FINITE ELEMENT METHOD FOR INCOMPRESSIBLE VISCOELASTIC MATERIALS

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ABSTRACT

A numerical method (F.E.)was derived for incompressible viscoelastic materials, the aging and environmental phenomena especially the temperature effect was considered in this method. A treatment of incompressibility was made for all permissible values of poisons ratio. A mechanical model represents the incompressible viscoelastic materials and so the properties can be derived using the Laplace transformations technique .A comparison was made with the other methods interested with viscoelastic materials by applying the method on a cylinder of viscoelastic material surrounding by a steel casing and subjected to a constant internal pressure, as well as a comparison with another viscoelastic method and for Asphalt Concrete problem exposed to constant pressure (vehicles load) was done.
The obtained results was very convenient , as well as, a large time steps can be taken than others methods.

الخلاصة

تم اشتقاق اسلوب حل عددي (F.E.M) للمواد اللزجة المرنة غير القابلة للانضغاط ،تم إدخال ظاهرة التقادم و الظروف البيئية وخاصة تأثير درجات الحرارة، إضافة إلى معالجة خاصية عدم الانضغاطية لكل القيم المسموحة من نسبة بويسن.تم اعتماد نموذج ميكانيكي يمثل المواد اللزجة المرنة غير القابلة للانضغاط واشتقاق خواص المادة منه باستخدام تقنية لابلاس. تم مقارنة اسلوب الحل العددي مع أساليب حل أخرى للمواد اللزجة-المرنة، وتطبيق ذلك على اسطوانة لزجة-مرنة ذات غلاف خارجي مرن(من الفولاذ) مع وجود ضغط ثابت، إضافة إلى مقارنة مع اسلوب حل لزج-مرن آخر لحالة الإسفلت المتعرض لضغط ثابت (وزن المركبات). أظهرت النتائج تقارب كبير مع باقي الأساليب مع وجود ميزة استخدام خطوات اكبر للزمن.

KEY WORDS/ Viscoelastic, Finite Element, Aging Effect, Incompressibility

INTRODUCTION

The finite element method for many years was used to solve problems depending on the Elastic and Quasi-Elastic theories, few researcher used the finite element depending on the Viscoelastic theories ,but the techniques still in short of the aging phenomena.

In this work the aging factor was applied depended on the temperature effect (physical aging)

For Viscoelastic materials aging may be due to a large number of causes: oxidation (with or without the stimulation of light); gradual loss of plasticizer or other low molecular weight additives [Boyer 1998]. Such chemical or physicochemical degradation processes are not considered in this research and only aging processes of a physical nature will be treated (i.e the type of aging that is due to inherent instability of the amorphous glassy state).

Physical aging is a reversible process in general, i.e. by re-heating the aged material to $T > T_g$ The original state of thermodynamic equilibrium is recovered and a renewed cooling to temperature $T< T_g$ will induce the same aging effects as before.

Temperature effects are extremely important in the analysis of viscoelasticity, temperature has three effects [Oza 2003]:

-temperature change causes thermal strains, which must be combined with mechanical strains,

-material module have different values at different temperatures,

-heat flow may occur.

Williams, Landel and Ferry [David Roylance 2001] have proposed that the variations in relaxation time are not primarily due to thermal activation, but to thermal expansion, i.e. the expansion of free volume V_f with increasing temperatures and by using an equation proposed by Doolittle. These authors derived the famous WLF equation:

$$\log a_T = -\frac{c_1 (T - T_S)}{c_2 + T - T_S} \quad (1)$$

Where Ts is the reference temperature (which represent material's specific constant for the position of the glass transition of the material).

C_1 , C_2 are constants relating to the choice of reference temperature.

Which will be used in the range of glass transition temperatures to coverage the Physical Aging phenomena.

The finite element technique, which was used to calculate displacements and stress for the elastic case, has been extended to provide analysis capability for the viscoelastic case in this research.

Many researcher have been used different methods to calculate displacement ,stress and strain for viscoelastic materials.

(Ghasak 2008) studied the rutting problem for Asphalt concrete which subjected to repeated axle loading using both elastic and viscoelastic model by finite element software (ANSYS 9).He found that the difference between the two approaches (Elastic; Viscoelastic) is about (12%). The rut depth was calculated and compared with two considerable models (Yassoub and Amjad models).

(O.C.Zienckwiecz 1968) developed a completely general method of numerical viscoelastic stress analysis with constant or temperature variable properties. He proved that numerical methods of elastic analysis (and in particular the finite element method) can be extended to deal with wide range of viscoelastic problems of the quasi-static type.

The method is checked against some known solutions. Examples from the field of propellant technology, concrete and rock behaviors are included.

The processes of numerical analysis have been incorporated into two- dimensional finite element analysis program.

An example of propellant is taken as cylinder of viscoelastic material which represents the rocket grain, surrounded by a steel casing and subjected to constant internal pressure P. Contours of maximum compressive stress is plotted at various time steps and the same contours is plotted but with a moving (burning up) inner boundary.

(Lakes, 2006) formulated a viscoelastic problem in a way that allows to the use of higher order differential equation solution techniques. In this literature the advantages of using Runge-Kutta integration formulae are indicated.
Examples for plane strain problem are developed under the assumption of:

-Linear viscoelasticity with a hereditary integral form of the stress – strain relation.
-Validity of the reduced time hypothesis.
-Bulk modulus constant in time.
-Homogeneous, isotropic material.

This formulation has the advantage that the increase in computational effect for each time step using the higher order formulae is, generally, more than offset by the increase in magnitude of the time step that can be used. This advantage is demonstrated with an example. Also with this approach a means of estimating the error involved in the integration is available. The process described in this literature is valid for a more general form of material representation.
An example of reinforced viscoelastic cylinder subject to constant internal pressure P is indicated here by using the fourth order Runge-Kutta method. Distribution of tangential and radial stress is plotted against the ratio of radius to outer radius for several time steps.
(Rogers 1988) solved stress analysis problems for linear Viscoelastic materials on basis of integral operator stress – strain relations by using the method of simple finite – difference numerical integration . They recommend to take the integral from 0 to t and consider the material is undisturbed for t<0 .
(Taylor and Pister 1990) developed a computational algorithm for the solution of uncoupled , quasi – static boundary value problem for a linear Viscoelastic solids undergoing thermal mechanical deformation, they showed that the stresses at a high temperature will decrease faster than at a lower temperature.

In this research a computational method based on finite element technique with using isoparametric element and local coordinate (natural coordinate) will be applied ,viscoelastic solution is obtained using Laplace transform technique.
As an applications of the method , a problem which studied by Zienkiewicz is examined ,as well as, a comparison with another viscoelastic method and for Asphalt Concrete problem exposed to constant pressure (vehicles load) was done, in order to know the efficiency of the procedure and make a comparison with the other methods.

MATHEMATICAL MODEL

Material Representation

For a viscoelastic material , a model can be used to relate components of strain to components of stress.
For incompressible Viscoelastic solid material , the more convenient famous model to represent is called "three parameter model"[Amada 1997], generally, this model used to represent most standard linear Viscoelastic solids as shown in Fig1.

This model which is consistently used in subsequent applications, it is useful to establish systematically its relaxation modules G and creep compliance J using Laplace transform techniques [Gibiansky 1997] as following in **Table 1** :

$$\bar{\varepsilon}(t) = \tilde{J}\bar{\sigma} \quad (2)$$

$$\tilde{J}(s) = \frac{1}{E_1} + \frac{1}{E_2 + s\mu} \quad (3)$$

$$\tilde{G}(s) = \frac{1}{\tilde{J}(s)} = E_1 \frac{s + E_2/\mu}{s + (1/\mu)(E_1 + E_2)} \quad (4)$$

Where: ε -strain σ -stress E -elasticity modulus μ -viscosity *s*- Laplace transform factor.

Applying the inverse Laplace transform and simplifying eqn (3),(4) can be reduced to :

$$J(t) = \left[\frac{E_1 + E_2}{E_1 E_2} - \frac{1}{E_2} EXP\left(-\frac{E_2}{\mu} t \right) \right] \quad (5)$$

$$G(t) = \left[\frac{E_1 E_2}{E_1 + E_2} - E_1 EXP\left(-\frac{E_1 + E_2}{\mu} t \right) \right] \quad (6)$$

METHOD OF SOLUTION

The displacement based finite element method is one such numerical procedure ,the effectiveness of the method is due to its conceptual simplicity, assuming that the nodal point displacement of the finite element mesh completely specify the displacement in the body.

This finite element technique , which has demonstrated to provide an excellent analysis method for elastic case , has been extended to provide analysis capability for the Viscoelastic case in this research .

The relation of stress- strain for plane strain case are [Hughes 1987] :

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - v(\sigma_{yy} + \sigma_{zz})) \quad (7)$$

$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - v(\sigma_{xx} + \sigma_{zz})) \quad (8)$$

$$\varepsilon_{xy} = \frac{2(1+v)}{E}\sigma_{xy} \quad (9)$$

But :

$$\upsilon = \frac{1 - \frac{2G}{K}}{2 + \frac{2G}{K}} \quad (10)$$

$$E = \frac{9K}{1 + \frac{3K}{G}} \quad (11)$$

Then the stress matrix {D}(matrix of properties) can be obtained from eqns (7), (8),(9) in term of relaxation G and bulk K moduli.

$$[\sigma] = [D]\ [\varepsilon] \quad (12)$$

$$[D] = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & 0 \\ 0 & 0 & G \end{bmatrix} \quad (13)$$

The global coordinate {X} of the node in terms of local coordinate (ξ, η) and displacement field $\{\delta\}$ in isoparametric element is [Zienkiewcz 1989] :

$$\{X\} = [N]\ \{X_{ii}\} = \begin{vmatrix} x(\xi,\eta) \\ y(\xi,\eta) \end{vmatrix} \quad (14)$$

$$\{\delta\} = [N]\ \{\delta_i\} = \sum_{i=1}^{n} \delta_i N_i \quad (15)$$

{N} is a matrix of shape function, which is a function of local coordinate ξ and η.

By differentiation of shape function with respect to global coordinate we can obtain strain quantities. This can be done by a transformation using Jacobian matrix{J} which can be obtained by differentiate Eqn 14 using chain rule.

$$\{J\} = \sum_{i=1}^{n} \begin{pmatrix} \frac{\partial Ni}{\partial \eta} xi & \frac{\partial Ni}{\partial \eta} yi \\ \frac{\partial Ni}{\partial \xi} xi & \frac{\partial Ni}{\partial \xi} yi \end{pmatrix} \quad (16)$$

Then local coordinates can be obtained as:

$$[J]^{-1} \begin{pmatrix} d\xi \\ d\eta \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} \quad (17)$$

For plane strain case the relation between strain and displacement is [Hughes 1987]

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \quad (18)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \quad (19)$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad (20)$$

Then the strain matrix {B} is obtained by writing eqns (18),(19),(20) in terms of matrix notation and using the following relations [Saabye 2000]:

$$\frac{\partial u}{\partial x} = \sum_{i=1}^{n} \frac{\partial Ni}{\partial x} ui \qquad (21)$$

$$\frac{\partial v}{\partial y} = \sum_{i=1}^{n} \frac{\partial Ni}{\partial y} vi \qquad (22)$$

$$[B] = \begin{bmatrix} \frac{\partial Ni}{\partial x} & o \\ o & \frac{\partial Ni}{\partial y} \\ \frac{\partial Ni}{\partial y} & \frac{\partial Ni}{\partial x} \end{bmatrix} \qquad (23)$$

It is incorrect to vary only stress matrix {D} with time (the Quasi – static solution) since properties of viscoelastic material varies with time, but it is convenient to differentiate this matrix with respect to time depending on the superposition theory of linear viscoelasticity :

So that :

$$[\frac{\partial D}{\partial t'}] = -\frac{\partial G(\tau - \tau')}{\partial t'} \begin{bmatrix} \frac{4}{3} & -\frac{2}{3} & 0 \\ -\frac{2}{3} & \frac{4}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \{\bar{D}\}$$

(24)

$\tau - \tau'$ is the current and past shifted time respectively which can be calculated from WLF eqn No.1.

From the chosen model in Fig.1 and for the incompressible linear viscoelastic material undergoes environmental temperature change, the total stress will expected to be as:

$$\sigma_{total} = \sigma_{elastic} + \sigma_{viscoelastic} + \sigma_{thermal}$$

(25)

So that:

$$\{\sigma(t)\} = [D]\{\varepsilon(t)\} + \int_0^t [\bar{D}]\{\varepsilon(t)\}dt - 3\alpha K[T(x_1 t) - T(x_1 0)]$$

(26)

α - thermal expansion which is constant in time .

By minimizing the equation of potential energy we can solve Eqn.- 26

The minimum potential energy M can be expressed as [Bath 1995]:

$$M = \frac{1}{2}\int_v [\sigma(t)]\,\varepsilon^T(t)dv - \int_v [\delta]^T Fv\, dv - \int_s [\delta]^T Fs\, ds$$

(27)

F_v : is the body force per unit volume

F_s : is the load of surface traction

By substituting Eqns 26 , 14 into Eqn 27 and minimization with respect to nodal displacements the total potential energy can be written as :

$$\frac{\partial M}{\partial[\delta^e]^T} = 0 = \int_{ve} B^T DBdv\{\delta^e\} + \int_{ve} B^T \bar{D}\left[\int_0^t \{\varepsilon(t)\}dt\right]dv - \int_{ve} N^T Fvdv - \int_{se} N^T Fsds - 3K\alpha\int_{ve} B^T(T(X,t) - T(X,0))dv$$

Solving Eqn 25 will give the values of displacements for all nodes in the structure of interest.

Then stresses can be obtained by solving Eqn 26.

For incompressibility conditions it is more convenient to separate the stress matrix {D} into two components (shear and bulk) [Amada 1997] as:

$$[D] = [D]^s + [D]^b \qquad (29)$$

And by applying a selective integration procedure [Saabye 2000] , which is third order Gauss rule for shear components and second order Gauss rule for bulk components.

This will make some equilibrium between shear and bulk components.

RESULTS AND DISCUSSION

The first step is obviously to test the rate of convergence and the other features of the process.The process of numerical analysis described in this research is applied into two problems which was solved by Zienkiewicz and Ghasak respectively.

The first problem shown in Fig 2 is a cylinder of Viscoelastic material surrounded by a case of steel and subjected to an internal pressure suddenly applied at t = 0 and maintained thereafter at a magnitude P_o.

The Viscoelastic material is assumed to be isotropic with the following properties [Zienckwiecz 1968]:

$$\frac{1}{Kcreep} = 0$$

$G_o = 2584.125 * 10^5$ N/m^2

K= $6891 * 10^5$ N/m^2

$G(t) = 2584.125*10^5 + 3*10^5 e^{-0.57\tau}$

T_s=75 C

The properties of steel case is taken as :

E= 206.73 GPa

$\nu = 0.3015$

The results obtained by applying the method of Viscoelastic technique which compared with the solution by Zienckwiecz as shown in Figs 3 and 4, where the variation of radial and circumferential stresses with time is shown.

There are very small differences from the values of the solution by Zienkiewicz.

The points from finite element solution are obtained by averaging stresses across the element boundaries .

The curves presented are obtained by taking a time step of Δ t = 0.5

It can be shown that the main computational advantage of this method over others lies in the fact that larger time steps can be taken. For example in the Quasi-Elastic solution (Zienkiewicz 1968) to obtain the curve in Fig3 at τ = 3,a thirty time steps is used, and this required thirty solutions of a set of equations, the same curve is obtained by the method of this research using six time steps, as well as, the method can cover the environmental phenomenon like aging and temperature effects.

The second problem shown in Fig 5 is a pavement subjected to constant pressure load with **tire print diameter=300 mm(actual contact area) and tire load=80 KN**, **Pressure=550 KPa** for the Single tire.

Data input in software of Asphalt concrete are exhibit in Table (3), the elastic solution is used for subbase and subgrade layers except asphalt material will be treat as a viscoelastic material **.**

The elastic properties for Subbase is E=350 MPa, υ=0.3and Subgrade is E=100 MPa υ=0.4 respectively, The mesh is shown in Figure 4 (1518 element-4-node).The results of rutting vs. number of load repetitions are shown in Figure 6

Figure 7 shows the comparison of rut depth for various number of axle load repetitions between the proposed technique and Ghasak method (Ghasak 2008). It can be seen that generally there is a small difference between the two techniques ranges from 5% to 7%.

Also this finite element model can be used for both the thermal and stress analysis (thermo-mechanical analysis) , the both thermal and force equilibrium are satisfied in each increment before the analysis proceeds to the next increment.

To capture the transient phenomenon for temperature displacements and applied loads , the time steps was taken small enough.

Using of the shifted time τ in Viscoelastic solution enables us to include the thermal effect by using WLF equation , as well as, using isoparametric element with local coordinates (ξ, η) enable us to use an element with curvilinear shape and cover the change in displacements with time.

The problem of incompressibility is distinguished by testing the ratio of bulk to shear modulus as following

$$\frac{Bulk\ \ mudulus(K)}{Shear\ \ mudulus(G)} = \frac{2(1+\nu)}{3(1-2\nu)}$$

For incompressible material ν approach 0.5 and bulk modulus becomes large

relative to shear modulus. It is note that the use of these values in the finite element codes have not been tailored for incompressibility analysis and lead to very serious numerical errors caused by the ill-conditioning resulting from the division by a value which is nearly zero, and more importantly," mesh locking" may occur ,this refers to the inability to of an element to perform accurately in an incompressible analysis, regardless how refined the mesh is due to an over-constrained condition and insufficient active degree of freedom.
It is noted that the element lock despite the fact that its area has remained constant ,resulting in the prediction of too small of a displacement and too large of stress.
Using of selective integration and separation of bulk from shear components will improve the values of results for all permissible values of Poisson's ratio(ν).

CONCLUSIONS:

Within the limitations of the present work and depending on the results of applying the proposed techniques the following conclusions can be inferred :

- It can be use more time steps with an accurate results in this procedure compare with other
method.
- It can coverage the aging phenomena and temperature effects.
- It can coverage the incompressibility phenomena and make a solution for it.
- It can be extend the procedure for most types of viscoelastic materials (compressible, incompressible, linear, non-linear ,…..etc.)

- It is recommended to extend the procedure for non-linear viscoelastic materials and using the procedure for rubber like materials.

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NOMENCLATURE

Symbol	Definition	Units
a_T	WLF shift factor	-
C_1	WLF eqn. constant	-
C_2	WLF eqn. constant	^{o}c
E	Elasticity modulus	N/m^2
G	Relaxation modulus	N/m^2
J	Creep compliance	m^2/N
K	Bulk modulus	N/m^2
P	Pressure	N/m^2
T_s	Reference temperature	^{o}c
t	Current time	hr
u	Horizontal displacement	mm
v	Vertical displacement	m

Greeks letters	Definition	Units
ε	Strain	m/m
σ	Stress	N/m^2
μ	Viscosity	$N.hr/m^2$
ρ	Density	kg/m^3
τ	Current shifted time	hr.
υ	Poisson ratio	-
ξ	Local horizontal coordinate	m
η	Local vertical coordinate	m

Matrices	Definition
[B]	Strain matrix
[δ]	Displacements matrix
[D]	Stress matrix
[F]	Elastic load vector
[J]	Jacobian matrix
[N]	Shape function matrix
[T]	Thermal load vector
[X]	Coordinate matrixP

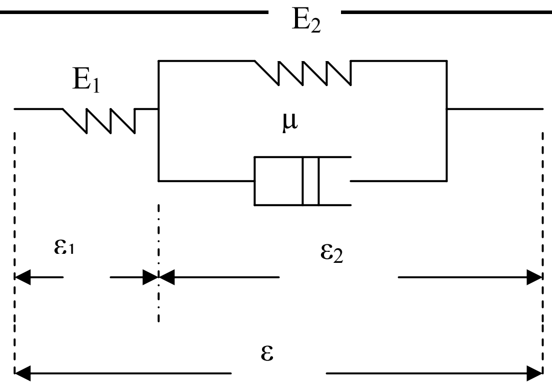


Fig 1. The three parameter model

Table1 : The Laplace transform technique

Constitutive equation.	Laplace transform
$\varepsilon_1 = \frac{\sigma}{E_1}$	$\bar{\varepsilon}_1 = \frac{\bar{\sigma}}{E_1}$
$\varepsilon_2 = \frac{\sigma'}{E_2}; \mu \frac{d\varepsilon_2}{dt} = \sigma''$	$\bar{\varepsilon}_2 = \frac{\bar{\sigma}'}{E_2}; s\mu\ \bar{\varepsilon}_2 = \bar{\sigma}''$
$\sigma' + \sigma'' = \sigma$	$\bar{\sigma}' + \bar{\sigma}'' = \bar{\sigma}$
$E_2\varepsilon_2 + \mu \frac{d\varepsilon_2}{dt} = \sigma$	$(E_2 + s\mu)\bar{\varepsilon}_2 = \bar{\sigma}$
$\varepsilon_1 + \varepsilon_2 = \varepsilon$	$\bar{\sigma}\left(\frac{1}{E_1} + \frac{1}{E_2 + s\mu}\right) = \bar{\varepsilon}$

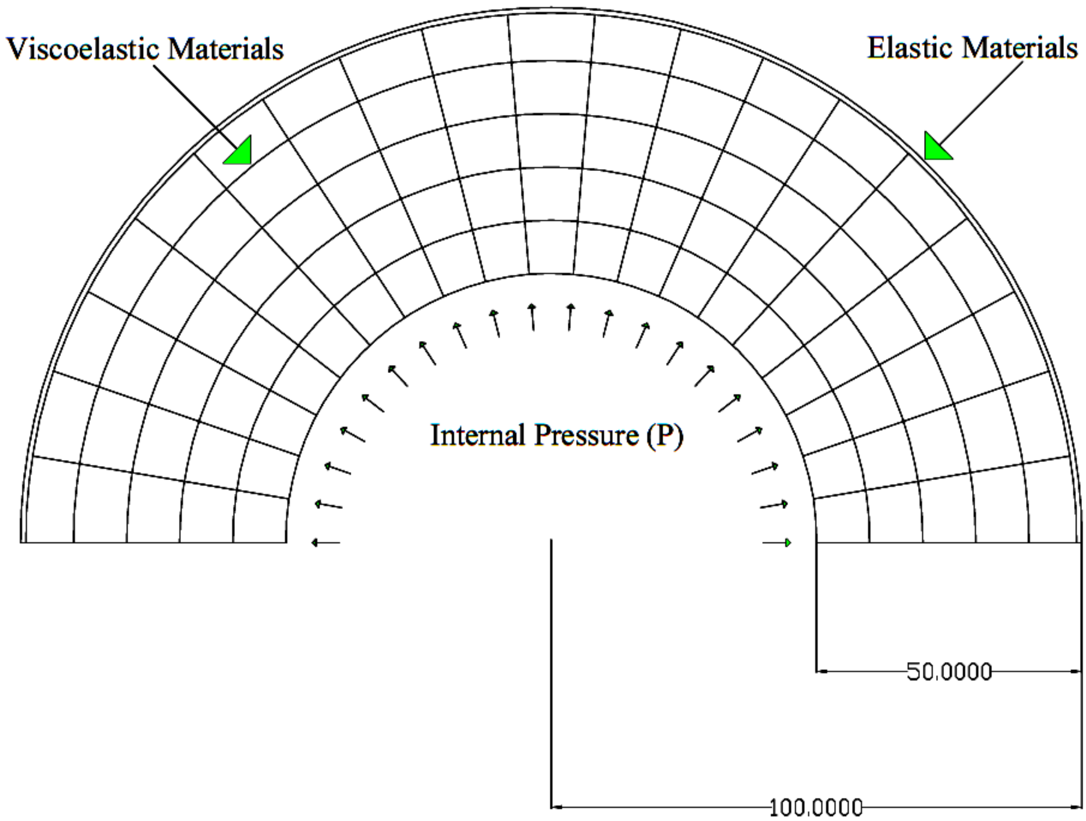


Fig 2: Viscoelastic cylinder surrounded by Elastic metal.

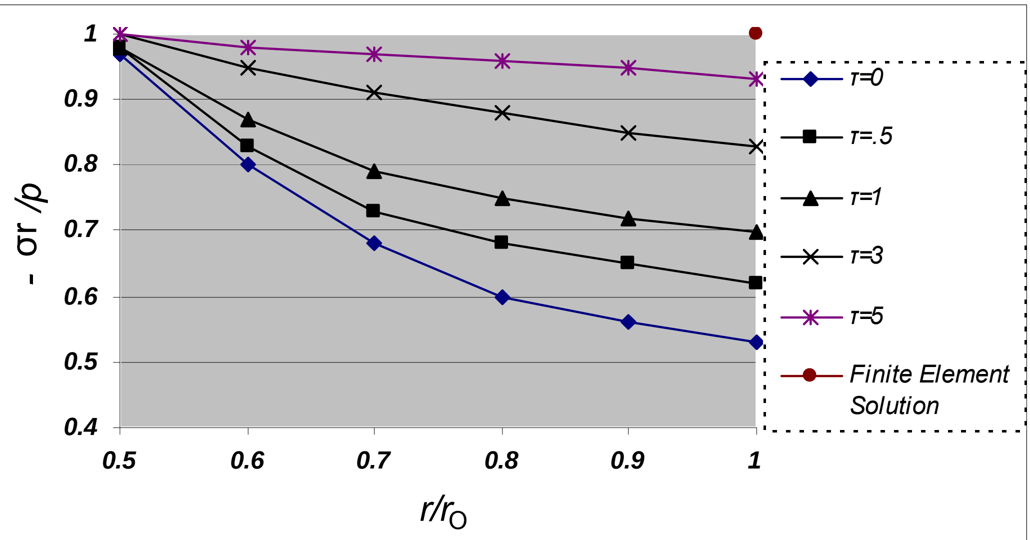
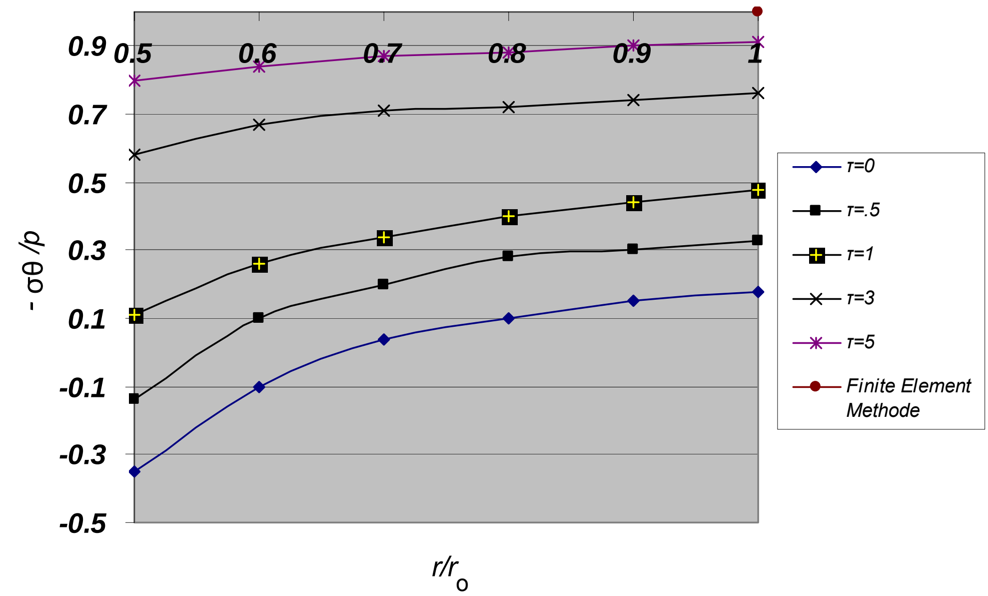


Fig3 Reinforced Cylinder Under Internal Pressure. Variation of Radial Stress



___Solution by Zienckwick

Fig4 Reinforced Cylinder Under Internal Pressure. Variation of Tangential Stress.

Table 2 : Comparison of results with the solution by Zienckwiecz at $\tau = 1$

r/r_o	Viscoelastic Solution FEM $-\frac{\sigma_r}{P}$	Solution by Zienkwiecz $-\frac{\sigma_r}{P}$	Viscoelastic Solution FEM $-\frac{\sigma_\theta}{P}$	Solution by Zienkwiecz $-\frac{\sigma_\theta}{P}$
0.5	0.96	0.97	0.12	0.125
0.6	0.87	0.875	0.24	0.247
0.7	0.79	0.80	0.33	0.34
0.8	0.77	0.78	0.41	0.415
0.9	0.75	0.756	0.45	0.452
1	0.74	0.748	0.48	0.49

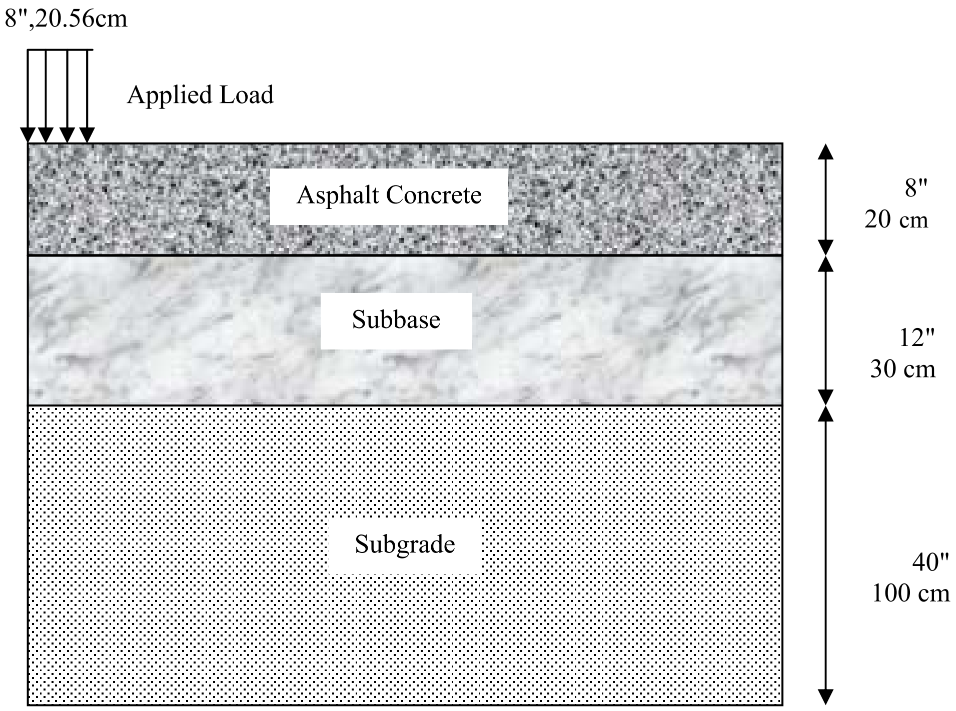


Fig. 5 : Pavement Layer configuration .

Table 2: Viscoelastic Material Properties for Asphalt Layer under Temperature T_s= 23 C ^{0}(Ghasak 2008).

Time of loading	Creep compliance J(t) (1/MPa)	Relaxation modulus G(t) (MPa)	Bulk modulus K(t)MPa
0.1	0.0048	208.43	231.6
0.25	0.0063	158.1	175.7
0.5	0.0084	118.12	131.25
1	0.0092	108.47	120.5
2	0.01	94.63	105.14
4	0.012	83.5	92.78
8	0.0132	75.5	83.9
15	0.017	58.37	64.9
30	0.0198	50.34	55.9
45	0.021	46.75	52

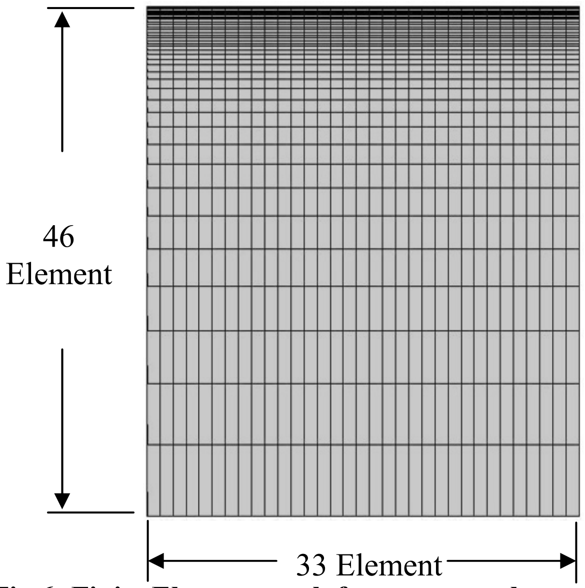


Fig 6 :Finite Element mesh for pavement layers configuration.

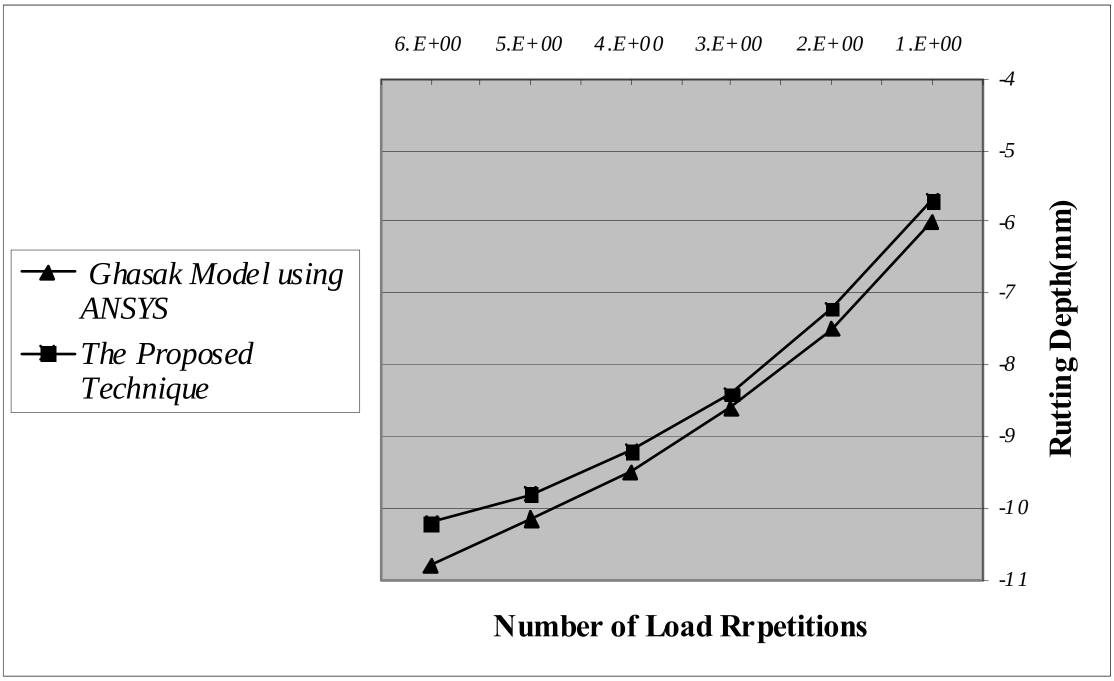
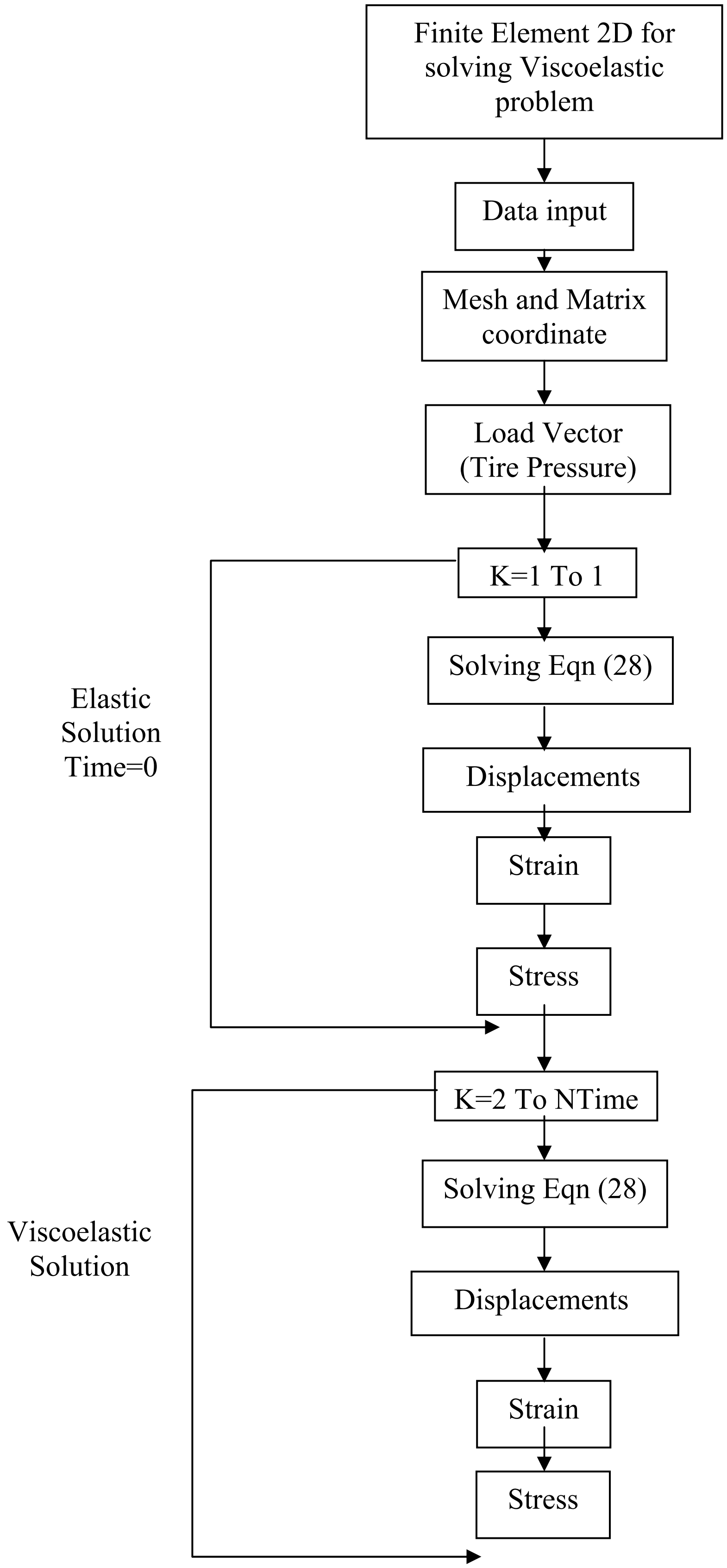


Fig 7 : Comparison between the Proposed procedure and Ghasak method.

APPENDIX



The Flow Chart