ANALYTICAL SOLUTION FOR THE DEVELOPED HYDRODYNAMIC PRESSURES IN RESERVOIRS DUE TO VERTICAL EARTHQUAKES WITH SEDIMENT EFFECT

Prof. Dr. Rafa Al-Suhaili
Env. Eng. Dept. Univ. of Bagdad.

Ass. Lecturer Mohammad Rashid Al-Gadsawi
Computer Center, Univ. of Bagdad

ABSTRACT

In a typical reservoir a sediment layer of considerable depth may be deposited on the top of exposed bedrock foundation. In this research an analytical solution was proposed to solve the case of the vertical acceleration with infinite reservoir with existence of such sediment layer.

The results indicate that as the depth of sediment increases, the pressure increases also. The behavior is found to be affected by the change of the degree of saturation of the sediment layer. As this degree of sediment decreases the developed Hydrodynamic pressure increase. A comparison between the case of stiff and soft foundation indicates that the pressure values are always greater for the stiff foundation.

حل تحليلي لايجاد الضغوط الهيدرونيماكية فى خزانات السدود الناجمة عن التعجيل العمودى للهزات الارضية مع وجود الرسوبيات

الخلاصة

ان دراسة استقرارية السدود عند تعرضها للهزة الارضية موضوع ذو اهمية لا تنكر فهنالك العديد من الدراسات حول هذا الموضوع ولكن معظمها تفترض ارضية الخزان خلف السلد صلدة. ونظراً لوجود طبقة رسوبيات على هذه الارضية فليس من الواقع ان نفرض كونها صلدة.

يشمل البحث دراسة تأثير حركة الارض العمودية التي تعطي تعجيلاً عمودياً لارضية الخزان في كلا الحالتين اعلاه تم دراسة تأثير نوعية ارضية الخزان **(Stiff)** صلدة و **(Soft)** رخوة وخصائص الرسوبيات مثل ارتفاعها ودرجة تشبعها.

تم استنباط حل تحليلي لمعادلة الموجة احادية البعد وتم استخدام معادلات لمعاملي الاخماد لتعجيل الهزة ومعامل امتصاص موجة الضغط في الشروط الحدودية للمسألة وعلى سطح الرسوبيات. وقد بيّن الحل بأن معادلة الضغط تتكون من حدين حد حقيقي وآخر خيالي في حالة وجود الرسوبيات .

بينت نتائج الدراسة ان قيم الضغوط الهيدروديناميكية في الاحوال جميعا تكون اعلى في حالة اعتبار ارضية الخزان صلدة (Stiff) مما هي عليه في اعتبارها رخوة (Soft) اما بالنسبة لخصائص الرسوبيات مثل عمق الرسوبيات فقد اظهرت نتائج البحث بأنه يمكن القول بصورة عامة بأن الضغط يزداد بزيادة عمق الرسوبيات في حالة التشبع الكلي. اما بخصوص درجة التشبع فكانت الضغوطات المتولدة تزداد كلما قلّت درجة التشبع.

INTRODUCTION

The design of a new dam and safety evaluation of an existing dam should be carried out with a high level of accuracy. The failure of such structures, particularly during an earthquake may, be disastrous. Most failures recorded of such structures were due to earthquakes (**Okamoto, 1973**). During earthquakes, a pressure in excess of hydrostatic is exerted on the dam face, called hydrodynamic pressure. Hydrodynamic force was hence developed and may cause failure. The response of a dam to an earthquake is affected by the compressibility of water in the reservoir, and the absorption of pressure waves at the bottom of the reservoir. Therefore, an accurate analysis should consider these two factors, in addition to other factors, such as, earthquake acceleration intensity and frequency, with the geometrical factors.

Reservoir sedimentation is a complex process varying with sediment production rate, transportation, and deposition as mentioned by (**Chen and Hung,1993**), serious deposition of sediment in reservoirs built prior to (1935) were identified. One-third of these reservoirs lost between one-fourth and one-half of their original capacity. About (14%) of the reservoirs lost between (50-70%) of their original capacity, and 10% were completely depleted.

(**Chopra *et al*., 1980**) investigated the response of concrete gravity dams, including the dynamic effects of impounded water and flexible foundation rock, to the transverse (horizontal) and vertical components of earthquake ground motion. The system was analyzed under the assumption of linear behavior for the concrete, foundation rock and water. The complete system was considered as composed of three substructures: the dam, represented as a finite element system. The fluid domain, as a continum of infinite length in the upstream direction, and the foundation rock region as a viscoelastic halfplane

(**Hall and Chopra,1982**) developed an analysis procedure in the frequency domain for determining the earthquake response of two-dimensional concrete gravity and embankment dams including hydrodynamic effect; response of the elastic dams and compressible water are assumed linear. The dam and fluid domain are considered to behave linearly, and the water as compressible. The dam and fluid domain are treated as substructure and modeled with finite elements. The fluid domain model approximately accounts for the interaction between the fluid and the underlying foundation medium through a damping boundary condition applied along the reservoir bottom. The sediment at the bottom of the reservoir can be roughly represented by introducing a so-called absorbing boundary at the bottom of the reservoir.

(**Lotfi and Tassoulas,1986**) developed a finite element method for the two-dimensional problems of the dynamics of dam-reservoir-foundation systems, taking into account all interactions rigorously. In this study the dam-reservoir-foundation region is partitioned into three subregions: the neighborhood of the dam, the solid hyperelement and the fluid-solid hyperelement. This study indicates that including the sedimentary layer decreases the response to vertical ground motion significantly. However, the response to horizontal ground motion is very little reduced. A comparison with the results obtained by the modified approximate treatment of reservoir-foundation interaction showed very good agreement.

(**Cheng,1986**) solved a simplified problem of a rigid dam impounded with water and subjected to vertical harmonic earthquakes. The reservoir bottom was modeled as a poroelastic sediment layer of finite extent sitting on the top of a semi-infinite elastic and impervious foundation. The reservoir bottom damping coefficients were obtained and explicitly expressed in terms of the earthquake excitation frequency and the material properties of the sediment.

(**Bougacha and Tassoulas,1991**) determined the response of a concrete gravity dam for a selected earthquake record in order to assess the potential significance of the sedimentary material accumulating on the reservoir bottom. The results were found to be sensitive to the direction of the horizontal ground motion. The significance of partially saturated sediment seems to be most pronounced when horizontal and vertical ground excitations are combined and less notable when only horizontal ground acceleration is applied. This study suggests that the reservoir bottom sediment may play a role in the response of dams to earthquakes. However, the importance of this role seems to hinge on the sediment properties, especially the pore-fluid compressibility.

(**Chen and Hung,1993**) used a two-dimensional model of poroelastic media to obtain the dynamic effect of reservoir sediment on the hydrodynamic pressure of an incompressible impounded water. The pressure wave equation for the pore water and the sediment is solved by an implicit finite difference method coupled with a fast Poisson solver. The sediment in this model has been treated as a fluid-filled mixture consisting of both solid and fluid phases. During the onset of ground acceleration, the ground rise of pore pressure associated with the compressibility of the pore water results in an incipient reduction of dynamic pressure in the impounded water. However, the dynamic pressure of the incompressible impounded water rises rapidly with the pore pressure associated with the pressure wave and inertia in the sediment.

(**Zhao,1994**) considered a simplified problem of a rigid dam impounded with water and subjected to vertical and harmonic earthquakes. The sediment of certain thickness at the bottom of a reservoir is assumed to be an elastic medium overlaying the top of a semi-infinite and impervious foundation. If the gravity dam is assumed to be rigid and the lateral scattering of reflected waves is negligible, a water-gravity dam-foundation system can be simplified as a one-dimensional wave problem.

(**Al-Suhaili,1997**) used a finite element model to investigate the transient pressures generated by earthquake in reservoirs, tunnels, and bottom outlets. Both infinite and finite reservoir cases were considered. The effect of tunnel diameter to reservoir height ratio, reservoir height to length of tunnel, and reservoir height to extent of reservoir ratio were studied. The effect of excitation frequencies was also investigated analytical solutions were also presented for two cases,namely dam-tunnel-reservoir with tunnel gate excited horizontally, and a reservoir and tunnel bottom excited vertically. All of those solutions were found to be simple and in good agreement with the verified finite elements results.

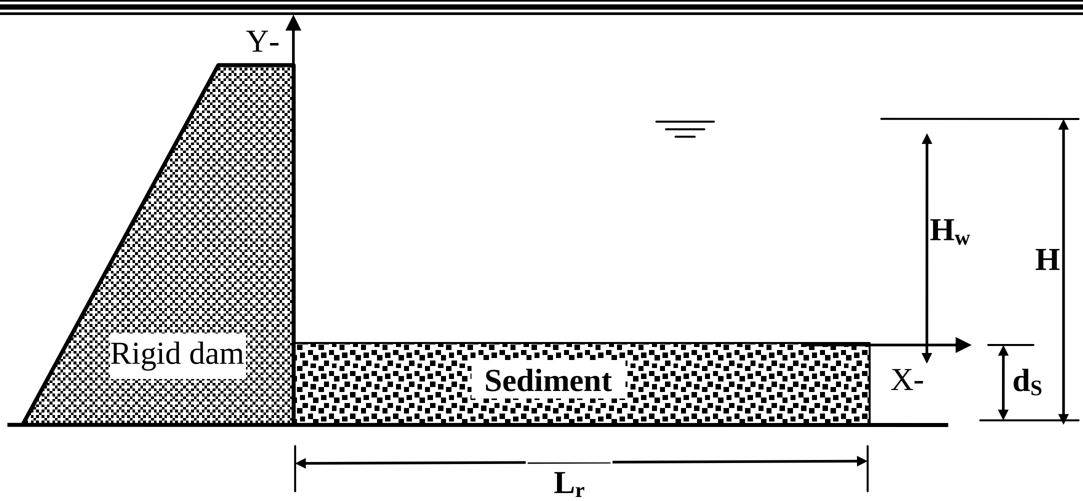

Fig -1-

The proposed analytical Solution

For the case of infinite reservoir and with the existence of a sediment layer at the bottom of the reservoir i.e., (ds ≠ 0) depth of sediment layer fig (1), no analytical solution could be found in the literature. It is proposed to solve this problem using the one-dimensional wave equation and the following boundary conditions:

$$\frac{d^2 \bar{P}}{d y^2} + \frac{\omega^2}{C_w^2} \bar{P} = 0 \tag{1}$$

$$\bar{P}(H_w, \omega) = 0 \tag{2}$$

$$\frac{d \bar{P}}{d y}(0, \omega) = -(\alpha_r + i\alpha_i)\rho_w \; ay + i\left(\frac{\omega}{C_w}\right)(\beta_r + i\beta_i)\bar{P}(0, \omega) \tag{3}$$

Where

Using the method of separation of variables, the solution is:

$$\bar{P}(y, \omega) = A \sin\left(\frac{\omega}{C_w} y\right) + B \cos\left(\frac{\omega}{C_w} y\right) \tag{4}$$

Applying the boundary condition Eq. (2) gives:

$$B = -A \tan\left(\frac{\omega}{C} H_w\right) \tag{5}$$

and the boundary condition Eq. (3) gives:

$$A \times \frac{\omega}{C} = -\left(\alpha_r + i\,\alpha_i\right)\ \rho_w\ ay + i\left(\frac{\omega}{C_w}\right)\left(\beta_r + i\beta_i\right)\bar{P}\left(0,\omega\right) \tag{6a}$$

$$A = -\left(\alpha_r + i\,\alpha_i\right)\ \rho_w\, ay\left(\frac{C_w}{\omega}\right) + i\left(\beta_r + i\beta_i\right)\bar{P}\left(0,\omega\right) \tag{6b}$$

$$A = \left(-\alpha_r\,\rho_w\, ay\left(\frac{C_w}{\omega}\right) - \beta_i\,\bar{P}\left(0,\omega\right)\right) + i\left(-\alpha_i\,\rho_w\, ay\left(\frac{C_w}{\omega}\right) + \beta_r\,\bar{P}\left(0,\omega\right)\right) \tag{6c}$$

Since $\bar{P}(0, \omega) = B$, Eq. (4) can be modified as:

$$A = \frac{-\alpha_r\,\rho_w\, ay\left(\frac{C_w}{\omega}\right) - i\,\alpha_i\,\rho_w\, ay\left(\frac{C_w}{\omega}\right)}{1 + i\,\beta_r\ tan\left(\frac{\omega}{C_w}H_w\right) - \beta_i\ tan\left(\frac{\omega}{C_w}H_w\right)} \tag{6d}$$

After substituting Eqs. (5), and (6d) in Eq. (4), the following equation will be obtained :

$$\bar{P}\left(y,\omega\right) = \frac{-\alpha_r\,\rho_w\, ay\left(\frac{C_w}{\omega}\right) - i\,\alpha_i\,\rho_w\, ay\left(\frac{C_w}{\omega}\right)}{1 + i\,\beta_r\ tan\left(\frac{\omega}{C_w}H_w\right) - \beta_i\ tan\left(\frac{\omega}{C_w}H_w\right)}\ sin\left(\frac{\omega}{C_w}y\right)\ - \frac{-\alpha_r\,\rho_w\, ay\left(\frac{C_w}{\omega}\right) - i\,\alpha_i\,\rho_w\, ay\left(\frac{C_w}{\omega}\right)}{1 + i\,\beta_r\ tan\left(\frac{\omega}{C_w}H_w\right) - \beta_i\ tan\left(\frac{\omega}{C_w}H_w\right)}\ tan\left(\frac{\omega}{C_w}H_w\right)\ cos\left(\frac{\omega}{C_w}y\right) \tag{7}$$

where :

$\bar{P}$ = steady state response hydrodynamic pressure (F/L^2);

ω = earthquake circular excitation frequency (1/T);

$C_w = (E_w / \rho_w)^{1/2}$ = pressure wave velocity of water (L/T);

E_w = Bulk modulus of elasticity of water (F/L^2);

ρ_w = mass density of water (M/L^3);

α and β are depended on the sediment and the foundation properties;

Results and Discussions

Tables (1), (2), (3) and (4) show the values of $(\overline{P}/\gamma H_w)$ calculated by Eq. (7) for infinite reservoir length, using $(\omega/\omega_1 = 0.5)$, for a wide range of properties of sediment, and for a unit vertical earthquake acceleration (1g).it is watch to mention is that the $(\overline{P}/\gamma H_w)$values for different acceleration (0.2g) can be obtained by multiply these values by(0.2g)

Table (1) hydrodynamic pressure found by the developed of analytical solution for standardized hydrodynamic pressures, $(\overline{P}/\gamma H_w)$, for unit earthquake vertical acceleration, (1g), with stiff foundation (Hw=112.5m), $(\omega/\omega_1 = 0.5)$, dS= 12.5m, (ρs/ρw=1.5)

Analytic solution for stiff foundation (L= ∞)			
Y / H	**Fully saturated sediment (S=100%)**	**Partially saturated sediment (S=99.5%)**	**Partially saturated (sediment S=99%)**
0.1	1.290	1.597	1.980
0.19	1.185	1.467	1.819
0.28	1.072	1.328	1.646
0.37	0.953	1.180	1.463
0.46	0.828	1.025	1.271
0.55	0.698	0.864	1.071
0.64	0.564	0.698	0.865
0.73	0.426	0.527	0.653
0.82	0.285	0.353	0.438
0.91	0.143	0.177	0.219
1	1.03E-19	1.04E-19	0

Table (2) hydrodynamic pressure found by the developed analytical solution for standardized hydrodynamic pressures, $(\overline{P}/\gamma H_w)$, for unit earthquake vertical acceleration (1g) with soft foundation (Hw=112.5m), $(\omega/\omega_1 = 0.5)$, dS= 12.5m, (ρs/ρw=1.5)

Analytical infinite solution for soft foundation			
Y/H	**Fully saturated sediment (S=100%)**	**Partially saturated sediment (S=99.5%)**	**Partially saturated sediment (S=99%)**
0.1	1.184	1.411	1.657
0.19	1.088	1.296	1.522
0.28	0.984	1.173	1.377
0.37	0.875	1.042	1.224
0.46	0.760	0.906	1.064

0.55	0.641	0.763	0.896
0.64	0.517	0.616	0.724
0.73	0.391	0.465	0.547
0.82	0.262	0.312	0.366
0.91	0.131	0.156	0.183
1	0.000	0.000	0.000

Table (3) Results of analytical solution for standardized hydrodynamic pressures, $\left(\bar{P}/\gamma H_w\right)$, for unit vertical acceleration, (1g), with stiff foundation (Hw=100m), $\left(\omega/\omega_1 = 0.5\right)$, dS=25m, (ρs/ρw=1.5)

Analytic solution for stiff foundation (L= ∞)			
Y / H	**Fully saturated sediment (S=100%)**	**partially saturated sediment (S=99.5%)**	**Partially saturated (sediment S=99%)**
0.2	1.331	2.73	8.04
0.28	1.222	2.511	7.389
0.36	1.106	2.272	6.688
0.44	0.983	2.020	5.945
0.54	0.854	1.75	5.165
0.6	0.720	1.47	4.354
0.68	0.581	1.19	3.516
0.75	0.439	0.902	2.656
0.84	0.294	0.604	1.77
0.92	0.147	0.303	.822
1	0	0	0

Table (4) Results of analytical solution for standardized hydrodynamic pressures, $\left(\bar{P}/\gamma H_w\right)$, for unit vertical acceleration, (1g), with soft foundation (Hw=100m), $\left(\omega/\omega_1 = 0.5\right)$, dS= 25m, (ρs/ρw=1.5)

Analytic solution for soft foundation (L= ∞)			
Y / H	**Fully saturated sediment (S=100%)**	**Partially saturated sediment (S=99.5%)**	**Partially saturated (sediment S=99%)**
0.2	1.188	1.949	2.72
0.28	1.091	1.790	2.50
0.36	.987	1.620	2.265
0.44	.878	1.440	2.013
0.54	.763	1.251	1.749

0.6	.643	1.055	1.474
0.68	.519	.851	1.190
0.75	.392	.643	.899
0.84	.262	.431	.602
0.92	.131	.216	.302
1	0	0	0

The developed solution (eq.7)can be used to investigate the effect of different factors on the values and distribution of hydrodynamic pressures developed due to earthquakes on dam faces. The effect of the type of reservoir bedrock foundation, degree of saturation of sediments, depth of sediments, density of sediments. The following sensitivity analysed is proposed to illustrate the effect of each of those parameters.

1- Effect of the Type of Reservoir Bedrock Foundation

To show the effect of the type of foundation on the hydrodynamic pressure, the relation between (Y/H) and the normalized hydrodynamic pressure are plotted as shown in Figs. (1), (2), and (3). These figures show that for the case of stiff foundation, the hydrodynamic pressures developed were always greater than those for the soft foundation. This result is in agreement with the results obtained by (**Cheng,1986**)

Changing the ratio of depth of sediments, (d_S), to depth of reservoir (H), in Figs. (4), (5), and (6) and the degree of saturation have no effect on this behavior. The reason for this behavior can be explained upon the terms incorporated into the boundary conditions. Since, for the stiff foundation, the coefficient of absorption of pressure wave is less than that for soft foundation. This is obvious from the value of the coefficient of absorption of pressure wave, which is lower for stiff foundation than that for soft foundation.

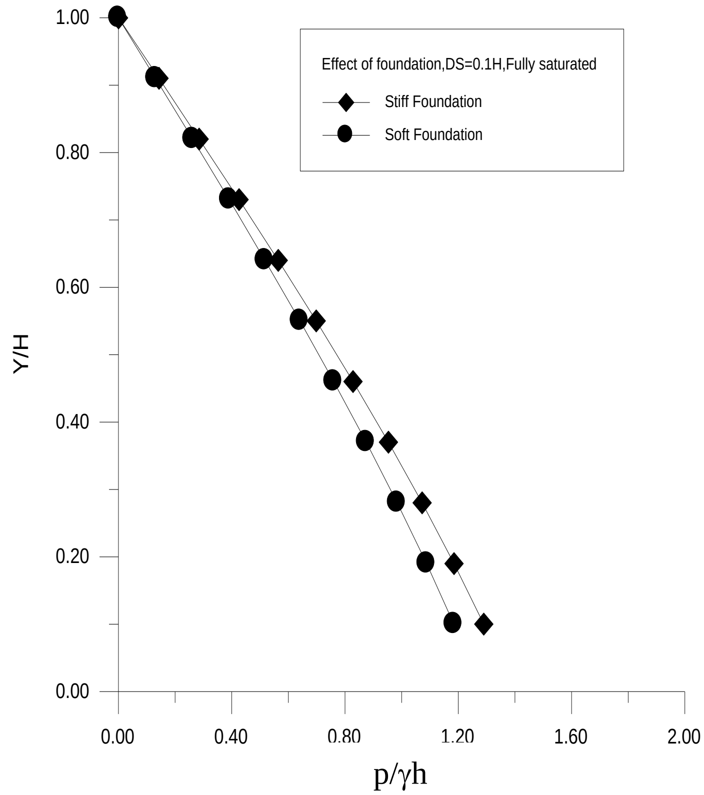

Fig.(1) Effect of the type of foundation on the hydrodynamic pressure developed due to vertical earthquake acceleration (1g) for ω/ω_1=0.5, with H_w=112.5m, d_S=12.5m, ρ_s/ρ_w=1.5,fully saturated sediments (100%)

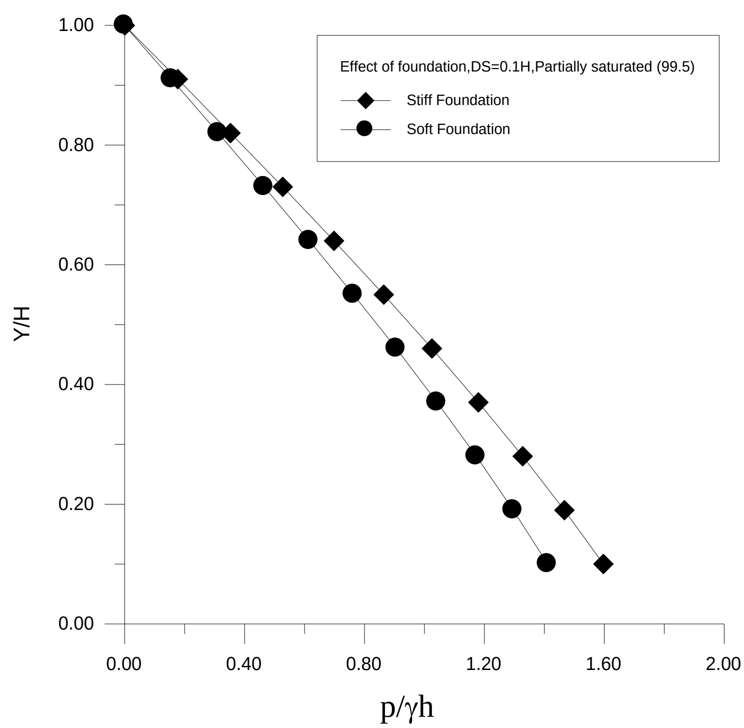

Fig.(2) Effect of type of foundation on hydrodynamic pressure developed due to vertical earthquake acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5,fully saturated sediments (100%)

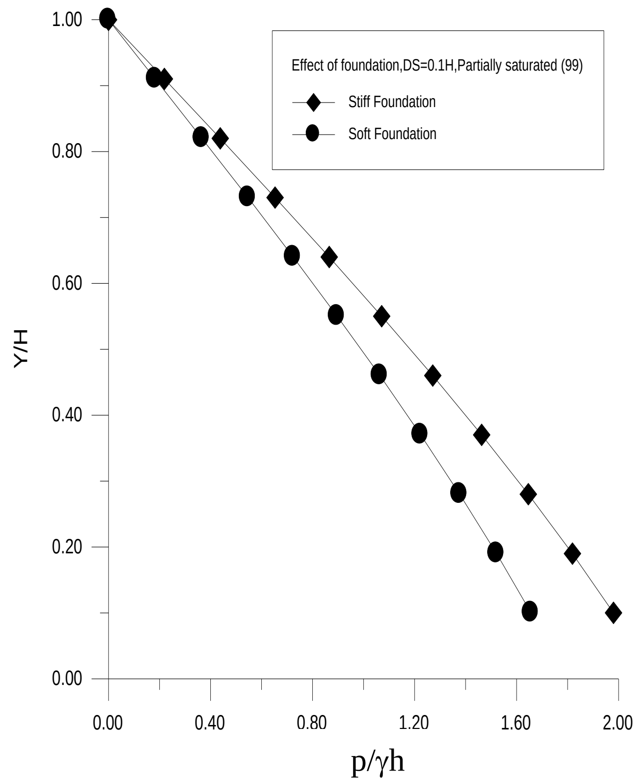

Fig.(3) Effect of type of foundation on hydrodynamic pressure developed due to vertical earthquake acceleration (1g) for ω/ω_1=0.5, with H_w=112.5m, d_S=12.5m, ρ_s/ρ_w=1.5,partially saturated sediments (99%)

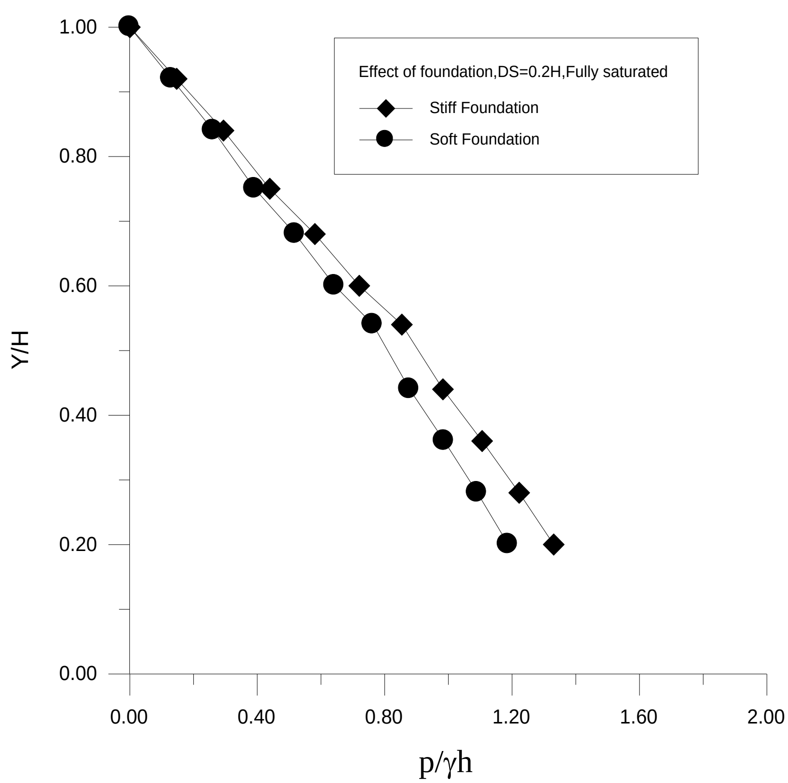

Fig.(4) Effect of type of foundation on hydrodynamic pressure developed due to vertical earthquake acceleration (1g) for ω/ω_1=0.5, with d_S=25m, ρ_s/ρ_w=1.5,fully saturated sediments (100%)

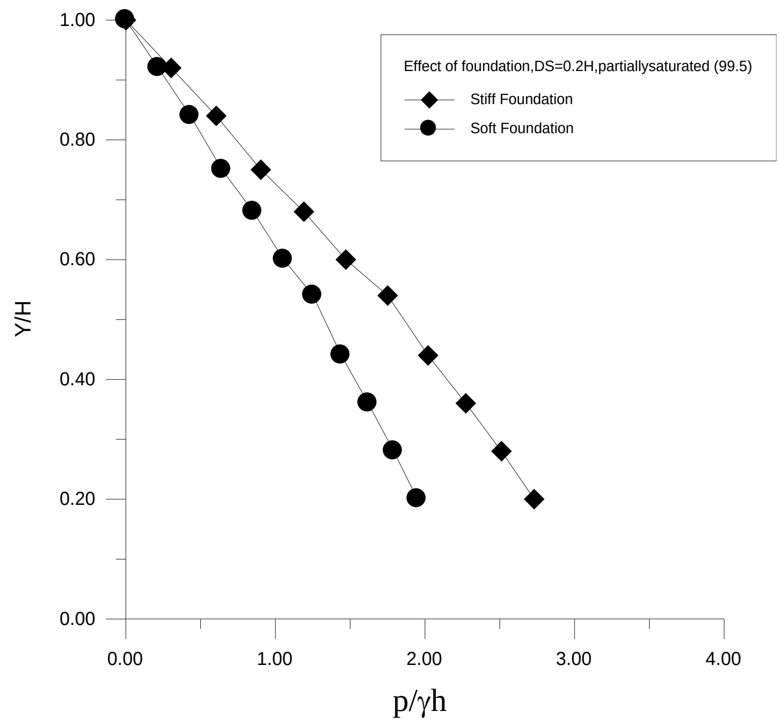

Fig.(5) Effect of type of foundation on hydrodynamic pressure developed due to vertical earthquake acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5,partially saturated sediments (99.5%)

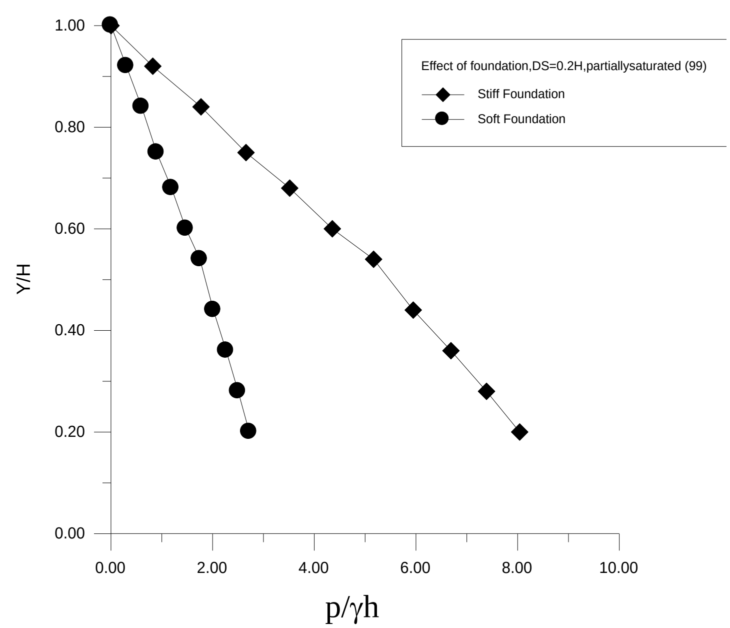

Fig.(6) Effect of type of foundation on hydrodynamic pressure developed due to vertical earthquake acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5,partially

saturated sediments (99%)

2- Effect of the Degree of Saturation of the Sediment

Figures (7), (8), (9) and (10) show the effect of the degree of saturation of the sediments on the hydrodynamic pressures developed due to vertical acceleration of (1g). It is shown that for (d_S/H) equals (0.1) and (0.2) the hydrodynamic pressure increases with the decrease of the degree of saturation. These results are in agreement with those found by (**Cheng,1986**), and (**Bougacha and Tassoulas,1991**).

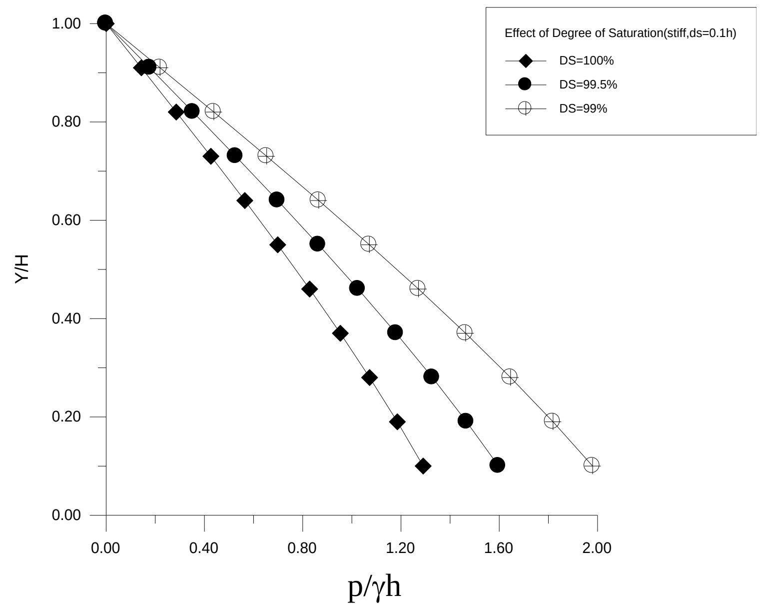

Fig. (7) Effect of the degree of saturation on the hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=112.5 m, d_S=12.5m, ρ_s/ρ_w=1.5, stiff foundation.

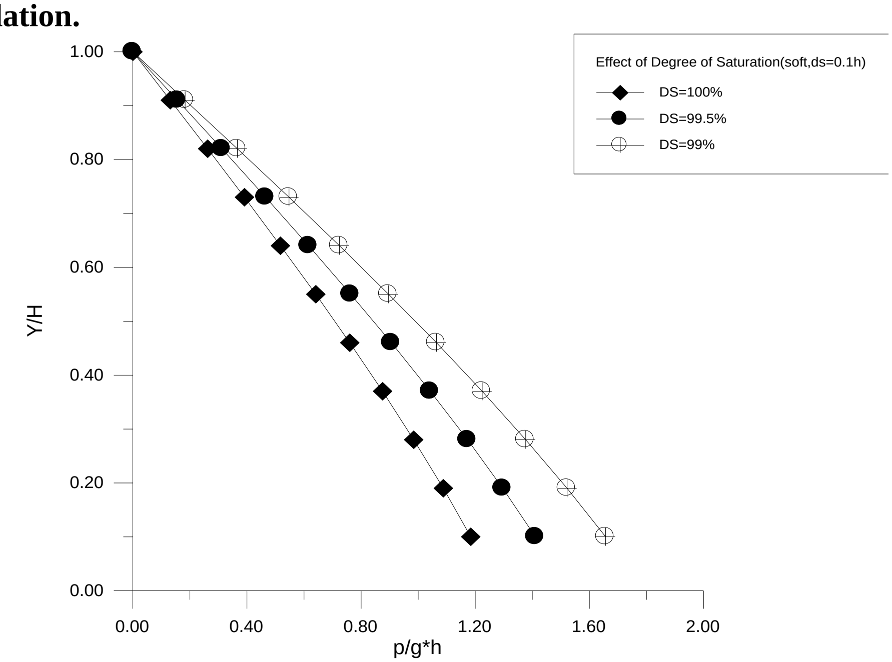

Fig. (8) Effect of degre p/γh on on hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=112.5 m, d_S=12.5m, ρ_s/ρ_w=1.5, soft foundation.

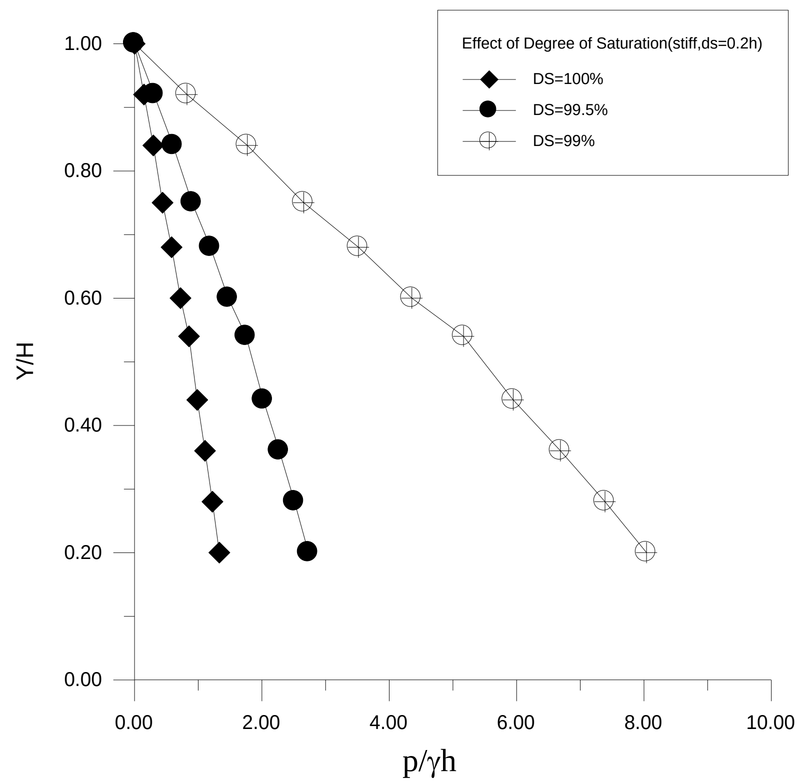

Fig. (9) Effect of degree of saturation on hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5, stiff foundation.

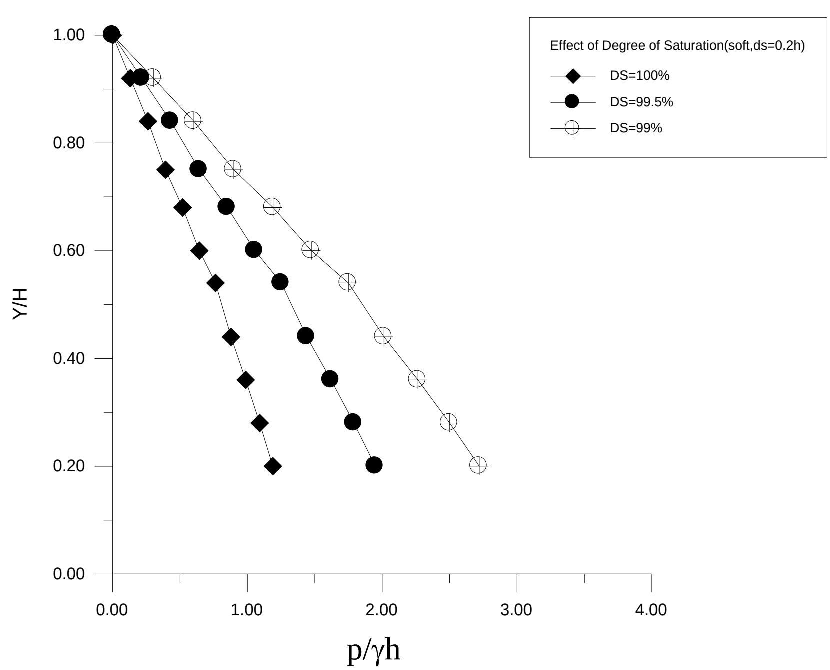

Fig. (10) Effect of degree of saturation on hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5, soft foundation.

3- Effect of the Depth of the Sediment

Figures (11), (12), and (13) show the effect of the depth of sediments on the hydrodynamic pressure developed due to vertical earthquake acceleration of (1g). It is shown that for the two types of foundation the hydrodynamic pressure increases when the depth of sediment increases for (99.5%) and (100%) degree of saturation. Figures (14), (15), and (16)

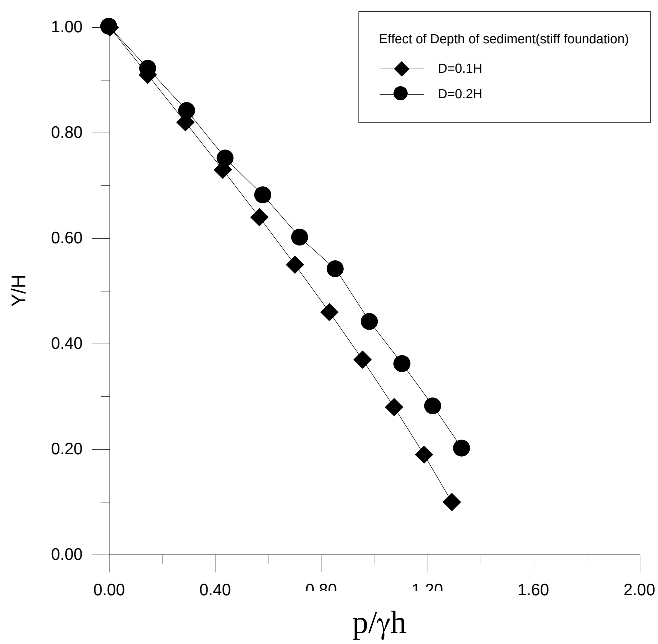

Fig. (11) Effect of the depth of sediment on the hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=112.5m, d_S=12.5m, ρ_s/ρ_w=1.5, stiff foundation,fully saturated sediments (100%).

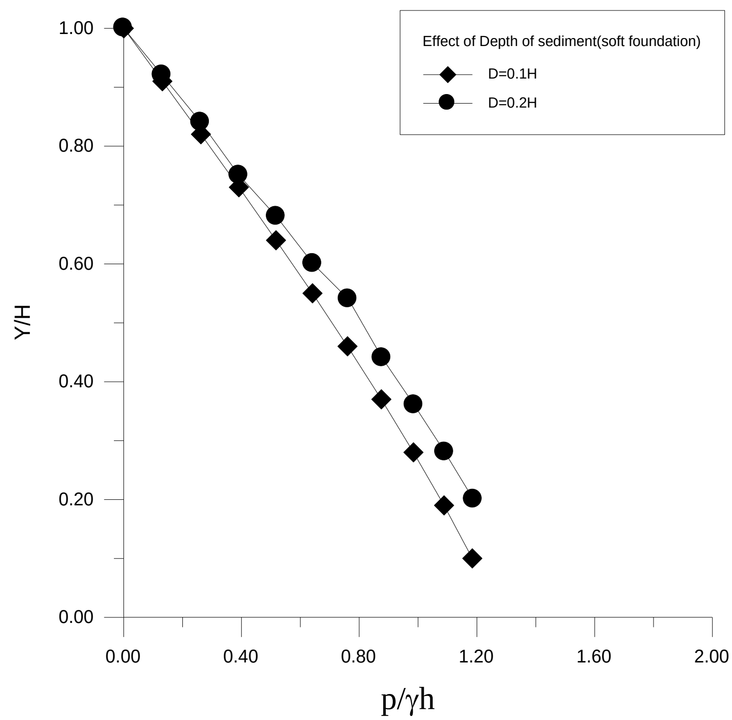

Fig. (12) Effect of the depth of sediment on the hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5, soft foundation,fully saturated sediments (100%).

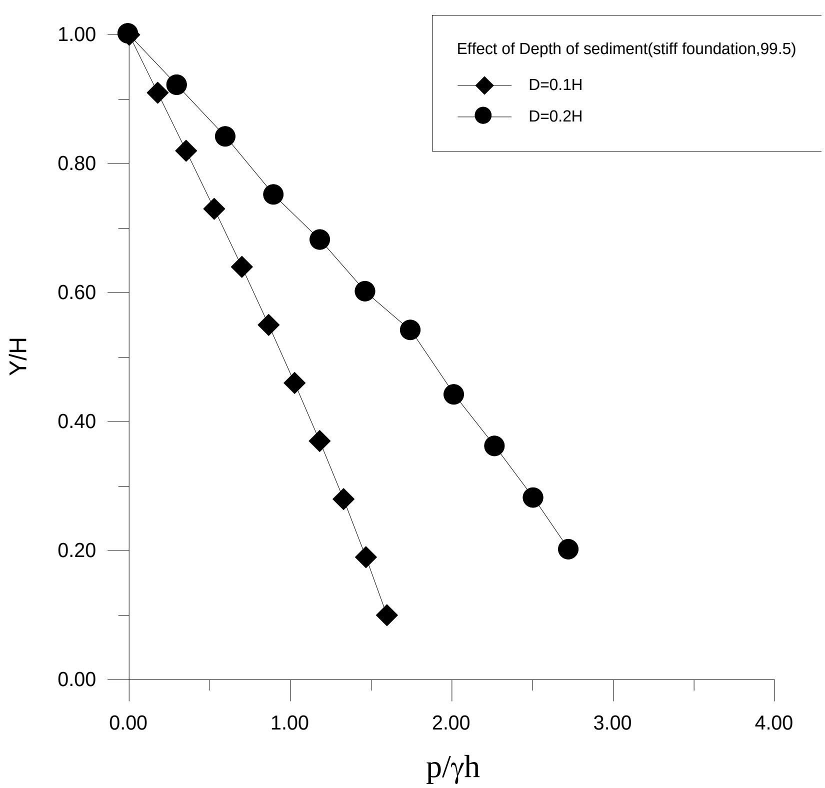

Fig. (13) Effect of depth the of sediment on the hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=112.5m, d_S=12.5m, ρ_s/ρ_w=1.5, stiff foundation,partially saturated sediments (99.5%).

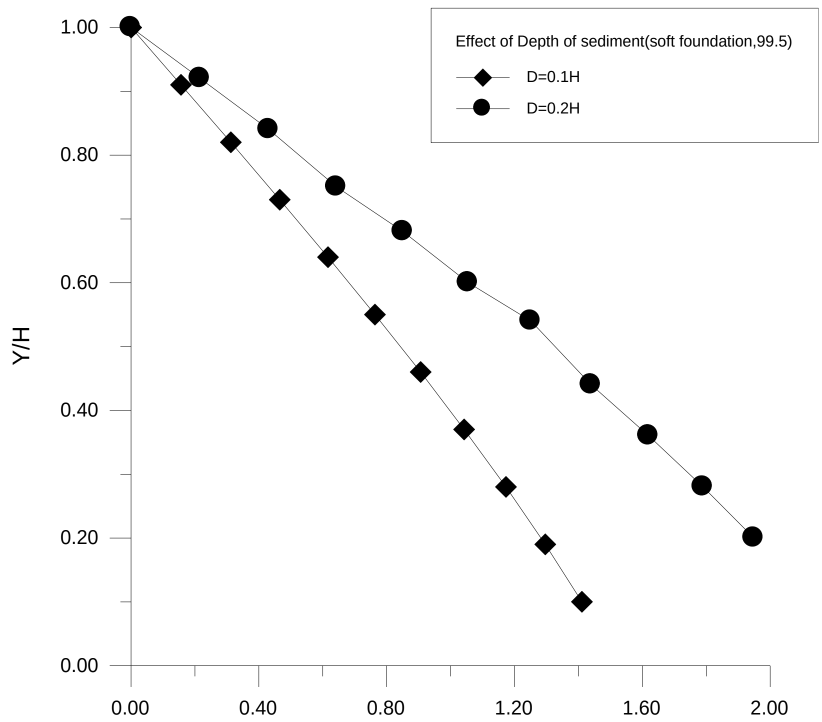

Fig. (14) Effect of the depth of sediment on the hydrodynamic pressures developed due to vertical acceleration (1g) for $\omega/\omega_1=0.5$, with $H_w=100m$, $d_S=25m$, $\rho_s/\rho_w=1.5$, soft foundation,partially saturated sediments (99.5%).

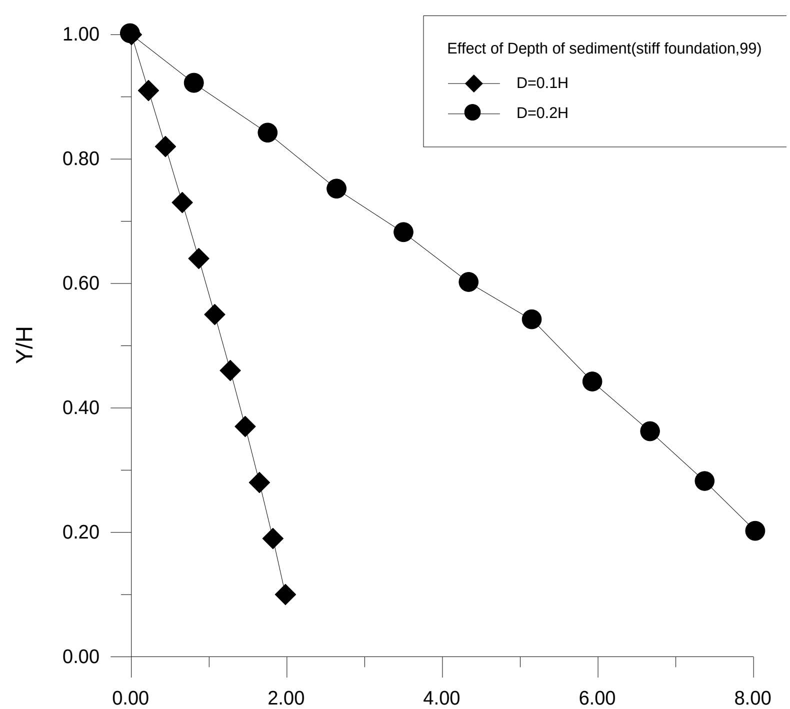

Fig. (15) Effect of the depth of sediment on the hydrodynamic pressures developed due to vertical

acceleration (1g) for ω/ω_1=0.5, with H_w=112.5m, d_S=12.5m, ρ_s/ρ_w=1.5, stiff foundation,partially saturated sediments (99%).

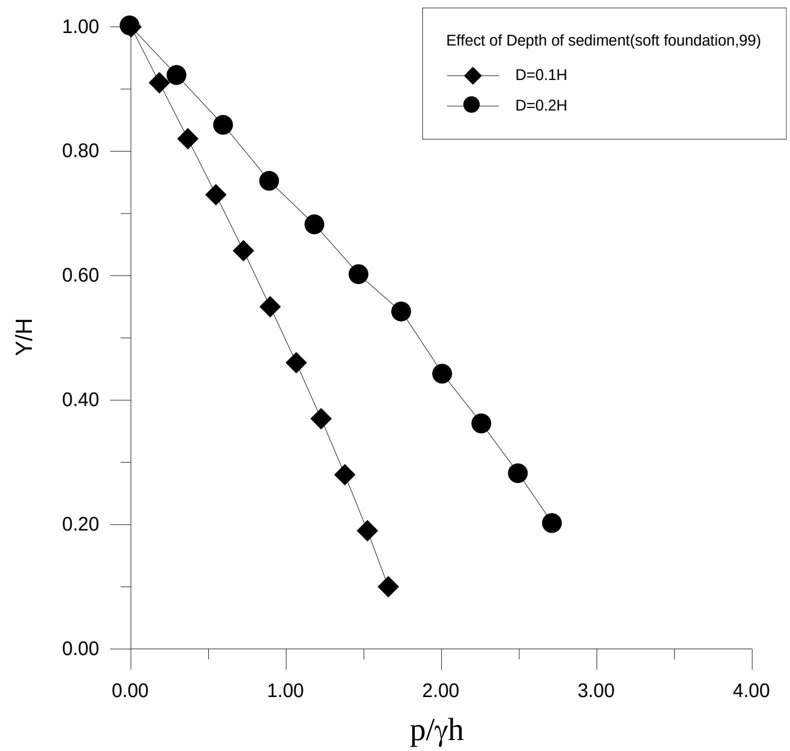

Fig. (16) Effect of the depth of sediment on the hydrodynamic pressures developed due to vertical acceleration (1g) for ω/ω_1=0.5, with H_w=100m, d_S=25m, ρ_s/ρ_w=1.5, soft foundation,partially saturated sediments (99%).

CONCLUSIONS

1. The values of the hydrodynamic pressure for the stiff foundation case are always larger than those for the soft foundation.
2. The hydrodynamic pressure increases with the decrease of the degree of saturation.
3. The hydrodynamic pressure increase when the depth of the sediment increase.

REFERENCES

Bougacha, S., and Tassoulas, J.L., (1991a): "Effects of sedimentary material on the response of concrete gravity dams." Earthquake Engng Struct. Dynamics Vol. 20, pp. 849-858, John Wiley & Sons, U.S.A.

Chen, B., and Hung, T., (1993): "Dynamic pressure of water and sediment on rigid dam." J. Engng Mech., ASCE, Vol. 119, pp.1411-1433.

Cheng, A. H., (1986): "Effect of sediment on earthquake–induced reservoir hydrodynamic response." J.

Engng Mech., ASCE, Vol. 112, pp. 654-663,.

Chopra, A. K., Chakrabarti, P., and, Gubta, S., (1980): "Earthquake response of concrete gravity dams including hydrodynamic and foundation interaction effects." Report No. UCB/EERC-80/01, Earthquake Engineering Research Center, Univ. of California, Berkeley.

Hall, J. F., and Chopra, A. K., (1982): "Two-dimensional dynamic analysis of concrete gravity and embankment dams including hydrodynamic effects." Earthquake Engng Struct. Dynamics Vol.10, pp. 305-332, John Wiley & Sons, U.S.A.

Okamoto, S., (1973): "Introduction to earthquake engineering" University of Tokyo Press.

Methods in Engineering, Vol. 21, pp. 1659-1669, John Wiley & Sons, U.S.A.

Al-Suhaili R.H., (1997): "Hydrodynamic pressures in dam-reservoir-tunnel system during earthquakes", Ph.D. Dissertation, College of Engineering, University of Baghdad.

Zhao, C., (1994): "Effects of reservoir bottom sediments on hydrodynamic pressure of gravity dams." Computers and Structures, Vol. 52, pp. 297-307, Pergamon, England.

Lotfi,v.,and Tassoulas,J.L..(1986):"analysis of the response of dams to earthquakes."report GR86-2,Geotechicnical Engineering

Research Cente,the University of texas,Austin.Texas.