THEORETICAL SIMULATION OF STRESS-STRAIN RELATIONS FOR SOME IRAQICLAYS USING THE ENDOCHRONIC MODEL

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ABSTRACT
A constitutive law can be defined as a mathematical functional relation between physical quantities such as stress and strain and may take other factors like time, temperature and additional material properties into account.

In this paper, the endochronic model is used to predict the stress-strain relations of two Iraqi clays. This model is a viscoplastic one but without introducing a yield surface. It encompasses material behaviour such that the current stress state is a function of strain history through a time scale called “intrinsic time” which is not the absolute time but a material property.

The simulation showed that the model overestimates the strains for all cases studied. This may be attributed to the material parameters which require a parametric study to determine their actual values for Iraqi clays.

KEYWORDS
Clay, End chronic Model, Stress, Strain, Model Parameters

INTRODUCTION
Endochronic theory was first introduced by Valanis in 1971. He coined this Greek name “Endochronic” that consists of two roots, endos (meaning inner) and chronos (meaning time). This theory encompasses material behaviour such that the current stress state is a function of the strain
history through a time scale called “intrinsic time” which is not the absolute time measured by a clock as in viscoplasticity but a material property. Hence, the endochronic theory is a “viscoplastic” one but without introducing a yield surface. Therefore, all the complexities and difficulties that develop in introducing a suitable yield criteria are avoided, (Valanis, 1971).

Bazant in 1974 and later with his coworkers extended Valanis theory to predict the behaviour of different engineering materials such as concrete, and soils.

**GENERALIZED CONSTITUTIVE RELATIONS:**

To generalize the uniaxial concept of the endochronic theory into three dimensions, first, the definition of the intrinsic time increment, $dz$, which is used in stead of real time increment, $dt$, is introduced. The intrinsic time for time-dependent behaviour is function of strain increments, $d\varepsilon_{ij}$ and time, $dt$. The dependence of $dz$ upon $d\varepsilon_{ij}$ is assumed to be gradual to exclude ideal plastic response. The function of $dz$ will be continuous, smooth, and monotonically increasing. Thus, function $(dz)^s$ with an appropriate exponent “$s$”, can be expanded in a tensorial power series in $d\varepsilon_{ij}$ and $dt$, i.e., (Bazant and Bhat, 1976):

$$dz = p + p_1 d\varepsilon_{ij} + p_2 d\varepsilon_{ij} + p_{ijkl} d\varepsilon_{ij} d\varepsilon_{ij} + dt + p_4 d\dot{t} + p_{ijklmn} d\varepsilon_{ij} d\varepsilon_{ij} d\varepsilon_{mn} + \ldots$$

(1)

where:

$\mathbf{P} = \text{coefficient matrices, the subscripts refer to the components in the Cartesian coordinates } x_i, i = 1, 2, 3, \text{ and number (4) refers to the time axis.}$

Since, $dz$ must vanish as $d\varepsilon_{ij} \rightarrow 0$ and $dt \rightarrow 0$, thus $P=0$. Setting $s=1$, and neglecting all quadratic terms, then $dz = P_4 dt$ which is of no interest, thus $P_4 = 0$. Setting $s=2$, and satisfying the conditions of isotropy, the quadratic form of Equation (1) can be written in terms of the first two invariants of $d\varepsilon_{ij}$, as follows, (Bazant and Bhat, 1976):

$$(dz)^2 = P_2 J_2 + (P_1 I_1 + P_3 dt)^2 + P_4 (dt)^2$$

(2)

where:

$P_0, P_1, P_2, P_3 = \text{non-negative coefficients.}$

$J_2 = \text{second deviatoric strain increment invariant, and}$

$I_1 = \text{first strain increment invariant.}$

Then, $dz$ must vanish for both instantaneous time, $dt = 0$, and pure volumetric deformation, $J_2 = 0$, hence $P_4 = 0$. Thus, the remaining terms in Equation (2) can be rewritten in the following form:

$$(dz)^2 = \left(\frac{d\xi}{Z_1}\right)^2 + \left(\frac{dt}{\tau_1}\right)^2$$

(3)

where:

$d\xi = f_i(\sigma_i, \varepsilon) \cdot d\xi$

(4.a)

$d\zeta = \sqrt{J} \zeta = \frac{1}{\sqrt{2}} d\varepsilon_{ij} \cdot d\varepsilon_{ij}$

(4.b)

$d\varepsilon_{ij} = \text{deviatoric strain increment tensor}$

$= d\varepsilon_{ij} - \frac{1}{3} \delta_{ij} \cdot d\varepsilon$

$\delta_{ij} = \text{Kronecker delta.}$

$d\varepsilon = \text{Volumetric strain increment} = d\varepsilon_{kk}$

$z_1, \tau_1 = \text{Constants.}$
$d\zeta$ is a scalar called "damage measure" that depends on strain increments and stresses to predict hardening and softening. $d\zeta$ is called "deformation measure" that depends only on strain increments. From Equations (3) and (4), $d\zeta$ and $dz$ represent geometrically the length of path traced by material states in a six-dimensional strain space for $d\zeta$, or in a strain-time space for $dz$. (Ansai et al., 1979).

Secondly, generalizing of equations to three dimensions using $dz$ instead of $dt$, and splitting the strain components into deviatoric and volumetric components to satisfy isotropy conditions, the following differential constitutive equations are deduced:

$$\frac{de_y}{2G} = \frac{dS_y}{2G} + S_y \frac{dz}{dz}$$

$$d\varepsilon = d\sigma_m \cdot \frac{dt}{3k} + d\lambda + d\varepsilon^0$$

where:

$de_y = d\varepsilon_y = -\frac{1}{3} \delta_y \cdot d\varepsilon$

$d\varepsilon = d\varepsilon_{11} + d\varepsilon_{22} + d\varepsilon_{33}$

$d\lambda$ = inelastic dilatancy,

$S_y$ = deviatoric stress tensor,

$S_y = \sigma - \delta_y \cdot \sigma_m$

$\sigma_m$ = mean stress = $\frac{1}{3} \sigma_{kk}$

$G, K$ = shear and bulk elastic moduli, and

$d\varepsilon^0$ = stress-independent inelastic strains (e.g. thermal strains).

Both of the first terms of Equations (5.a) and (5.b) represent the elastic strain increments, while the remaining terms represent the inelastic strain increments. For instance, the term $(\sigma_m \cdot dt / 3K\tau_i)$ represents the time-dependent inelastic volumetric strain, i.e. creep, while $d\lambda$ represents the time-independent volumetric strain.

To develop a quasi-linear elastic incremental constitutive law for simplicity, the plastic stress increment tensor $d\sigma_{ij}^p$ can be obtained from Equations (5) by multiplying Equation (5.a) by $2G$, and Equation (5.b) by $3K$, hence:

$$d\sigma_{ij}^p = 2G \cdot de_{ij}^p + \delta_{ij} (3K \cdot d\varepsilon^p)$$

$$= S_y \cdot dZ + \delta_{ij} (\sigma_m \cdot dt \cdot \tau_i + 3K \cdot d\lambda + 3K \cdot d\varepsilon)$$

The stress increments $d\sigma_{ij}$ are related to the elastic strain increments $d\varepsilon_{ij}^e$ by the following equations:

$$d\sigma_{ij} = 2G \cdot de_{ij}^e + \delta_{ij} (3K \cdot d\varepsilon^e)$$

Hence, the summation of Equations (7) and (8) yields:

$$d\sigma_{ij} + d\sigma_{ij}^p = D_{ijkl} \cdot d\varepsilon_{kl}$$

where:

$D_{ijkl}$ = elastic coefficient matrix
THE NUMERICAL PROCEDURE:
The basic constitutive law, Equation (5), is of a differential form, and the variables that govern inelastic deformations are \(dz\) and \(\lambda d\lambda\). Bazant and Bhat (1976) used the step-by-step integration or step-iterative algorithm in which for each loading step, a number of iterations are performed till satisfaction of equilibrium of stresses and strains occurs. This is assured when the change in values of \((dz)\) and \((\lambda d\lambda)\) for the same loading step becomes very small.

In this algorithm, the values of \((dz)\) and \((\lambda d\lambda)\) computed from the previous loading step provide an initial estimate for the next loading step.

Endochronic Hardening Functions and Parameters:
The function \(f_1\) in Equation (4) that accounts for hardening or softening, should decrease as the inelastic strains accumulate, because \(d\xi\) is adopted as a measure of the accumulated inelastic strain, hence:

\[
d\xi = \frac{d\eta}{f(\eta)} ; \quad d\eta = F(\sigma,\varepsilon) \cdot d\xi
\]  

(9)

where:

\(f(\eta) = \) Strain-hardening function.

\(F(\sigma,\varepsilon) = \) Strain-softening function.

Thus, the function \(f(\eta)\) has a significant effect on the non-linearity of the stress-strain relations, while the function \(F(\sigma,\varepsilon)\) allows for a gradual decrease of these relations on approach to peak stress. Both functions depend mainly on material type.

Hardening Functions and Dilatancy for Normally Consolidated Clays:
The function \(F\) in Equation (9) is determined semi-empirically from experimental data. The function \(F\) is governed by the effective confining stress \(\sigma_1^e\), the volume change, \(\varepsilon_1^e\), and the second deviatoric strain invariant, \(J_2^e\). Bazant et al. (1979) introduced the following formulation for function \(F\):

\[
F(\sigma,\varepsilon) = a + \frac{\left|\frac{1-a^1}{1+a^1} + \frac{\varepsilon_1^e}{1+a^1} \right|}{\sigma_1^e / Pa}
\]  

(10)

where: \(a^1\) = material constants.

\(Pa =\) atmospheric pressure = 101.3 kN/m²

The division of \(\sigma_1^e\) in Equation (10) by \(Pa\) is to make the relation dimensionless. Constant “a” must be positive to ensure irreversible strain increment for the critical case of no hardening or softening, (Bazant et al., 1979).

The function \(f(\eta)\) represents the limiting critical case of no hardening or softening. Thus, for large values of \(\eta\), this function, \(f(\eta)\), must converge to one. The function \(f(\eta)\) takes the following form:

\[
f(\eta) = 1 + \frac{\beta_1}{1 + \beta_2 \eta}
\]  

(11)

where: \(\beta_1\) and \(\beta_2\) = constants.

The dilatancy or densification function \(d\lambda\) of clays depends on shear and volumetric stresses and strains. Hence, the function \(d\lambda\) depends on \(J_2^e\), \(I_1^e\) and \(I_1^e\). Moreover, \(d\lambda\) depends on \(\lambda\) itself because the volumetric strain increment should decrease monotonically till zero as a limit in the case of failure. Hence \(d\lambda\) is equal to (Bazant et al., 1979):
\[ d\lambda = \frac{C_o \| + C_1 I_1 \| d\zeta}{(1 + C_2 I_\eta^\prime / Po)(1 + C_3 J_2^\prime)(1 + C_4 \lambda)} \]  \hspace{1cm} (12)

where:  \( c_o, c_1, c_2, c_3, c_4 \) = material constants.

\( d\lambda \) is determined empirically from tests and it depends on the clay type, stress path and stress history.

The tensile strengths of soils are very small and hence neglected.

The elastic moduli \( G \) and \( K \) of the soil element change during loading, and thus the accumulated densification-dilatancy measure \( \lambda \) and the effective normal stress also change. Thus, the effect of void ratio is:

\[ \frac{de}{e_o} = \frac{\epsilon_v (1 + e_o)}{e_o} = \frac{3(1 + e_o) \lambda}{n} \]

\[ = \frac{3\lambda}{n} \hspace{1cm} (13) \]

where:  \( e_o \) = initial void ratio

\( \epsilon_v \) = volumetric strain = \( \epsilon_{kk} \)

\( n \) = porosity.

while the effect of normal stress is the ratio \((I_{1^\prime}^\sigma - I_1^{\sigma^*}) / I_1^{\sigma^*}\), where \( I_1^{\sigma^*} \) is the initial first stress invariant. Hence, the elastic moduli will be equal to:

\[ G = G_o (1 + b_1 \frac{I_{1^\sigma} - I_1^{\sigma^*}}{I_1^{\sigma^*}} + b_2 \frac{3\lambda}{n}) \]

\[ = \frac{2}{3} G (1 + \nu)(1 - 2\nu) \hspace{1cm} (14) \]

where:  \( b_1 \) and \( b_2 \) = constants,

and \( K = \frac{2}{3} G (1 + \nu)(1 - 2\nu) \hspace{1cm} (15) \)

**MODEL PARAMETERS OF CLAYS**

All material parameters in the previous equations are based on best fit of experimental results.

Constant “a” in Equation (10) affects the value of the peak stress. Constant \( a_3 \) which is called “distortion coefficient” is determined by the following correlation proposed by Ansal et al. (1979).

Based on general pattern of results:

\[ a_3 = 153.8(e_p Pa / Po) + 34.62 \]  \hspace{1cm} (16)

where:

\( Po \) = consolidation pressure.

Similarly, the plasticity coefficient \( Z_1 \) in Equation (3) that accounts for rigidity and deformibility of clays, is determined from the following correlation:

\[ Z_1 = 0.00294(e_p Pa / P_o)^2 - 0.0177(e_p Pa / P_o) + 0.0396 \]  \hspace{1cm} (17)

Ansal et al. (1979) determined an approximate correlation for densification coefficient \( C_o \) in Equation (12), softening coefficient \( \beta_2 \) in Equation (11), and the elastic modulus \( E \), as shown in Figure (1). This correlation depends on the consolidation pressure \( P_o \), and the liquidity index of the clay \( I_L \), where (Mitchel, 1993):

\[ I_L = \frac{w_{sat} - w_p}{I_p} \]

\[ = \frac{w_{sat} - w_p}{I_p} \hspace{1cm} (18) \]

where:
\( w_{nt} \) = natural water content.
\( w_p \) = plastic limit
\( I_p \) = plasticity index = \( w_L - w_P \)
\( w_L \) = liquid limit.

Choice of appropriate ratio of the liquidity index to the consolidation stress is tempered by judgment in the absence of test results.

All other constants are determined experimentally. The values of the parameters as proposed by Bazant et al. (1979) are shown in Table (1):

**Table (1) – Material parameters of endonchronic model for normally consolidated clays.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>4</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>500</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.75</td>
</tr>
<tr>
<td>( \beta_l )</td>
<td>5n (n = porosity)</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>2500</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0.25</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>1000</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>9000</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0.1</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Computer Program**
The computer program **Endoch**, coded in Fortran language, was written by the authors. The algorithm used in the endochronic model incorporates an iterative procedure. The program computes stresses, strains, all functions like \( F, f(\eta) \), and variables like \( \lambda, \eta \), at mid-step loading. Iterations are then performed till the tolerance of the values of \( dz \) and \( d\lambda \) becomes less than 0.05 %.
The values of strain increments, \( d\varepsilon \), intrinsic time, \( dz \), and inelastic dilatancy, \( d\lambda \), or the previous step are taken as an estimate for the current step.

**APPLICATIONS:**
This model have been applied for simulating stress-strain relationships of two Iraqi soils:
i) **First application**    Al-Mufty (1990) carried out a series of tests on al-Fao soft clay.
    Block samples were obtained from an area close to the river Shatt-Al-Arab.
    The top layer of Fao soil was found to be stiff to very stiff brownish gray silty clay with a desiccated crust. This layer is followed by a soft to very soft gray silty clay.
    According to the unified classification system, the soil from both layers may be classified as CL-CH, inorganic clays of medium to high plasticity. According to, AASHTO M145-73, the soil is classified as A-7-6 (16).
a) Approximate correlation for:

b) Softening coefficient, $\beta_2$.

c) Elastic modulus, $E$. 
The average properties of the soil at sampling depths 1.25 m and 3 m respectively are listed in Table (2).

Table (2). - Average properties of the soft clay from Al-Fao, (from Al-Mufty, 1990).

<table>
<thead>
<tr>
<th>Property</th>
<th>1.25 m depth</th>
<th>3 m depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total unit weight $\gamma_t$, kN/m$^3$</td>
<td>17.9</td>
<td>17.7</td>
</tr>
<tr>
<td>Water content w %</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td>Liquid limit $w_L$ %</td>
<td>54</td>
<td>50</td>
</tr>
<tr>
<td>Plasticity index $I_p$ %</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>Liquidity index $I_L$</td>
<td>0.11</td>
<td>0.79</td>
</tr>
<tr>
<td>Specific gravity G</td>
<td>2.7</td>
<td>2.72</td>
</tr>
<tr>
<td>Sand size fraction %</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Silt size fraction %</td>
<td>58</td>
<td>60</td>
</tr>
<tr>
<td>Clay size fraction %</td>
<td>33</td>
<td>28</td>
</tr>
<tr>
<td>Activity A</td>
<td>0.82</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Among the tests carried out by Al-Mufty (1990) unconsolidated undrained triaxial compression tests on samples compacted by the standard compaction test to the maximum dry density and optimum moisture content. These results are compared with those predicted by the endochronic model in Figures (2) to Figure (6).

Fig (2) and (3) represent the samples that are taken from the top layer, and the figures from (4) to (6) represent the samples that are taken from the layer below the top layer.
Fig.(2). - A comparison between the stress-strain relationships predicted by the endochronic model with laboratory tests of Al – Mufty (1990), $\sigma_3 =300$ kPa.

![Graph showing stress-strain comparison](image1)

Fig.(3). - A comparison between the stress-strain relationships predicted by the endochronic model with laboratory tests of Al – Mufty (1990), $\sigma_3 =300$ kPa.

![Graph showing stress-strain comparison](image2)

Fig.(4). - A comparison between the stress-strain relationships predicted by the endochronic model with laboratory tests of Al – Mufty (1990), $\sigma_3 =100$ kPa.
Fig. (5). - A comparison between the stress-strain relationships predicted by the endochronic model with laboratory tests of Al – Mufty (1990), $\sigma_3 = 200$ kPa.

Fig (6). with laboratory tests of Al – Mufty (1990), $\sigma_3 = 300$ kPa.

It can be observed in these figures that the model overestimates the strains for all the cases studied under high stress increments.

In addition, there is no definite yield point can be obtained. Thus it is approximately suitable for normally consolidated clays where ductile behaviour of the stress-strain is expected.

ii) Second application

Al- Saady (1989) carried out laboratory tests on an A-6 soil during construction of a road embankment. A representative area located at Al – Zafarania (south of Baghdad), was chosen for the research. The site covers an area of soil composed of silty clay with varying thickness. This stratum behaves as normally or slightly overconsolidated soil, have an upper desiccated crust 0.5-0.75 m thick.

The distribution of the particle sizes indicated:

Clay fraction = 45 %, silt fraction = 37 %, sand fraction = 18 %.

It is classed as “CL” in a Casagamde classification chart.

Among the tests carried out by Al- Saady (1989) consolidated undrained triaxial test which was designated as series D as shown in Table (3).

In addition, unconsolidated undrained triaxial test which was designed at as series G as shown in Table (4).

Consolidated undrained triaxial test results are compared with those predicted by the endochronic model in Figures (7) to (12) which show a comparison between the stress-strain relationships predicted by the endochronic model with laboratory tests of Al – Saady, (series, D).

Consolidated drained triaxial test results are compared with those predicted by the endochronic model in Figures (13) to (18). Figures (19) to (24) show a comparison between the volumetric strain–axial strain relationships predicted by the endochronic model with laboratory tests of Al-Saady, (series, G).
Table (3). - The results of series (D), (from Al-Saady, 1989).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\sigma'_0$ kN/m$^2$</th>
<th>$c_0$</th>
<th>$w_c$ %</th>
<th>$(\sigma_1 - \sigma_0)$, kN/m$^2$</th>
<th>$(\sigma / \sigma_0)$, kN/m$^2$</th>
<th>$\Delta u_f$ kN/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79</td>
<td>0.76</td>
<td>26.0</td>
<td>123.24</td>
<td>3.50</td>
<td>30.81</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0.70</td>
<td>24.3</td>
<td>123.00</td>
<td>3.55</td>
<td>52.22</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>0.74</td>
<td>25.6</td>
<td>189.21</td>
<td>3.30</td>
<td>72.45</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>0.69</td>
<td>24.6</td>
<td>219.60</td>
<td>3.25</td>
<td>104.45</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>0.75</td>
<td>25.4</td>
<td>279.00</td>
<td>3.25</td>
<td>176.68</td>
</tr>
<tr>
<td>6</td>
<td>376</td>
<td>0.73</td>
<td>26.0</td>
<td>348.01</td>
<td>3.30</td>
<td>224.07</td>
</tr>
</tbody>
</table>

Fig. (7). - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 1, Series D.

Fig. (8) - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 2, Series D.
Fig. (9) - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 3, Series D.

Fig. (10) - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 4, Series D.

Fig. (11) A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 5, Series D.

Fig. (12) A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 6, Series D.
Table (4). - The results of series (G), (from Al-Saady, 1989).

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\sigma'_e$ kN/m$^2$</th>
<th>$e_o$</th>
<th>$w_c$ %</th>
<th>$(\sigma - \sigma_3)_f$ kN/m$^2$</th>
<th>$\left(\frac{\Delta V}{V_0}\right)_f$ kN/m$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79</td>
<td>0.66</td>
<td>23.5</td>
<td>198.87</td>
<td>2.300</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>0.69</td>
<td>24.7</td>
<td>281.18</td>
<td>2.283</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>0.75</td>
<td>26.0</td>
<td>348.03</td>
<td>3.026</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>0.75</td>
<td>27.0</td>
<td>405.03</td>
<td>3.016</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>0.69</td>
<td>25.2</td>
<td>752.55</td>
<td>3.590</td>
</tr>
<tr>
<td>6</td>
<td>376</td>
<td>0.72</td>
<td>25.0</td>
<td>913.52</td>
<td>3.710</td>
</tr>
</tbody>
</table>

Fig. (13). - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 1, series G.

Fig. (13). - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 2, series G.
Fig. (14). - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test2, series G.

Fig.(15) - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test3, series G.
Fig. (16) - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test4, series G.

Fig. (17) - A comparison between the stress-strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test5, series G.

Fig. (18) - A comparison between the volumetric strain – axial strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test1, series G.
Fig.(19) - A comparison between the volumetric strain – axial strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test2, series G.

Fig.(20) - A comparison between the volumetric strain – axial strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test3, series G.

Fig.(21) - comparisons between the volumetric strain – axial strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 4, series G.
Fig.(22) - A comparison between the volumetric strain – axial strain relationship predicted by the endochronic model with laboratory tests of Al – Saady, Test 5, series G. The same behaviour is noticed in this clay. The predicted volumetric strains are closer to measured strains under small stress increments. At large stresses, the predicted strains became larger.

CONCLUSIONS:
1- The endochronic model overestimates the strains for all the cases simulated under high stress increments.
2- There is no definite yield point can be obtained when simulating the laboratory tests. This means that this model can be adopted for normally consolidated clays where ductile behaviour of the stress-strain is expected.
3- The error in simulation may be attributed to the model parameters, which need to be evaluated by carrying out parametric study for Iraqi clays.

REFERENCES:
NOTATION:

\[ d_{ij} \] Strain increments
\[ dt \] Time increments
\[ P \] Coefficient matrices
\[ J_2 \] second deviatoric strain increment invariant
\[ I_1 \] first strain increment invariant
\[ d\varepsilon_{ij} \] deviatoric strain increment tensor
\[ \delta_{ij} \] Kronecker delta
\[ d\varepsilon \] Volumetric strain increment
\[ z_i, \tau_j \] Constants
\[ d\xi \] damage measure
\[ d\zeta \] deformation measure
\[ d\lambda \] inelastic dilatancy
\[ S_{ij} \] deviatoric stress tensor
\[ \sigma_m \] mean stress
\[ G \] shear elastic moduli
\[ K \] bulk elastic moduli
\[ d\varepsilon^{\prime\prime} \] stress-independent inelastic strains
\[ d\sigma_{ij} \] The stress increments
\[ D_{ijkl} \] elastic coefficient matrix
\[ d\varepsilon_{ij}^{\prime} \] elastic strain increments
\[ f(\eta) \] Strain-hardening function.
\[ F(\sigma,\varepsilon) \] Strain-softening function.
\[ I_1^e \] effective confining stress
\[ I_1^v \] the volume change
\[ J_2^e \] the second deviatoric strain invariant
\[ a's \] material constants
\[ \beta_1 \] constants
\[ c's \] material constants
\[ \beta_2 \] softening coefficient
\[ e_0 \] initial void ratio
\[ \varepsilon_v \] volumetric strain
\[ n \] porosity
\[ b's \] constants
\[ P_0 \] consolidation pressure
\[ C_0 \] densification coefficient
\[ E \] elastic modulus
\[ I_L \] the liquidity index of the clay
\[ w_{ant} \] natural water content.
\[ w_p \] plastic limit
\[ I_p \] plasticity index
\[ w_L \] liquid limit.