows the WMR at the starting with large initial errors at the first GPS-point-1. After that, the WMR tracking error converged to the zero error at the second GPS GPS-point-2.

The kinematic controller parameters were found by trial and errors  $k_\rho = 2$ ,  $k_\alpha = 1$ ,  $h = 0.3$ , and time-stepping 0.1 seconds. The linear speed is limited at 4 m/s, as shown in **Fig. 9**. The velocity profile shows the compensated velocity by the proposed controller before reaching the limited velocity. **Fig. 10** shows the profile of the angular velocity of the WMR; the angular velocity converges to zero because of the linear tracking path. This case is used when the starting point and final point are specified, and it does not matter how the path reaches the goal. The control law produced an acceptable stable path with a specified speed.

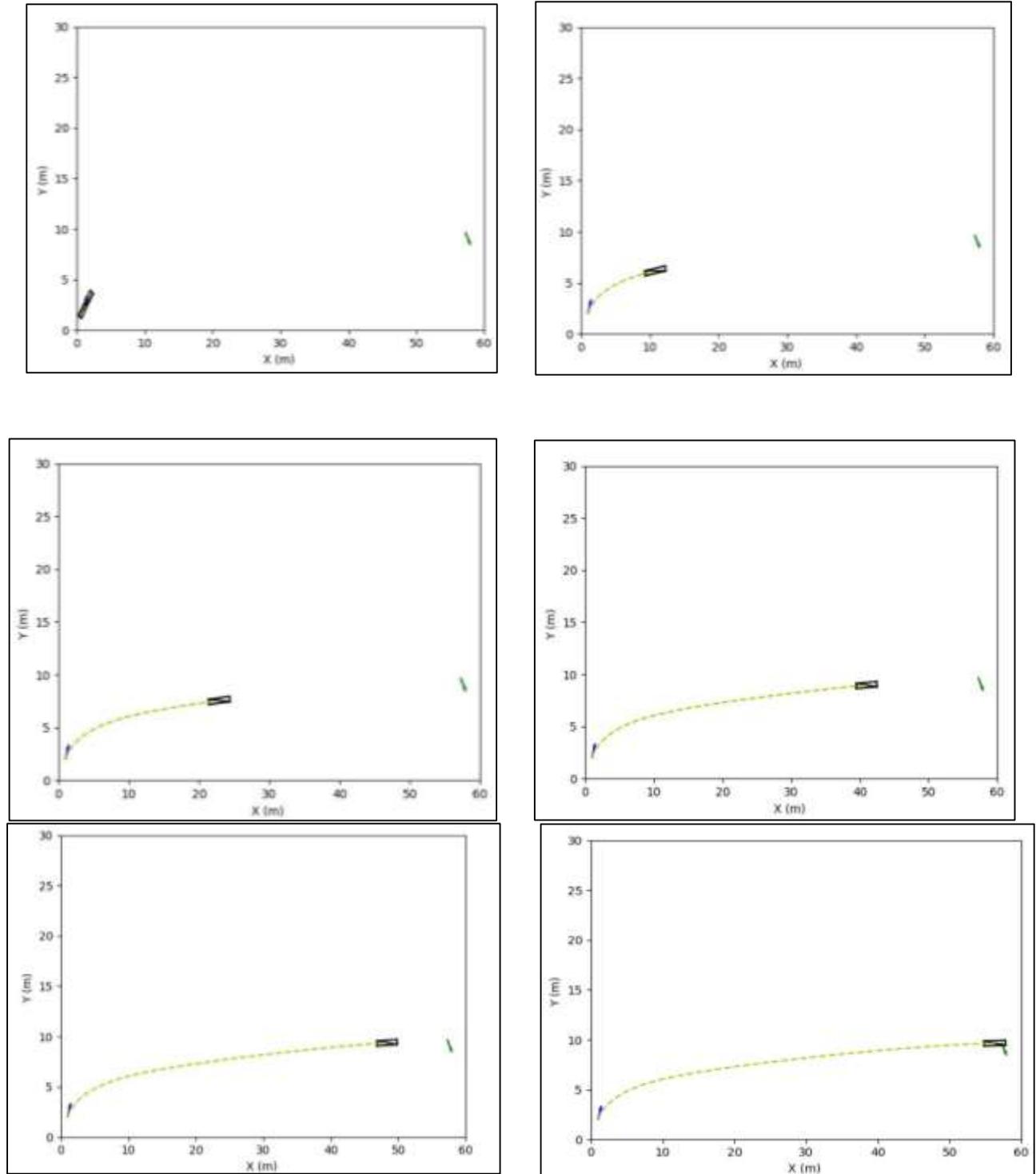


Figure 8. Simulation sense of the tracking performance for the proposed mobile robot.

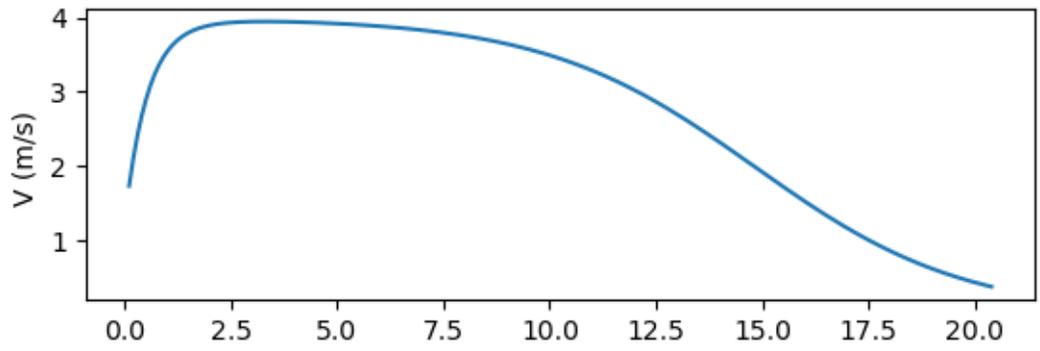


Figure 9. Linear velocity profile.

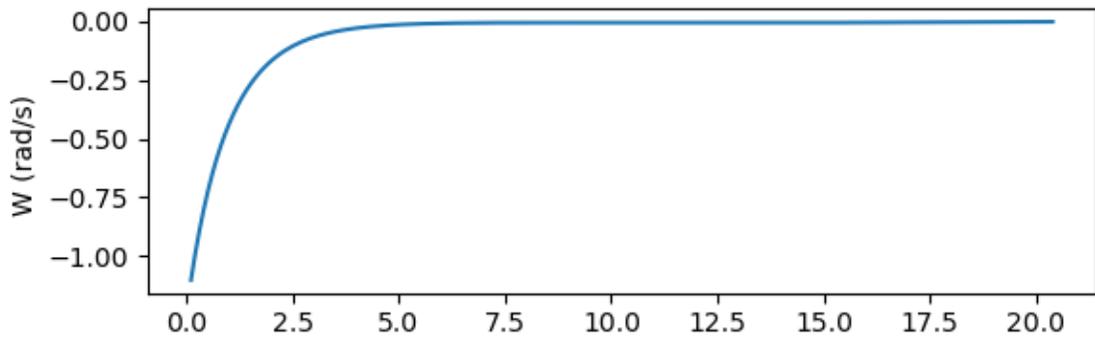


Figure 10. Angular velocity profile.

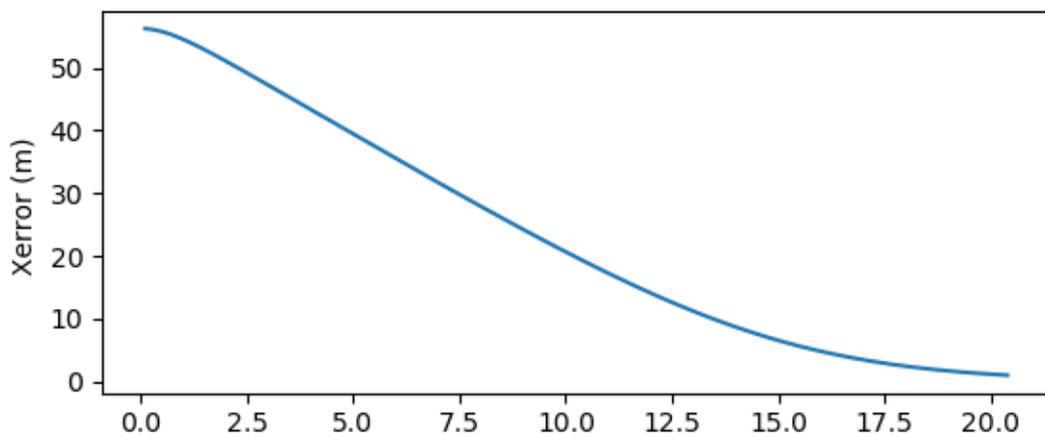


Figure 11. Tracking error in the x-direction.

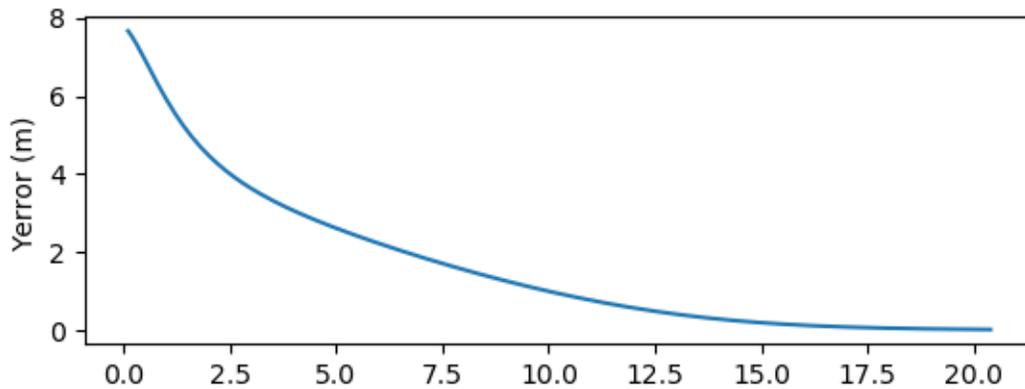


Figure 12. Tracking error in the y-direction.

## 5. CONCLUSIONS

In this paper, a modified stabilized kinematic controller for point to point in the polar coordinate system was proposed. It was applied in tracking control of non-holonomic WMR. The mathematical model of the WMR was derived in the polar coordinate successfully. The controller produces smooth compensated linear and angular velocities, and it forced the WMR to flow the desired path. The globally asymptotic stability of the controller had been proved by the Lyapunov theory. The errors of the posture in the second case were obtained from GPS data points. The data of GPS were converted in polar coordinate, using the Haversine formula, to be suitable for use in the controller. The point to point path tracking limited with 4 m/s, as indicated in Fig. 5. Simulation results demonstrate the effectiveness and stability of the proposed scheme. As future work, the presented work, will be implemented in real experimental and dynamic trackin.

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