SLOPE STABILITY OF EMBANKMENTS BY THE FINITE ELEMENT METHOD

Dr. Yousif J. Al-Shakarchi
Professor, University of Baghdad, College of Engineering, Civil Eng.Dept

Dr. Mohammed Yousif Fattah
Lecturer, University of Technology, Building and Construction Dept.

Maysam Th. Al-Hadidi
M.Sc student, University of Baghdad, College of Engineering. Civil Eng. Dept.

ABSTRACT
Conventional methods of slope analyses are based on the concept of limit equilibrium and enable only the calculation of the critical height or the minimum factor of safety of a slope. These methods are not usually concerned with the stress distribution within a slope either above or below assumed surfaces of failure. This task can be done through the finite element method. In this paper, this method is used for the analysis of embankments. It was found that a good agreement could be obtained between the simple non-linear elastic-plastic finite element analysis, adopted in this paper, and sophisticated analyses which deal with plastic behavior of soils. This can be done when choosing the appropriate material parameters.

الخلاصة
إن اكثر الطرق الشائعة لتحليل المنحدرات مبنية على مبدأ معادلات الاتزان الحدي (limiting equations) التي تعطي إمكانية حساب الارتفاعات الحرجة (critical heights) للسدة الترابية أو أقل معامل أمان للمنحدر. هذه الطرق عادة لا تأخذ بنظر الاعتبار توزيع الاجهادات خلال المنحدر تحت أو أعلى سطح الفشل المفترض. هذه المهمة يمكن أجراؤها عن طريق التحليل بطريقة العناصر المحددة .
في هذا البحث أستخدمت هذه الطريقة في تحليل السداد الترابية حيث وجد أن افضل النتائج يمكن أن تتحقق من خلال اعتماد مبدأ (التحليل اللاخطي المرن-اللدن المبسط) مقارنة مع الطرق الأكثر تعقيداً التي تأخذ بنظر الاعتبار (التحليل اللدن للتربة) باختيار المعاملات المناسبة للتربة.

KEY WORDS
Slope stability, Embankment, Finite element, Displacement

INTRODUCTION
It is common practice among geotechnical engineers to use conventional limit equilibrium methods for the evaluation of slope stability in designing earth and rockfill dams and other slopes.
Despite its widespread use, the limit equilibrium stability analysis method is subjected to several theoretical shortcomings. This led to the development of the finite element method of stability analysis, which eliminates most of the limitations found in limit equilibrium methods. Theoretical objections that apply in general to all limit equilibrium analysis methods, either total or effective stress, have been reported by Wright et al. (1973). These methods also have a number of common characteristics, (Adikari and Cummins, 1985).

Duncan and Wright (1980) studies and others have frequently compared limit equilibrium methods that are in existence and have reported their relative merits and shortcomings.

Tavenas et al. (1980) summarise four major shortcomings of existing effective stress analysis methods as below. These arise from the use of oversimplistic assumptions which do not correspond to the actual stress conditions in-situ and which in fact are in contradiction with the objective of effective stress analysis:

1- The effective normal stress used in the stability analysis may differ significantly from those prevailing just prior to failure.
2- The indiscriminate use of simple Mohr Coulomb criterion ignores the existence of limit state and critical state of the materials in question.
3- The degree of mobilization of the material strength is expressed mathematically in a way that implies a specific stress path up to failure.
4- The equation between local and overall values of the factor of safety is valid only under specific circumstances that are generally impossible to meet.

Despite these shortcomings, many investigators have found that the classical stability analyses result in a surprisingly small error on the computed factor of safety when applied to case histories, (Adikari and Cummins, 1985).

METHODS OF ANALYSIS

Two methods of analysis are adopted in this research:

1- Bishop's simplified and modified method of slices.
2- (Non - linear) finite elements method.

Bishop's Method of Slices

Despite that Bishop's simplified method does not perform all conditions of equilibrium, the little iterations and the accuracy of its results make it more attractive to use. This method of solving is applied in this research by using the computer program (STABGM).

The STABGM program was written at University of California Berkeley. It was then modified in 1974 and, in 1981.

The program will be used to compare the results obtained by the finite element analysis .

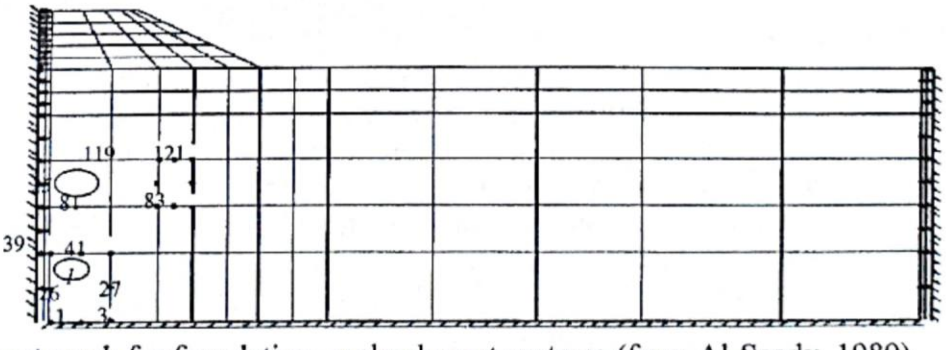
Non - Linear Finite Elements Method:

Previous studies on embankment analysis, i.e. Duncan and Seed (1966), Al-Damluji (1981) have shown that non-linear incremental finite element analysis appear to model actual behavior of the embankments much more closely than do linear analysis.

This method consists of using the internal stresses determined by performing analysis of the slopes to determine, (Wright et al., 1973):

1- The variation of the normal stress and the factor of safety along the shear surface.
2- The overall factor of safety of each slope.

Al-Saady, (1989) used the finite element mesh for foundation-embankment system as shown in **Fig. (1)**



- Typical finite element mesh for foundation- embankment systems (from Al-Saady, 1989)

Eight-node quadrilateral isoparametric plane strain elements are used, and a computer program edited by Smith and Griffiths (1988) (program, 6.1) was used with the following modifications:

1- The program was modified to consider different types of geometries and materials.
2- The initial stresses in the soil were taken into account. A procedure called "gravity turn-on analysis" which had been improved by Dunlop and Duncan (1970) was used in this program. In the plane strain case, the in-situ stresses are calculated by considering the lateral earth pressure coefficient (K_o).
3- The program considers gravity loading only. It was modified to consider external loads too.
4- The input data was simplified by adding subroutines (which were written for this purpose) to calculate:

a- the (half-band width) value of the stiffness matrix.
b- number of nodes in the mesh.
c- total number of degrees of freedom in the mesh.
d- the freedom data for restrained nodes.

5- The program was modified to examine whether elements were yielded and decide if that element failed or not by comparing the shear strength value (Cu), (which is equal to shear strength at failure) with shear stress at each element which is equal to $((\sigma_1 - \sigma_3)/2)$

If (stress level$= \dfrac{\tau_{peak}}{\tau_{element}} = \dfrac{c_u}{(\sigma_1 - \sigma_3)/2} = 1$), this means that the element is failed.

6- The program was modified to calculate the factor of safety, internal stresses, and displacements at each node and decide what region is failed.

The problem to be analyzed is a slope of (Mohr-Coulomb) failure criterion material subjected to gravity loading and external loads. The loads in this program are applied in a single increment and a "trial factor of safety loop" is considered. The material behaviour is elastic-plastic.
Keeping the loads constant, several values of (F) are attempted until the algorithm fails to converge. For simplification, the yield and ultimate failure surfaces are considered identical, (Smith and Griffiths, 1988).

DISCRETIZATION OF EMBANKMENTS:

Discretization of a system such as an embankment on a "rigid" foundation is straightforward because the boundaries are defined, but in problems involving "infinite" media, e.g., embankment foundations in deep geologic masses, finite boundaries must be established. It is helpful if a relatively hard material is encountered at shallow depth, in which case the boundary for the finite element mesh is established at the interface of the relatively hard material, (Desai and Christian, 1977)
It has been thought that relatively hard material can be defined as one, which has a modulus equal to 500 to 1000 times that of the overlying material, (Desai and Christian, 1977). When a relatively hard material is not present, it is necessary to establish finite boundaries within which the significant influence of the construction operations occurs.
Dunlop and Duncan, (1969) suggested that the extension from the toe of slope should be at least three times the foundation depth (3D).
Fig. (2) summarizes the results of several studies to determine the minimum extents required for modeling a finite media.

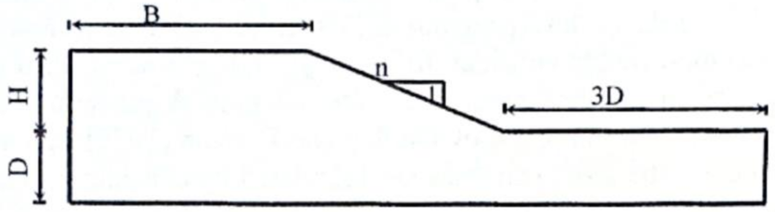


Fig. (2) Discretization of embankments.

STABILITY EVALUATION:

In addition to compute the stresses and displacements, the results of the finite element solutions can be used effectively to:

1- Isolate zones of local yielding.
2- Compute the overall factor of safety of an embankment.

Utilizing an appropriate failure criterion during construction, one can follow the progression of the failure zones.

The overall stability can be evaluated by evaluating the computed mobilized shear stresses along a critical shear surface and comparing these stresses with the strength along the surface. The factor of safety is essentially the same or slightly larger than the factor of safety computed from the Bishop's method of slices and that the factor of safety increases slightly with increasing values of Poisson's ratio.

CASE STUDY EXAMPLE:

In order to apply the slope stability finite element program in solving problems, a case history will be presented below. A comparison is made between the slope stability finite element program's results and the published results, which are obtained using another finite element program adopting triangular elements. The case is taken from Desai and Christian (1977)- example (3-2).

In this problem, the stability of an embankment is investigated using associated and non- associated flow rules. The results of embankment studies are expressed in terms of a factor of safety with respect to collapse, and in this case (Desai and Christian, 1977) follow a similar procedure by determining the minimum value of cohesion for which a stable solution can be obtained. The angle of friction was held constant at (20^o), and the cohesion was reduced until failure occurred. The results of this analysis are presented in the form of cohesion verus displacement curve.

Using the same properties and dimensions of this case, a mesh is made, and reducing the value of cohesion until the failure occurs does the analysis.

Fig. (3) shows the finite element mesh used in the analysis. The left and right boundaries are considered free to move vertically. Material properties are also presented in this figure.

Fig. (4) shows the embankment deformation resulted from gravity loading only.

In this Figure the vertical displacement for point (A) is varied with reduction of cohesion. When comparing the results of (the slope stability finite element program) with those obtained by (Desai and Christian, 1977), one can observe that there is a good agreement between the two results. It is noticed from **Fig. (4)** that the values of the factor of safety obtained are very close to those obtained by Desai and Christian (1977). This reflects the suitability of the algorithm convergence adopted in the program. The visco - plastic material behavior following the (Zienkiewicz and Cormeau, 1974) method is successful. Through this method, the material is allowed to sustain stresses outside the failure criterion for finite periods.

The factor of safety which was obtained from the stability analysis using the two methods (Bishop's method represented by STABGM program, and the non - linear finite elements program) are shown in **Table (1)** for c =3 kN/m^2 .

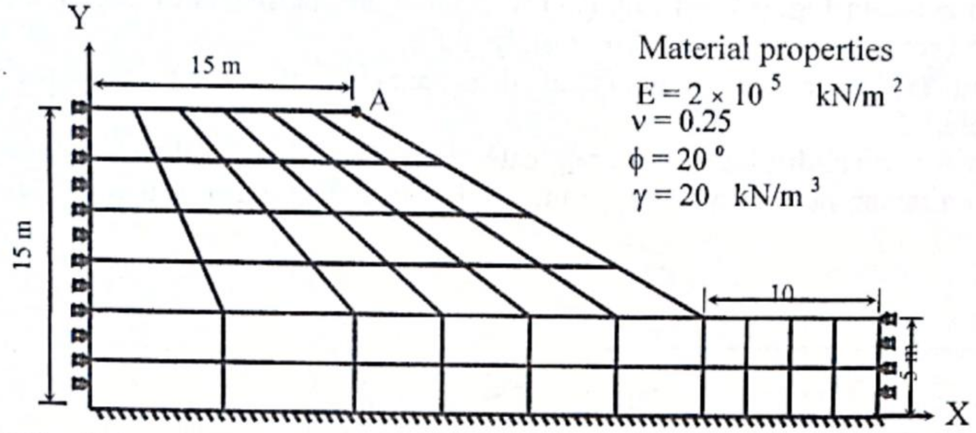


Fig. (3)- The finite element mesh for the embankment, and the material properties, (Desai and Christian, 1977).

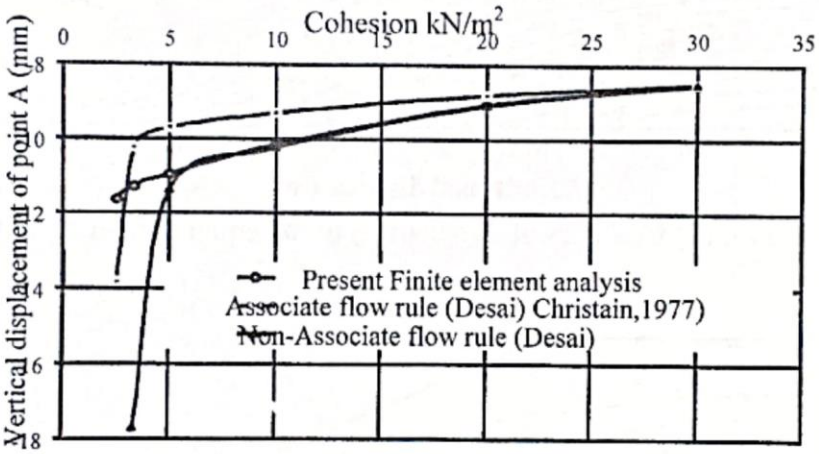
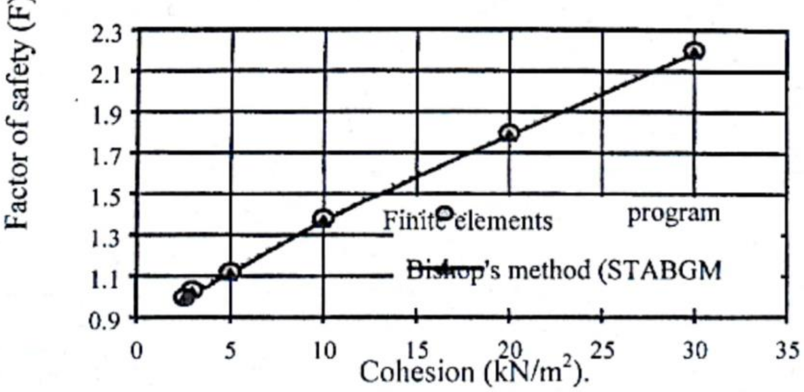


Fig. (4) - Embankment deformations resulting from reduction of cohesion, (application of gravity loading only).

Table (1) Results of stability analysis.

Method	(Bishop's Method of Slices) (STABGM) program	Finite Elements
F	1.034	1.032

Fig. (5) shows the variation of the value of the factor of safety with the value of cohesion obtained from the non-linear finite elements program, and from STABGM program.



Variation of the value of the factor of safety with the value of cohesion.

Figs. (6) and **(7)** show the contours of stresses at failure in y and x-directions, representatively.
A comparison between **Fig. (6)** and **Fig. (7)** reveals that the coefficient of lateral stresses (the ratio of horizontal to vertical stresses) is approximately (0.5).
Figs. (8) and **(9)** show the contours of displacements at failure in y and x-directions, representatively.
It is noticed that vertical displacements are greater near the surface, while horizontal displacements concentrate in a region near the toe, which means that spreading out of soil will take place near the toe.

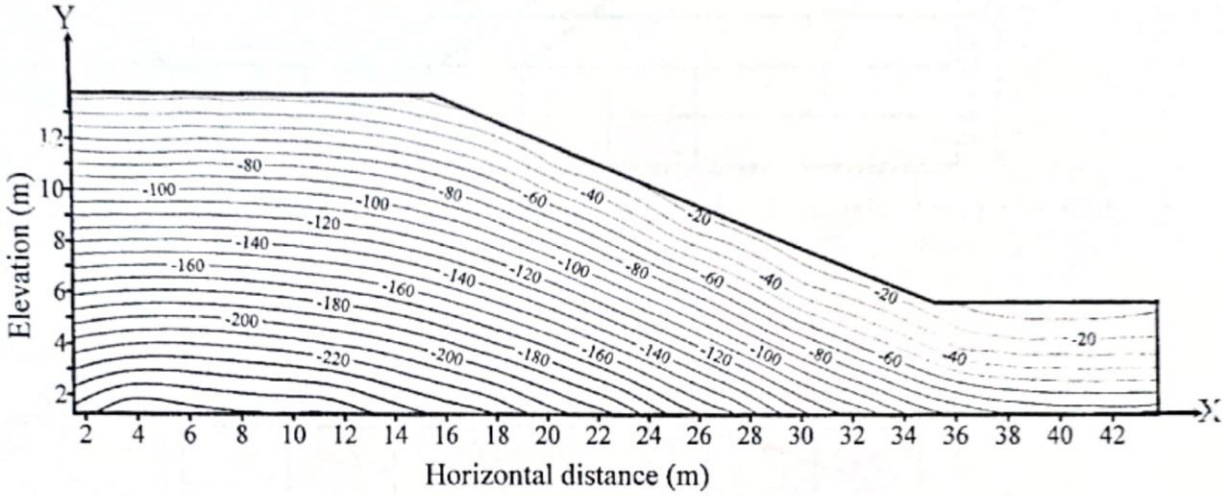


Fig. (6) Contours of σ_y (kN/m^2) in the embankment at failure.

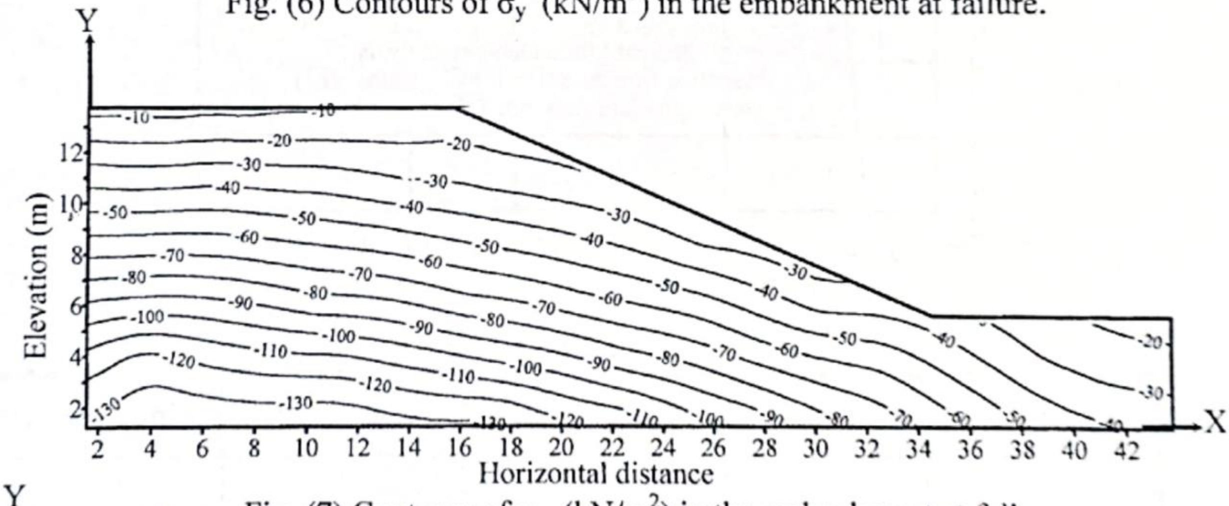


Fig. (7) Contours of σ_x (kN/m^2) in the embankment at failure.

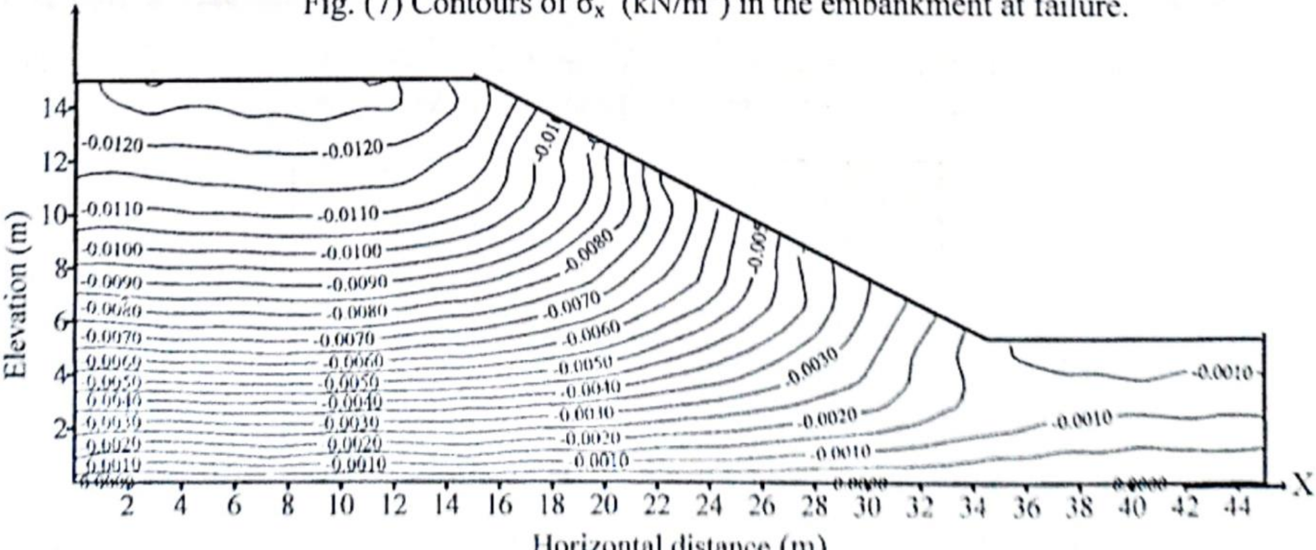


Fig. (8) Vertical displacement contours (m) in the embankment at failure.

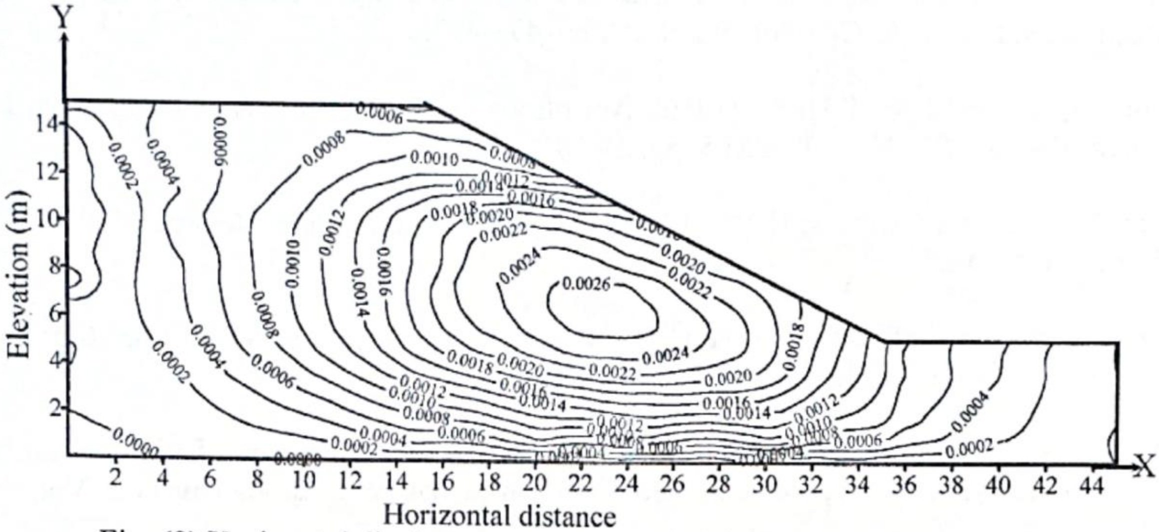


Fig. (9) Horizontal displacement contours (m) in the embankment at failure.

CONCLUSIONS:

1- The computer program developed for this work which adopts finite element analysis gives good results (factor of safety) for slope stability problems compared with those obtained by Bishop's method of slices.

2- A good agreement can be obtained between the simple non-linear elasto-plastic finite element analysis adopted in this paper and sophisticated analyses which deal with plastic behavior of soils.

The algorithm of convergence followed here for attaining the material non-linearity; constant stiffness method gives good results. The visco-plastic material behavior following the (Zienkiewicz and Cormeau, 1974) method is successful.

Through this method, the material is allowed to sustain stresses outside the failure criterion for finite periods.

REFERENCES:

Adikari, G. S. and Cummins, P. J., (1985), An Effective Stress Slope Stability Analysis Method for Dams, Proceedings of the 11 th ICSMFE, San Francisco, Vol. 2, p.p. 713-718.

Al-Damluji, O. F., (1981), Non- Linear Soil Behavior by the Finite Element Method, M.Sc. thesis, University of Baghdad.

Al-Saady, N.H., (1989), Analysis of an A-6 Soil During Construction of Road Embankment, M.Sc. thesis, University of Baghdad.

Desai, C. S., and Christian, J. T., (1977), Numerical Methods in Geotechnical Engineering, Chapter 3,McGraw-Hill Book Comp. New York.

Duncan, J. M. and Wright, S. G., (1980), The Accuracy of Limit Equilibrium Methods of Slope Stability Analysis, Engineering Geology, Vol. 16, p.p. 5-17.

Dunlop, P., and Duncan, J. M. (1969), Slopes in Stiff- Fissured Clays and Shale, J. Soil Mech. Found. Div., ASCE, Vol., 95. SM 2, pp. 467-491.

Dunlop, P. and Duncan, J. M., (1970), Development of Failure Around Excavated Slopes, J. Soil Mech. Found. Div., ASCE, Vol., 96. SM 2, pp. 471-493.

Duncan, J. M. and Seed, H. B., (1966), Anisotropy and Stress Reorientation in Clay, J. Soil Mech. Found. Div., ASCE, Vol., 92. SM 5, pp. 21-50.

Smith, I. M., and Griffiths, D.V., (1988), Programming the Finite Element Method, 2nd ed., John Wiley, New York.

Tavenas, F., Trak, B. and Leroueil, S., (1980), Remarks on the Validity of Stability Analysis, Canadian Geotechnical Journal, Vol. 17, p.p. 61-73.

Wright, S. G., Kulhawy, F. H. and Duncan, J. M., (1973), Accuracy of Limit Equilibrium Slope Stability Analysis, ASCE, Journal of Soil Mechanics and Foundations Division, Vol. 99, p.p. 783-791.

Zienkiewicz, O. C., and, Cormeau, I. C., (1974), Visco- Plasticity and Creep in Elastic Solids. A Unified Numerical Solution Approach, Int. J. Num. Meths. Engrg., Vol. 8, pp. 3201-3845.

NOTATION

B half width of the crest of embankment
C_u undrained cohesion
D depth of the foundation
F factor of safety
H height of embankment
K_o at- rest lateral earth pressure coefficient
n side slope of embankment (n horizontal to 1 vertical)
σ_x, σ_y lateral stress (x- direction) and (y- direction) respectively
σ_1, σ_3 principal stresses