THEORETICAL AND NUMERICAL ANALYSIS OF FUEL DROPLET COMBUSTION PROCESS WITH CONVENTIONAL AND TRANSIENT MODELS

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ABSTRACT

The study introduces a theoretical analysis and numerical solution for the combustion of a hydrocarbon fuel droplet inside the combustion chamber. The study employs two mathematical models to analyze the combustion process. Conventional (Classical) model and Transient model. The combustion process of a stagnant droplet in the steady state was analyzed in the classical model, while, In the transient model, it was assumed that there is a period of time in the stages of the droplet combustion in which the droplet is heated before combustion. The effect of change in temperature on the thermo-physical properties of the fuel was adopted through the two models. For the classical model, a convenient approximation was adopted for the heat transferred inside the droplet.

KEY WORDS

 droplet, combustion, numerical, modeling, transient.

INTRODUCTION

Droplet burning has relevance to many practical combustion devises, including, rockets, and gas-turbine engines, as well as oil-fired boilers, furnaces, and process heaters. In these devices, spray combustion is the dominant feature [Turns, 2000]. The problem of droplet burning includes both the evaporation and the combustion process. Solving problems concerning droplet evaporation and combustion needs the use of mass transfer and thermodynamic laws and equations. In all the above systems, liquid fuels and/or oxidizer are usually injected into the combustion chamber as a spray of droplets, which undergoes process [Lamnna, et.al. 2004]. Research on the gasification, oxidation,

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and dynamics of fuel droplet commands both practical and fundamental interest to energy and combustion science. On the practical aspect one recognizes that petroleum oil constitutes a significant share of the world energy supply [Law, 1982]. According to law, droplet combustion is a problem involving complex chemically reacting multicomponent two-phase flows with phase change, rich in physical and chemical phenomena typically to be studied. Theories and studies concerning the evaporation and combustion fuel droplets took into account a spherical droplet (motionless or moving). It is cold and is placed in a hot stagnant, gravity free environment, [Law, 1982]. No forced or natural convection is concerned. Heat is transferred from the ambience towards the cold surface of droplet, where some of it is used to gasify the droplet and the remaining is transferred to the interior of droplet to heat it up. According to Law, through the above mechanism, a liquid mass can be converted to vapor and eventually dispersed to ambience, with the rate controlling process as heat and mass diffusion the ignition of the droplet causes a reaction or oxidation of the fuel with heat generated to be transferred both outward and inward. With D^2-law [Godsave, 1953, Spalding, 1953, Goldsmith & Penner, 1954] with its simple assumptions and some inadequacies. Then different approaches were conducted to develop the D^2-law. Some included the unsteady droplet heating and combustion [Law & Sirignano, 1977]. Others included internal circulation, transient heating, multicomponent fuel, radiation, high pressure, sprays, etc., introducing modern computational models [Williams & Sirignano,1983, Abraham,2001, Dombrovsky & Sazhin,2003, Hohmann & Ranz,2003,Sazhin,et.al.,2004,Sazhin,et.al.,2005,andYan&Aggarwal,2006].

THEORIES

Classical Droplet Combustion Model:

D^2-law theory for droplet combustion established the basic fundamentals for droplet combustion model. One of the assumptions of that theory is that thermophysical properties are constant for all temperature of droplet surface and it is surrounding, so that it's more suitable to consider the dependence of thermophysical properties to a reference temperature which vary with the variation of surface and environmental temperature. This behavior will be considered later through this work to obtain its effects on the main parameters of combustion process such as mass burning rate. Mass burning rate represents the amount of fuel that is burned during the combustion process. It is dependent on the mass vaporization, rate which is the amount of fuel that evaporates from the surface of the droplet as a result of heat that is transferred to the surface (or droplet) because of the burning fuel in the process. Mass burning rate represents the combustion process and affects other parameters in the process such as flame radius, which Is the radius of flame front that is surrounding the burning fuel droplet. The flame radius can be expressed as flame standoff ratio. It is a ratio of the radius of flame to the radius of the individual liquid droplet, and that can show the position of flame front from the surface of the droplet at any time of droplet combustion process. Flame temperature is the temperature of the flame front that is surrounding the fuel droplet. Knowing this temperature can provide primary knowledge to the heat that is transferred inward and outward the fuel droplet, and that heat is dependent on the mass burning rate. In the classical model for

combustion, mathematical model will be considered and solved for mass and heat transfer for the gas-phase of droplet combustion process. The solution will provide the basic equations that relates the main characteristic parameters, which will be solved numerically to evaluate their values overall the combustion process. A suitable approach for the heat that is transferred to the droplet interior will be considered.

TRANSIENT DROPLET COMBUSTION MODEL:

The study of transient combustion process is required to understand the effects of heat up period on the combustion parameters. Heat up period is the period of time that the droplet surface is raised from its initial temperature to a steady temperature (boiling temperature) by gaining heat from its surrounding flame. In heat up period, heat that is transferred to the droplet can be divided into two parts, one part used for heating the droplet liquid to its steady temperature, and the other part for vaporizing the liquid fuel, causing dominant effects on the main parameters of combustion process such as mass burning rate. Surface temperature of the droplet varies with time until it reaches to a steady temperature. This variation can affect latent heat of vaporization that can be considered to become slowly –time varying latent heat of vaporization and can be expressed as a function of surface temperature, which, in turns, can be expressed as a function of time, and the numerical substitution can be used to simplify this dependence. Numerical computation will be used to solve the basic equations which relate the basic parameters evaluated from the solution of mass and heat transfer equations for the mathematical model.

MATHEMATICAL WORK

Classical Droplet Combustion Model: Problem Statement

Spherical fuel droplet that has the initial radius r_{so} is burning and is surrounded by thin flame sheet that has a radius r_f and is surrounded by infinite medium of oxidizing atmosphere to the radius ∞. Liquid fuel is vaporized at the droplet surface, the droplet is the source of vapor in the system, vapor is convicting and diffusing outward toward the flame. The source of convicting species at the droplet surface is the specific volume change associated with phase change at the surface, while the oxidizer diffuses inward from the ambient environment to the flame sheet [Kuo, 1986]. The fuel and the oxidizer react stoichiometrically ,and the chemical kinetics are assumed to be infinitely fast, so that the fuel vapor mass fraction Y_F is maximum at the droplet surface and monotonically decrease to zero at the flame sheet, where the fuel is totally consumed . The oxidizer mass fraction Y_{ox} mirrors this, being a maximum far from the flame and decrease to zero at the flame sheet . If the fuel is burning at a medium of pure oxygen then the value of mass fraction of the oxidizer at infinity is unity. Most of combustion systems are working in air rather than in pure oxygen, so that the environment will be considered as air in the calculations and Y_{ox} will be calculated. Heat is generated at the reaction on the flame sheet causing the flame temperature T_f to be the highest temperature in the system. Heat is transported via conduction and radiation outward from the flame to infinity and inward back to the droplet surface causing vaporization. The heat deposited at the droplet surface is balanced by the evaporation process at the vapor-liquid interface.

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There are three important temperatures in the system, the droplet surface temperature T_s, the flame temperature T_f, and the temperature of the medium at infinity T_∞. The combustion products have their maximum concentration at the flame sheet (unity) diffusing both inward toward the droplet and outward away from the flame [Turns, 2000].

The major assumptions of this model are [Law, 1982]:

* Spherically symmetric: forced and natural convection are neglected, this reduces the analysis to one dimension.
* No spray effects: the droplet is an isolated one, immersed in an infinite oxidizing environment.
* Isobaric process: at 1 atm.
* Flame-sheet combustion: chemical reaction rates are much faster than gas-phase diffusion rates such that the flame is of infinitesimal thickness and can be treated simply as a sink for the reactants and a source of chemical heat released.
* Gas-phase is quasi-steady.
* Simultaneous fuel gasification and consumption: this assumes that the amount of fuel gasified at the surface is instantly consumed at the flame; it neglects the accumulation of fuel vapor between the droplet and the flame.
* Single fuel species: thus it is unnecessary to analyze liquid-phase mass fraction.
* The fuel has zero solubility for gases and phase equilibrium prevails at the liquid-gas interface.
* No soret, Dufour, sooting, and radiation effects.

* Thermophysical properties are functions of temperature only.

CONSERVATION OF MASS:

With the assumption of quasi-steady burning, the mass flaw rate $\dot{m}(r)$ is constant, independent of radius; thus:

$$\dot{m} = \dot{m}_F = \rho v_r A = constant$$

For the inner region bounded between the zone ($r_s <$ r $< r_f$), Fick's law of diffusion can be applied to the inner region:

$$\dot{m}_A'' = Y_F(\dot{m}_A'' + \dot{m}_B'') - \rho D_{AB} \nabla Y_A$$

This can be simplified to;

$$\frac{dY_F}{dr} = \frac{-\dot{m}_F}{4\rho\pi D} \frac{1-Y_F}{r^2} \tag{1}$$

With the boundary conditions;

$$Y_F(r_s) = Y_{Fs} \tag{2.a}$$

$$Y_F(r_f) = 0 \tag{2.b}$$

The outer region is that bounded by the zone ($r_f < r < \infty$), the important diffusing species is the oxidizer, Fick's law of diffusion can be applied to the outer region:

$$\dot{m}_{ox}'' = Y_{ox}(\dot{m}_{ox}'' + \dot{m}_{Pr}'') - \rho D_{AB} \nabla Y_{ox}$$

It can be written as:

$$\frac{dY_{ox}}{dr} = \frac{\dot{m}_F}{4\pi\rho D} \frac{(Y_{ox} + v)}{r^2} \tag{3}$$

With the boundary conditions;

$$Y_{ox}(r_f) = 0 \tag{4.a}$$

$$Y_{ox}(\infty) = Y_{ox\infty} \tag{4.b}$$

CONSERVATION OF ENERGY:

The Shvab-Zeldovich form of the energy equation is used. Since the chemical reactions occur at the boundary, i.e., the flame sheet, the reaction rate term is zero both inside the flame and outside the flame [Turns, 2000]:

$$\frac{1}{r^2}\frac{d}{dr}[r^2(\rho vr \int CpdT - \rho D\frac{d}{dr}\int CpdT)] = -\sum h° f_i \dot{m}_i{}'$$

This equation can be re-arranged and written as:

$$\frac{d_2T}{dr^2} = \left(\frac{\dot{m}_F Cp_g}{4\pi k_g r^2} - \frac{2}{r}\right)\frac{dT}{dr} \tag{5}$$

With the boundary conditions for inner region:

$$T(r_s) = T_s \tag{6.a}$$

$$T(r_f) = T_f \tag{6.b}$$

And for outer region:

$$T(r_f) = T_f \tag{7.a}$$

$$T(\infty) = T_\infty \tag{7.b}$$

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ENERGY BALANCE AT DROPLET SURFACE:

Heat is conducted from the flame through the gas-phase to the droplet surface. Some of this heat is used to vaporize the fuel, while the remaining is conducted into the droplet interior. Mathematically:

$$\dot{Q}_{g-i} = \dot{m}_F h_{fg} + \dot{Q}_{i-l} \tag{8}$$

Where:

$$\dot{Q}_{g-i} = -\left[4\pi \, r_s^{\,2} \, k_g \, \frac{dT}{dr}\Big|_{r_s}\right]$$

The droplet is assumed to be consisting of two zones, an interior zone existing uniformly at its initial temperature T_o, and then surface layer at the surface temperature T_s. For this it is called "onion-skin model ":

$$\dot{Q}_{i-l} = \dot{m}_F \, Cp_l(T_s - T_o)$$

So that, equation (8) becomes:

$$-k_g 4\pi \, r_s^{\,2}\frac{dT}{dr}\Big|_{r_s} = \dot{m}_F\big(hfg + Cp_l(T_s - T_o)\big) \tag{9}$$

ENERGY BALANCE AT FLAME SHEET:

Since the flame temperature is the highest temperature in the system, heat is conducted both toward the droplet, $\dot{Q}_{f-i}$, and away to infinity, $\dot{Q}_{f-\infty}$ The chemical energy released at the flame is taken into account by using absolute enthalpy fluxes for the fuel, oxidizer, and products. A surface energy balance at the flame sheet can be written as:

$$\dot{m}_F \Delta h_c = \dot{Q}_{f-i} + \dot{Q}_{f-\infty} \tag{10}$$

Where $\dot{Q}_{f-i}$, $\dot{Q}_{f-\infty}$ can be represented as:

$$\dot{Q}_{f-i} = -k_g 4\pi \, r_f^{\,2}\frac{dT}{dr}\Big|_{r_f^{-}}$$

$$\dot{Q}_{f-\infty} = -k_g 4\pi \, r_f^{\,2}\frac{dT}{dr}\Big|_{r_f^{+}}$$

SOLUTION:

solving equation (1) with the boundary conditions (2.a, b) for $\dot{m}_F$ gives:

$$\dot{m}_F = \frac{4 \ln\left(\frac{1}{1 - Y_{Fs}}\right) \pi \rho D r_s}{1 - \frac{r_s}{r_f}} \tag{11}$$

Solving equation (3) with the boundary conditions (4.a, b) for r_f presents:

$$r_f = \frac{-\dot{m}_F}{4 \pi \rho D \ln\left(\frac{v}{v + Y_{ox\infty}}\right)} \tag{12}$$

Solving equation (5) with the boundary conditions (6.a,b) gives the temperature distribution of inner region, which can be derived with respect to r, and the resulted equation can substituted into equation (9) which can be solved for T$_s$ to obtain:

$$T_s = f(\dot{m}_F, T_f, r_s, r_f) \tag{13}$$

Solving equation (5) with the boundary conditions (7.a,b) gives the temperature distribution of outer region, which can be derived with respect to r, and the resulted equation can substituted into equation (10) which can be solved for T$_f$ to obtain:

$$T_f = f(\dot{m}_F, T_s, r_s, r_f) \tag{14}$$

To estimate the relation that describe the equilibrium between the liquid and vapor-phase of the fuel at the surface, and applying Antoine equation gives :

$$Y_{Fs} = \frac{exp\left(A - \frac{B}{T_s + C}\right) Mw_F}{exp\left(A - \frac{B}{T_s + C}\right) Mw_F + \left[P - exp\left(A - \frac{B}{T_s + C}\right)\right] Mw_{Pr}} \tag{15}$$

The droplet life time was obtained using the mass balance, which states that the rate at which the mass of the droplet decreases is equal to the rate at which the liquid is vaporized [Williams, 1965]:

$$\frac{dm_d}{dt} = -\dot{m}_F \tag{16}$$

Where:

$$m_d = \rho_l V = \frac{4}{3} \pi r_s^3 \rho_l$$

And $\dot{m}_F$ can be expressed as a function of r_s :

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$$\dot{m}_F = 4\,\pi\rho D r_s \ln\left(\frac{\frac{1}{1-Y_{Fs}}}{\nu + Y_{ox\infty}}\right)$$

Equation (16) can be written as:

$$\frac{dr_s}{dt} = \frac{-3\rho D}{r_s \rho_l} \ln \frac{\frac{1}{1-Y_{Fs}}}{\nu + Y_{ox\infty}} \tag{17}$$

TRANSIENT DROPLET COMBUSTION MODEL

Problem Statement:

In order to formulate the model that describes the droplet heating and combustion, the major assumptions will be [Chigier, 1981]:

1. Constant, stagnant, unbounded atmosphere.
2. Combustion process is spherically symmetric and isobaric.
3. The combustion process is following the double-film theory.
4. Fuel and oxidizer meets in a satoichiometric ratio at the flame front.
5. The reaction kinetics is fast compared with gas-phase transfer.
6. Physical properties will be calculated at reference temperature.
7. Single component droplet with no liquid-phase flows.
8. No soot formation, negligible radiation.

The problem is analyzed as follows. At time $t = 0$, the droplet with radius $r_s = r_{so}$ and temperature $T = T_{so}$ is igniting in constant, unbounded atmosphere characterized by its temperature T_∞, pressure P_∞, and oxidizer mass fraction $Y_{ox\infty}$. The outwardly diffusing fuel vapor reacting stoichiometrically and completely with the inwardly diffusing oxidizer gas at a thin flame front located at r_f. For analyzing the inner and outer regions, mass and heat balance will be obtained considering the non-dimensional quasi-steady phase equations [Law, 1976].

CONSERVATION OF MASS:

Inner region is expressed by non-dimensional equation [Law, 1976]:

$$\hat{m}_F Y_F - \hat{r}^2 \frac{dY_F}{d\hat{r}} = \hat{m}_F \tag{18}$$

Where:

$$\hat{m} = \frac{\dot{m}}{4\pi\rho D r_s}, \hat{r} = \frac{r}{r_s}$$

With the boundary conditions:

$$Y_F(1) = Y_{Fs} \tag{19,a}$$

$$Y_F(\hat{r}_f) = 0 \tag{19,b}$$

Outer region is represented by the non-dimensional equation [Law, 1976]:

$$\hat{m}_F Y_{ox} - \hat{r}^2 \frac{dY_{ox}}{d\hat{r}} = -\hat{m}_F v \tag{20}$$

With the boundary conditions:

$$Y_{ox}(\hat{r}_f) = 0 \tag{21,a}$$

$$Y_{ox}(\infty) = Y_{ox\infty} \tag{21,b}$$

CONSERVATION OF ENERGY:

for the inner region:

$$\hat{m}_F(\hat{T} - \hat{T}_s) - \hat{r}^2 \frac{d\hat{T}}{d\hat{r}} = -\hat{m}_F \tilde{Q} \tag{22}$$

Where:

$$\tilde{Q} = \frac{Q}{h_{fg}}, \hat{T} = \frac{CpT}{h_{fg}}$$

With the boundary conditions:

$$\hat{T}(1) = \hat{T}_s \tag{23,a}$$

$$\hat{T}(\hat{r}_f) = \hat{T}_f \tag{23, b}$$

For the outer region:

$$\hat{m}_F(\hat{T} - \hat{T}_s) - \hat{r}^2 \frac{d\hat{T}}{d\hat{r}} = -\hat{m}_F(\tilde{Q} - \tilde{H}) \tag{24}$$

Where:

$$\tilde{H} = \frac{\Delta h_c}{h_{fg}}$$

And the boundary conditions:

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$$\hat{T}(\hat{r}_f) = \hat{T}_f \tag{25, a}$$

$$\hat{T}(\infty) = \hat{T}_\infty \tag{25, b}$$

SOLUTION:

Solving equation (18) with the boundary conditions (19.a, b) for $\hat{m}_F$ and expressing with dimensional form:

$$\dot{m}_F = \frac{4 \ln(1 - Y_{Fs})\pi\rho \, Dr_s}{\frac{r_s}{r_f} - 1} \tag{26}$$

After solving equation (20) for r_f and applying boundary conditions, equation (21, a, b), expressing with dimensional form:

$$r_f = \frac{-\dot{m}_F}{4 \ln\left(\frac{v}{v + Y_{ox\infty}}\right)\pi\rho D} \tag{27}$$

T_f Can be expressed in dimensional form after solving equation (22) with boundary conditions (23, a, b) as:

$$T_f = \frac{Cp_g \, T_s - Q + \exp\left(\frac{-m_F}{4\pi\rho D \, r_f}\right) Q \exp\left(\frac{m_F}{4\pi\rho D \, r_s}\right)}{Cp_g} \tag{28}$$

Where, Q expressed after solving equation (24) with the boundary conditions (25, a, b), in dimensional form:

$$Q = \frac{Cp_g(T_f - T_s) - \Delta h_c + \left(\Delta h_c + Cp_g(T_s - T_\infty)\right)\exp\left(-\frac{m_F}{4\pi\rho D \, r_f}\right)}{\exp\left(-\frac{m_F}{4\pi\rho D \, r_f}\right) - 1} \tag{29}$$

The relation of surface temperature with droplet radius can be found using the expression of time varying latent heat of vaporization as:

$$\tilde{Q} = 1 + \frac{\frac{\dot{Q}_{sen}}{m}}{h_{fg}} \tag{30}$$

Where:

$$\frac{\dot{Q}_{sen}}{m} = \frac{\frac{4}{3}\pi r_s^3 \rho_l \, Cp\frac{dT}{dt}}{-4\pi\rho_l \, r_s^2 \frac{dr_s}{dt}}$$

Chain rule can be used to reduce one of the derivatives as :

$$\frac{dT_s}{dt} = \frac{dT_s}{d\sigma}\frac{d\sigma}{dr_s}\frac{dr_s}{dt}$$

Where:

$$\sigma = \frac{r_s^2}{r_{so}^2}$$

Substituting into equation (30), and simplifying:

$$\frac{d\hat{T}_s}{d\sigma} = \frac{3(1-\hat{Q})}{2\sigma} \qquad\qquad (31)$$

In order to solve this equation to obtain $\hat{T}_s$ as function of σ (or r_s), the term $\hat{Q}$ which is function of $(\hat{T}_s, \hat{m}_F, \hat{T}_f,$ and $\hat{r}_f)$ must be expressed as function of $\hat{T}_s$ only, the vapor is saturated at the droplet surface, an independent relation between Y_{Fs} and $\hat{T}_s$ was given by [Law,1976]:

$$Y_{Fs}(T_s) = P_\infty^{-1} \exp\left[\frac{Cp}{R}\left(\hat{T}_b^{-1} - \hat{T}_s^{-1}\right)\right]$$

Using computational substituting is necessary obtain $\hat{Q}$ as function of $\hat{T}_s$, then substituting the resulted equation into (31) gives:

$$\frac{d\hat{T}_s}{d\sigma} = \frac{3}{2\sigma}\left(1 - f(\hat{T}_s)\right)$$

With boundary condition:

$$\hat{T}_s(1) = \hat{T}_o$$

Solving this equation presents $\hat{T}_s = f(\sigma)$, which can be expressed as dimensional form as:

$$T_s = f(r_s) \qquad\qquad (32)$$

The droplet lifetime could be evaluated as follows:

$$\hat{m}_F = \frac{m}{4\pi\rho_g \, D \, r_s}$$

This can be expressed as:

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$$\hat{m}_F = -\frac{4\pi\rho_l \ r_s^2 \frac{dr_s}{dt}}{4\pi\rho_g \ D \ r_s} \tag{33}$$

Where dt can be expressed as dimensionless form as:

$$\hat{t} = \frac{2\rho_g \ D}{\rho_l \ r_{so}^2} t$$

Differentiating:

$$dt = \frac{\rho l \ r_{so}^2}{2\rho_g \ D} d\hat{t}$$

This equation can substituting into equation (33) and r_s can be expressed as dimensional from σ, rearranging the resulted equation gives:

$$d\hat{t} = -\frac{d\sigma}{\hat{m}_F} \tag{34}$$

CALCULATIONS

For both models, the following relations were used for calculate the terms and thermo-physical properties as; for Cp, k, and µ [Andrews & Biblarz, 1981]:

$$Cp = \sum_1^7 a_i \ T^{i-1}$$

$$k = \sum_1^7 b_i \ T^{i-1}$$

And:

$$k_g = 0.4k_F(\bar{T}) + 0.6k_\infty(\bar{T})$$

Where k_∞ for air, For binary diffusion coefficient [Reid, et.al. 1987] :

$$D_{AB} = \frac{3}{16} \frac{(4\pi \ k_B \ T/Mw_{AB})^{\frac{1}{2}}}{(P/RuT)\pi \ \sigma_{AB}^2 \ \Omega_D} f_D$$

The parameters of this equation are listed in reference above, Mass fraction of the oxidizer (Oxygen) at the infinite medium (air) was suggested as followed:

$$Y_{ox\infty} = \frac{mol\ oxidizer \times Mw_{ox}}{mol\ air \times Mw_{air}}$$

The molecular weights of the fuel and the products that used in the analyses, Mw_F and Mw_{Pr}, were calculated as:

$$CxHy + \left(x + \frac{y}{4}\right)(O_2 + 3.76N_2) \rightarrow xCO_2 + \frac{y}{2}H_2O + 3.76\left(x + \frac{y}{4}\right)N_2$$
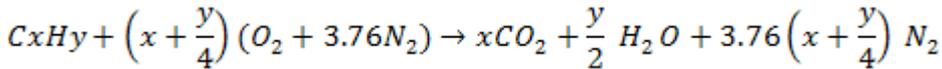
So:

$$Mw_F = 12x + y$$

$$Mw_{Pr} = 44x + 9y + 105.28\left(x + \frac{y}{4}\right)$$

The above relations were obtained at reference temperature as[Sparrow & Gregg,1958]:

$$T_r = T_s + \frac{1}{3}(T_\infty - T_s)$$

NUMERICAL WORK

Computer programs were created to evaluate the required properties, solving the ordinary differential equations evaluated from heat and mass Transfer balances, then constructs systems of non-linear equations as follows; for the classical model, the program fixed the value of surface temperature as T_{boil} , This is a reasonable assumption when the droplet is burning vigorously after its initial heat-up transient.[Turns,2000, Kuo,1986, kotake & Okazaki,1969, Law, 1982]. The program substituted equation (13) into equation (14) and resolving for T_f , then used the resulted equation with equations (11)and(12) to construct system of non-linear equation, calculating the Jacobian matrix for the system, then solving the system with Newton-Raphson method using the Jacobian. The droplet lifetime was obtained by solving equation (17) using Range-Kutta 4 method with appropriate time interval. For transient model, the program solves dimensionless equations for mass and heat balances, then made numerical substitution to obtain $T_s = f(r_s)$, constructing system of non-linear equations using equations (26),(27),(28),(29),(32) and the equation that related $Y_{Fs} = f(T_s)$, then solving the system with Newton-Raphson method using the Jacobian. Droplet lifetime can be obtained using equation (34).

RESULTS AND DISSCUSION

The droplet size used in calculations are; d=100µm as an initial diameter, and d=0 as a final diameter. The environment conditions are taken as 1500K, and pressure 1 atm.. Figures (1) and (2), shows the behavior of mass burning rate with the size ratio of the two fuels and for both models. The droplet size changes from r_{so} to zero. It is expressed in terms of a size ratio so it changes from 1

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to zero. It is evident from these figures, For the classical model, the burning rates greater than that for the transient model due to the classical theory, which assumes that all the heat gain to the droplet is used in the vaporization of the fuel droplet at the surface. This will lead to an increase in mass vaporization rate, which in turn, will increase the mass burning rate. Actually, some of the heat gained to the droplet is used for heating the droplet in early stages of its lifetime. This decreases the amount of fuel vaporized from the droplet surface, and this will lead to a decrease in the mass burning rate.
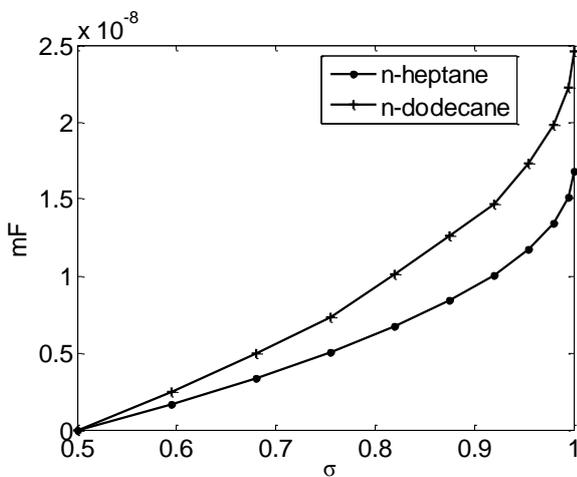
Fig.1 Burining rate vs.size ratio (Classical)
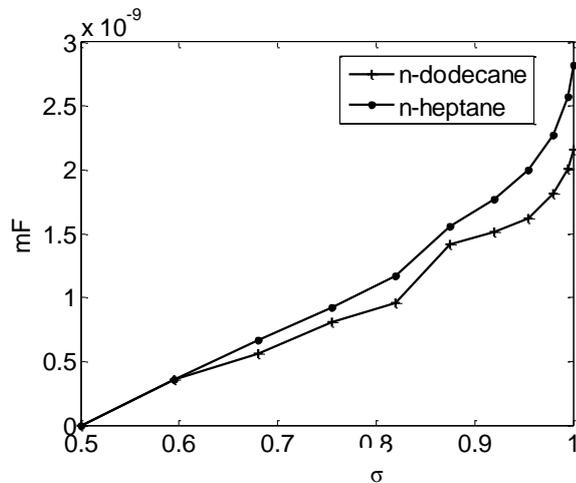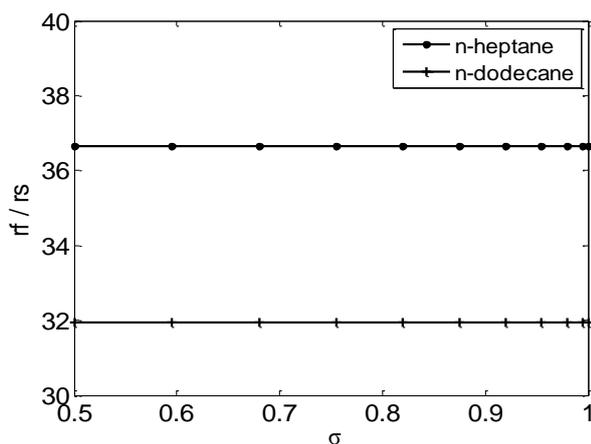

Fig.2 Burining rate vs.size ratio (Transient)

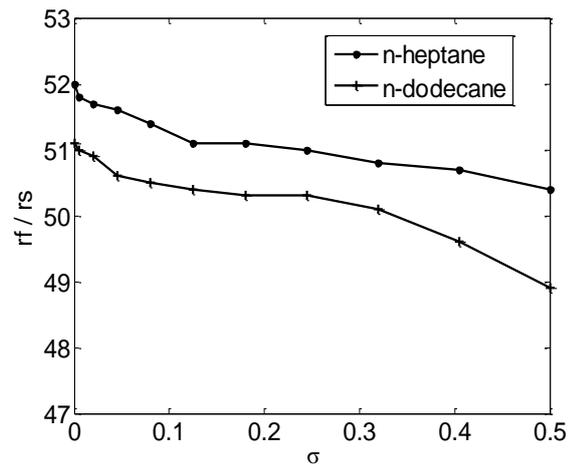**Fig.3** Stand off ratio vs.size ratio (Classical) (Transient)

Fig.4 Stand off ratio vs.size ratio
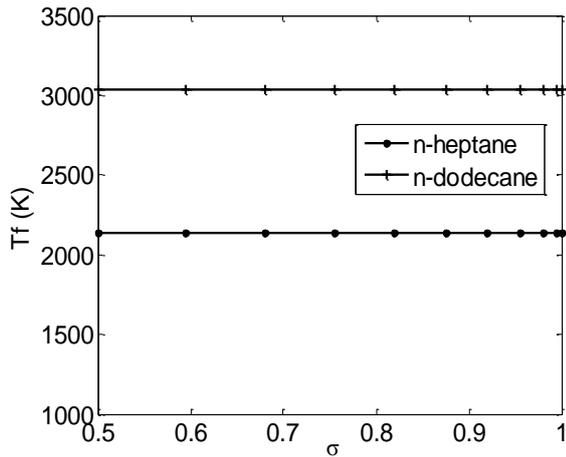
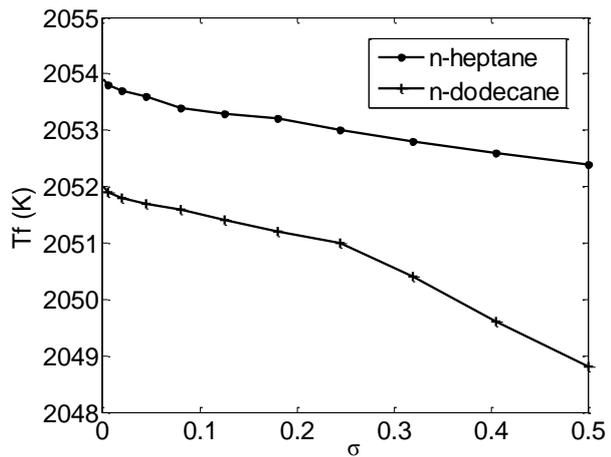**Fig.5** Flame temprature vs.size(Classical)

Fig.6 Flame temprature vs.size(Transient)
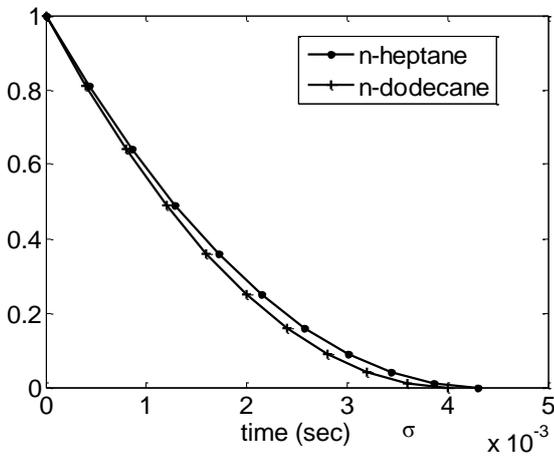

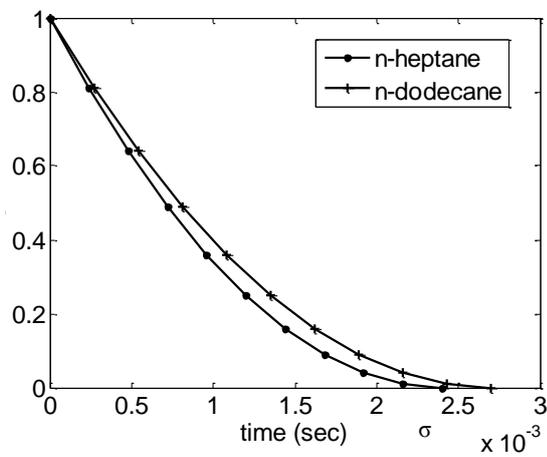
Fig.7 size ratio vs. time (Classical)

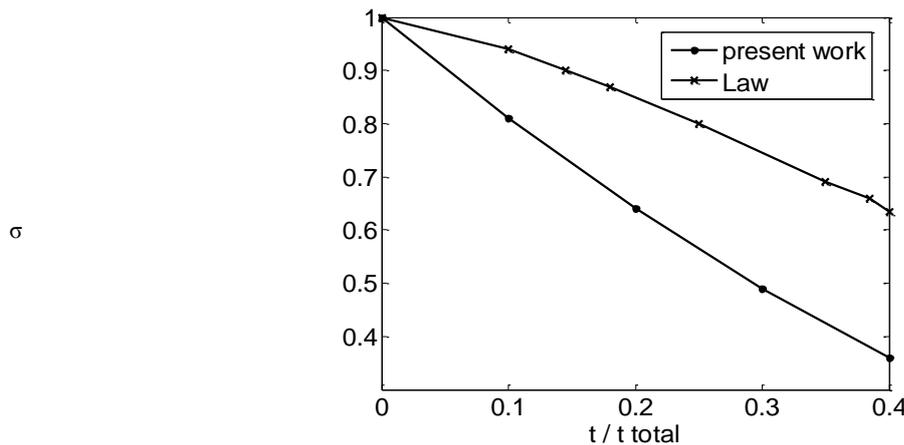

Fig.8 size ratio vs. time (Transient)

Fig.9 comparison of present Transient model

With model of Law

Figure (3) and (4) represent the variation of flame radius in terms of flame stand off ratio for the two fuels and two models as a function of size ratio. It can be seen that the values of stand off ratio

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for the transient model is higher than that for the other model. The assumption of quasi-steady burning of the fuel in the stagnant case resulted in raising the values of flame stand off ratios much more than its original values.

Figures (5) and (6) show the variation of flame temperature with size ratio, the flame temperature increases with the decrease in the size of droplet for the transient model. This can be attributed to the assumption of variable surface temperature.The flame temperature values for the classical model is higher than that of the transient model for the same condition. The burning rate of this model is higher than that of the transient model. This will lead to the burning a larger amount of fuel.

Figures (7) and (8) show the relationship between the time of the combustion process with the size ratio for both models and for the same condition. From these figures, it can be seen that the droplet size decreases with the increases in time. This is attributed to the consumption of the liquid fuel during the combustion process. This behavior continues through the whole process and the complete consumption of the droplet. The final time of the process is reached when the droplet radius diminishes completely.

Regarding and referring to the results expressed in the figures, two fuels were considered, namely: n-heptane and n-dodecane. The difference encountered between the two fuels can be attributed to the fact that the two fuels have different boiling point. The boiling point for n-heptane and n-dodecane are (371.4) and (489.3) respectively. For the transient model, the amount of heat that is used to reach the boiling point of n-dodecane is larger than that for n-heptane, and the fraction of heat used to heat up the droplet for n-dodecane is larger than that of n-heptane. As a result, the heat required to complete the vaporization will be less. Consequently, the vaporization rate will be less. The reduction in the burning rate causes a reduction in the flame conditions, which are position and the temperature. The position of the flame or radius of the flame. The flame temperature may increase as the burning rate becomes larger, since large amount of fuel is burning. Referring to the figure (8), it can be seen that the fuel of the largest burning rate has the shortest lifetime. Increase the burning rate causes the fuel to consume more rapidly. For that reason the figures of transient model is exactly opposite to the arrangements of fuel in the figures of other model.

COMPARISON WITH OTHER WORK:

for the transient model, Figure (9) shows the comparison between the transient combustion models with that of Law [Law, 1976]. Figure (9) shows the droplet size ratio variation with the variation of non-dimensional time (time to final time). Good agreement between Law's model and the present model can be seen with some difference, this difference can be attributed to the more complicated and elongated model used by Law and his use of the finite difference method.

CONCLUSION

The two models show that thermophysical properties are strong functions to reference temperature even when the droplet surface temperature is assumed constant. For classical model shows that the variation of surface temperature can be neglected and this temperature can be assumed constant. For transient model shows that the period of droplet heat up is dominant in the process and caused the mass burning rate at the early stages to be less than that expected in classical theory droplet combustion.

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NOMENCLATURES

A, B, C **:** Antoine equation constant**s**

$Cp(\frac{J}{kgK})$ **:** Specific heat

$D\left(\frac{m2}{s}\right)$ **:** Binary mass Diffusivity co-efficient

$h\left(\frac{J}{kg}\right)$ **:** Enthalpy

$h_{fg}(\frac{J}{kg})$ **:** Latent heat of vaporization

$k(\frac{W}{mK})$ **:** Thermal conductivity

$m(kg)$ **:** Mass

$\dot{m}(\frac{kg}{s})$ **:** Mass flow rate

$\dot{m}''(\frac{kg}{m^2 s})$ **:** Mass flux

$Mw\left(\frac{kg}{kgmol}\right)$**:** Molecular Weight

$P(atm)$ **:** Pressure

Pr **:** Prandtl number

$\dot{Q}(W)$ **:** Heat transfer rate

$Q(\frac{J}{kg})$ **:** Time varying latent heat of vaporization per unit mass

$r(m)$ **:** Radius

$t(s)$ **:** Time

$v_r(\frac{m}{s})$ **:** Radial velocity

x **:** Number of carbon atoms in fuel molecule

$Y\left(\frac{kg}{kg}\right)$ **:** Mass fraction

y **:** Number of hydrogen atoms in fuel molecule

GREEK SYMBOLS

$\Delta h_c\left(\frac{J}{kg}\right)$ **:** Heat of combustion

σ **:** Size ratio

$v\left(\frac{kg}{kg}\right)$ **:** Oxidizer to fuel stoicheometric ratio

SUBSCRIPTS AND ABBREVIATIONS

b**:** Boiling

d **:** Droplet

F : Fuel

f : Flame

g : Gas

$g-i$ **:** Gas–phase to interface region

$i-l$ **:** Interface region to liquid-phase

ox **:** Oxidizer

Pr **:** Products

ref **:** reference

s **:** Droplet surface

sen **:** Sensible

o **:** Initial

∞ **:** Environment