FREE VIBRATION OF BEAM ELASTICALLY RESTRAINED AGAINST TRANSLATION AND ROTATION AT ENDS

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ABSTRACT

An approximate solution of the vibration of an elastically restrained, uniform and non-uniform beams with translational and rotational springs is obtained using Rayleigh-Ritz approach. The frequencies are presented for wide range of restrained parameters and some of these have been compared with those available in the published literature illustrating the accuracy and versatility of the approach. It is believed that the results present in this paper will be of use in design of beams, shaft, and piping under dynamics consideration.

الخلاصة

في هذا البحث تم دراسة الاهتزاز الحر للعتبات ذات ظروف الإسناد المرنة حيث استخدمت طريقه الحل التقريبي (Rayleigh-Ritz) . الإسناد المرن الذي تمنت دراسته هو الانتقالي والد وراني للعتبات ذات المقطع العرضي المنتظم وغير المنتظمة المقطع العرضي. النتائج التي حصل عليها لمدى واسع من معامل مرونة الإسناد الخطب والد وراني وبعد المقارنة مع نتائج البحوث المنشوره أثبتت دقة هذه ألطريقه وكفاءتها. كما بينت النتائج التي تم الحصول عليها في البحث امكانية ووثوقية استخدام طريقة (Rayleigh-Ritz) في تصميم الأعمدة والمساند والأنابيب الواقعة تحت تأثير الأحمال الديناميكية.

KEY WARDS

Free vibration, beam, shaft, frequency parameter, elastically restrained, Rayleigh-Ritz

INTRODUCTION

There is a large number of technical studies dealing with vibrating beam taking into account several complicating effects such as axial forces, elastic constraints, variable cross-section rotary inertia of the tip mass.

Maurizi, Rossi and Reyes(1976) studied the problem of free vibration of a beam supported by a rotational spring at one end and having a translational spring at the other end. Sundarajan(1979) derived a simple algebraic expression for un upper bound to the fundamental frequency of beams with unsymmetrical springs.

Freeman and Soreensen(1980) have studied the response of elastically restrained cantilever Bernoulli-Euler beams and presented exact frequencies and mode shapes for the first four modes of vibration for a number of restraint parameters. In an attempt to estimate the fundamental frequencies of vibration fuel rods, Passig(1970) derived an exact frequency equation for a beam supported by symmetrical springs at either end of the beam. Abbas(1984) has studies the problem of elastically restrant Timoshenko beams and presents some results for degenerate case of Bernoulli-Euler beam.

In the present paper the problem of free vibration of partially restrained Bernoulli-Euler is solved by using Rayleigh-Ritz approach. A simple algorithmic procedure is applied for determining the frequency parameter of the restraint beam.

APPROXIMATE SOLUTIN

The classical application of the Rayleigh-Ritz method is based upon the selection of a deflection function which is a specific function of the position .
However, as was a first shown by Rayleigh one can use a polynomial function with undefined parameters. The strain .energy of the structural system under consideration **Fig. (1).**

$$U_{max} = \tfrac{1}{2} E \int_0^{\ell} I(x) \left(\frac{\partial^2 W}{\partial x^2} \right)^2 dx + \tfrac{1}{2} \phi_r \left(\frac{\partial W}{\partial x} \right)^2 + \tfrac{1}{2} K W^2 \Bigg|_{x=x1}^{x=x2} \qquad (1a)$$

be equal to its maximum kinetic energy

$$V = \tfrac{1}{2} \rho \omega^2 \int_0^{\ell} A(x) W^2 dx \qquad (1b)$$

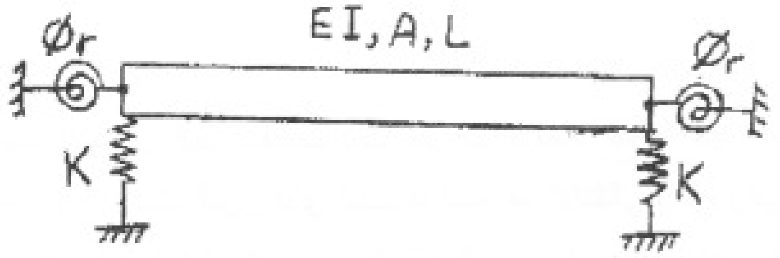


Fig.(1) The structural element under investigation, showing the support system.

Where E is the Young's modulus I(x) is the variable moment of inertia, A(x) is the variable cross-section area, ϕ_r and K are the coefficient of rotational and translational springs. The boundary conditions for the beam can be written at x=0

$$EI \frac{\partial^3 W(0,t)}{\partial x^3} = -KW(0,t)$$
$$EI \frac{\partial^2 W(0,t)}{\partial x^2} = \phi_r \frac{\partial W(0,t)}{\partial x} \qquad (2)$$

and at x=L as:-

$$EI \frac{\partial^3 W(0,t)}{\partial x^3} = KW(L,t)$$
$$EI \frac{\partial^2 W(0,t)}{\partial x^2} = -\phi_r \frac{\partial W(L,t)}{\partial x} \qquad (3)$$

The displacement deflection shape can now be approximate by means of polynomial

$$W \approx W(x,t) = (1 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4) \left(\sum_{n=0}^{N} A_n X^n \right) e^{\omega t} \qquad (4)$$

Where α 's are determined from the boundary conditions of equations (2) and (3). Substitution of expression (4) into equation (2) and differentiated with respect to the undefined constant A's would yield a characteristics equation of the form

$$[K - \omega^2 M]\{A\} = 0 \tag{5}$$

In which K and M are the stiffness and mass matrices of the structure, and ω is the circular frequency.

NUMERICAL RESULTS AND DISSCUSSION

The first case studied is that of uniform cantilever beam (clamped at x=0) with rotational spring supported located at the free end of the beam. The first five frequency parameters are shown in **Table(1)** and compared with the exact values from the work of Lau[1], The table shows excellent agreement with exact results .

The second case studied is that of determination of the natural frequency parameter for an uniform, single span, spring hinged beam.

Table (1) Comparison of frequency parameters $(\omega^2 ml^4 / EI)^{1/2}$ for a uniform cantilever beam with translational spring and a rotational spring at the same point (K=ϕ=10).

x/l	Mode Sequence Number				
	1	2	3	4	5
0.2	2.136 (2.1268)	5.125 (5.0431)	8.211 (8.0236)	11 . 1 32 (11 ,002)	14.431 (14.206)
0 . 6	2.786 (2.7462)	5.032 (4.9737)	8.082 (8.0698)	11 . 356 (11 .283)	14,. 165 (14. 146)
1.0	2.744 (2.7146)	5.402 (5.3348)	8.412 (8.6607)	11.512 (11.4375)	14.532 (14.5271)

Numbers in parentheses are exact values.

Table (2) Comparison of frequency parameters $(\omega^2 ml^4 / EI)^{1/2}$ for a uniform cantilever beam with translational spring and a rotational spring at the same point (K=ϕ=10).

	Mode Sequence Number				
	1	2	3	4	5
10,10	17.273 (17.2 6 9)	49.682 (49.601)	101 .891 (101.318)	172.025 (171.748)	262.124 (261.527)
100,100	19.266 (19.272)	54.961 (54.509)	110.783 (108.773)	182.642 (182.180)	275.430 (274.921)

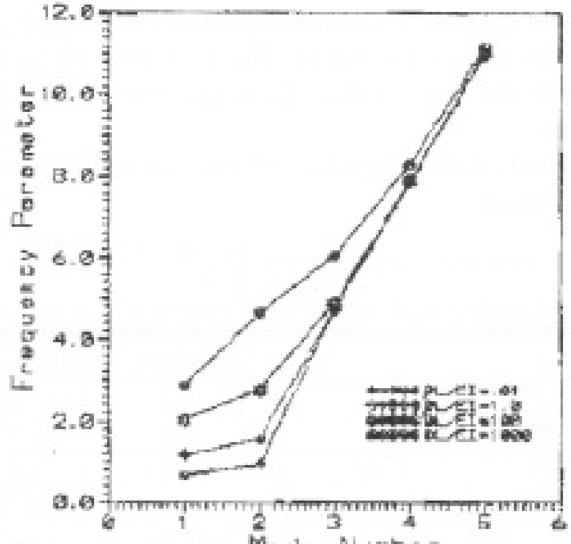
Numbers in parentheses are exact values

Beam where both ends elastically restrained against rotational and fully translationally restrained. The results of the frequency parameters for two combinations of rotational stiffness are considered together with exact results computed by Gorman[8] as shown in **Table (2)**. Again close agreement with exact results is achieved.

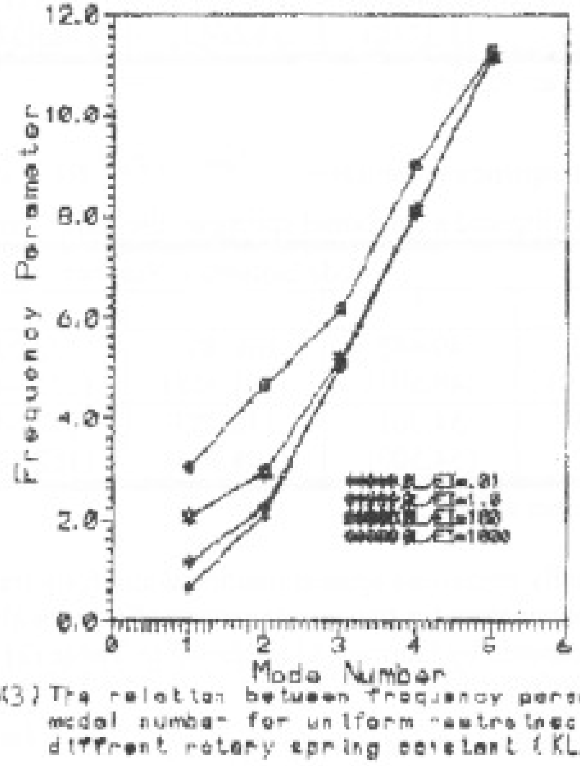
Figs. (2,3,4 and 5) show the relation between the frequency parameter $(\omega^2 ml^4 / EI)^{1/2}$ and the mode number under different values rotational spring constant for uniform single span beam.

Figs. (6,7,8 and 9) show the relation between the frequency parameter $(\omega^2 ml^4 / EI)^{1/2}$ and the mode number under different values off rotational spring constant for three span beam, which is symmetrically parabolas tapered in both width and depth where the central dimensions one half of end dimension and has both end translationally and rotationally spring supported.

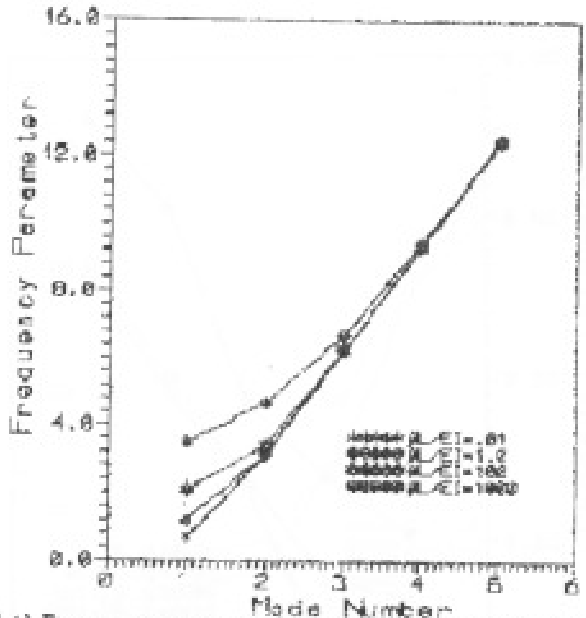
It can seen from the figures presented in this paper that both the translational and rotational spring constants have significant effect on thee lower mode of vibration. This study also, shows that in general the translational spring constant has relatively greater effect on the frequencies than the rotational spring constant.



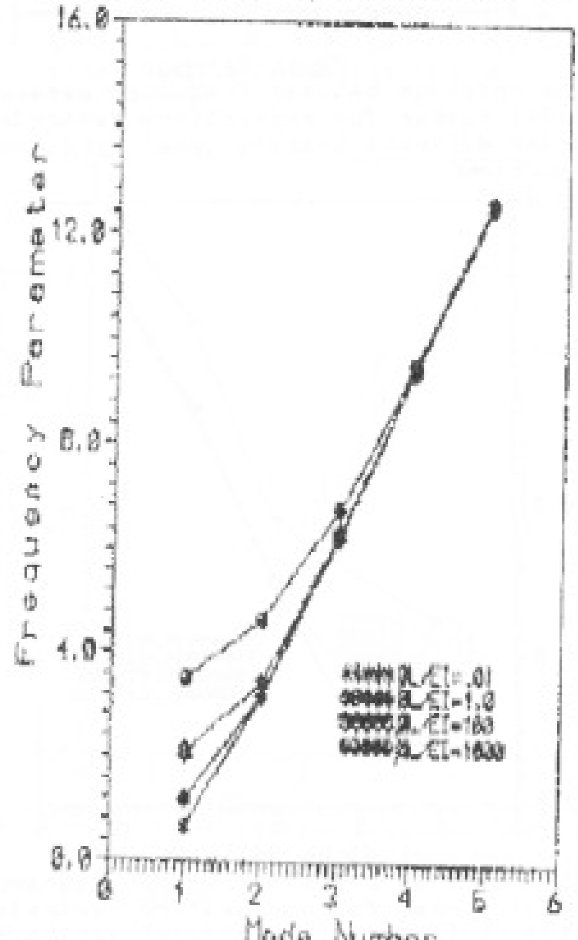
Figure(2) The relation between frequency parameter and model number for uniform restrained beam under diffrent rotary spring constant (KL/EI=0.1)



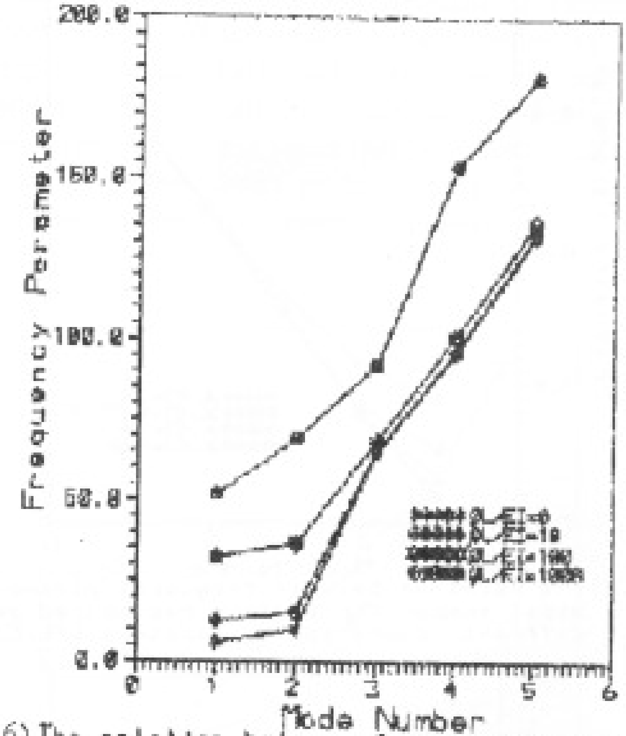
Figure(3) The relation between frequency parameter and model number for uniform restrained beam under diffrent rotary spring constant (KL/EI=1.0)



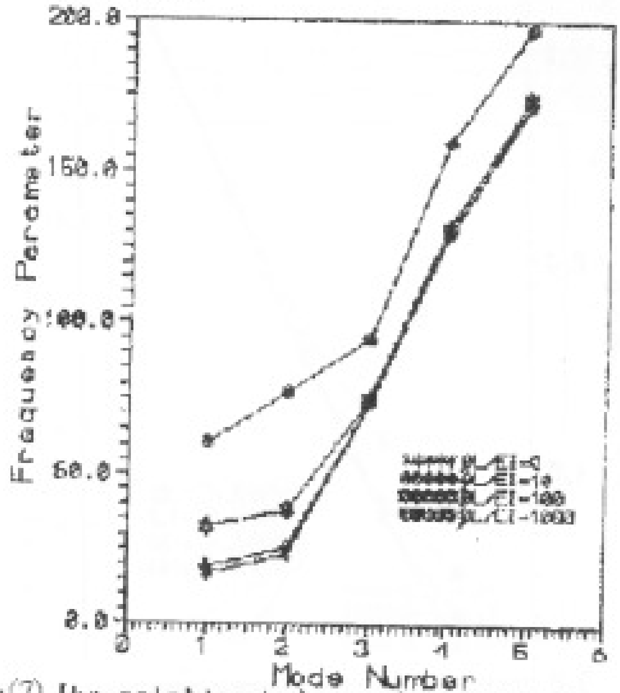
Figure(4) The relation between frequency parameter and nodal number for uniform restrained beam under diffrent rotary spring constant (KL/EI=100)



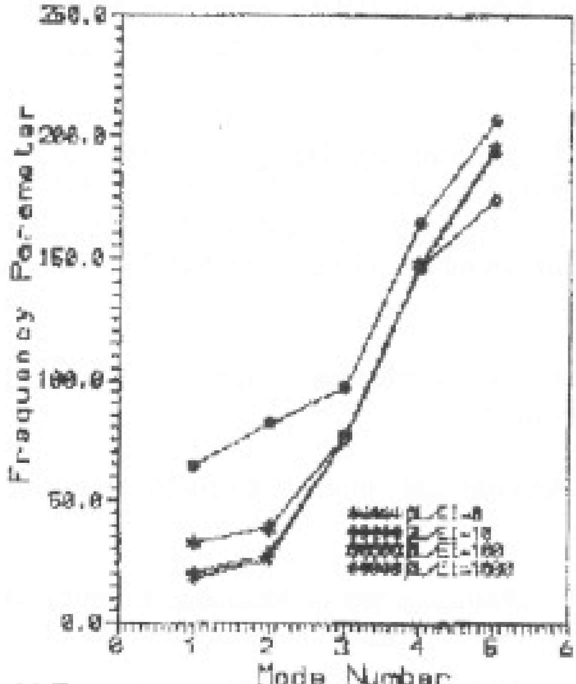
Figure(5) The relation between frequency peremeter and model number for uniform restrained beam under diffrent rotary spring constant (KL/EI=1000)



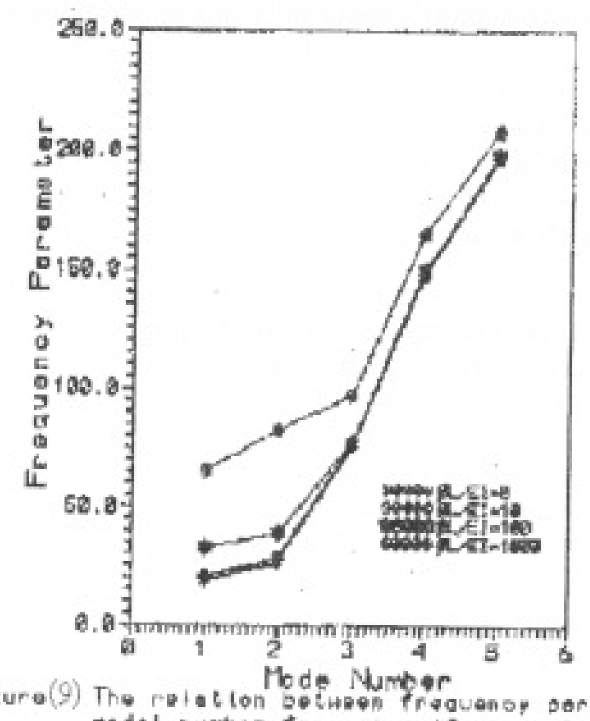
Figure(6) The relation between frequency parameter and modal number for non-uniform restrained beam under diffrent translational spring constant (KL/EI=0)



Figure(7) The relation between frequency parameter and modal number for non-uniform restrained beam under diffrent translational spring constant (KL/EI=10).



Figure(8) The relation between frequency parameter and modal number for non-uniform restrained beam under diffrent translational spring constant (KL/EI=100)



Figure(9) The relation between frequency parameter and modal number for non-uniform restrained beam under diffrent translational spring constant (KL/EI=1000).

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NOMENCLATURE

I(x)- variable moment of inertia (m^4)
A(x)- variable cross-section area (m^2)
L- beam's length (m)
E- Young's modulus of elasticity (N/m^2)
ρ - density of the beam (kg/m^3)

ϕ_r and K - are the coefficient of rotational and translational springs.
K and M- are the stiffness and mass matrices of the structure
ω . is the circular frequency (rad/s).