DAMAGE DETECTION AND LOCATION FOR IN AND OUT-OF-PLANE CURVED BEAMS USING FUZZY LOGIC BASED ON FREQUENCY DIFFERENCE

Asst. Prof. Dr. Nabil H. Hadi 1*and M. Sc. Hayder S. Ashoor 2**
*University of Baghdad / College of Engineering / Mechanical Engineering Department
**University of Baghdad / College of Engineering / Mechanical Engineering Department

ABSTRACT

In this study, structures damage identification method based on changes in the dynamic characteristics (frequencies) of the structure are examined, stiffness as well as mass matrices of the curved (in and out-of-plane vibration) beam elements is formulated using Hamilton's principle. Each node of both of them possesses seven degrees of freedom including the warping degree of freedom. The curved beam element had been derived based on the Kang and Yoo's thin-walled curved beam theory in 1994. A computer program was developing to carry out free vibration analyses of the curved beam as well as straight beam. Comparing with the frequencies for other researchers using the general purpose program MATLAB. Fuzzy logic system (FLS) applied in two stages to calculate the damage extent and location in simply in and out-of- plane curved beam, the damage deduce by reduction in stiffness for three levels (20%, 40%, 60%). At the first stage the output faults of the fuzzy system represented by four levels of damage in curved beam (undamaged, slight, moderate, and severe), and at second stage indicate damage location at element with two defuzzification methods (centroid and middle of maximum).

The results show that the frequency difference method is efficient to indicate and quantify damage with accuracy about (99.5%) for slight and moderate damage about (100%) for severe damage. Consequently fuzzy logic performs well for detecting, locating and quantifying damage in curved beam.

الخلاصة

في هذا البحث يتم التعرف على الضرر في الهياكل من خلال فحص التغيير في الصفات الديناميكية (الترددات) للهيكل،تم صياغة مصفوفتي الجساءة والكتلة لعنصر العتبة المقوسة (بالاتجاه الموازي والعمود على المستوي) باستعمال مبدأ هاملتون. كل عقدة تحوي على سبع درجات من الحرية مع الاخذ بنظر الاعتبار الاعوجاج (warping).أشتق عنصر العتبة المقوسة بالإعتماد على نظرية كانك وياو (1994) للعتبة المقوسة ذات الجدران الرقيقة، وطور برنامج ماتلاب (MATLAB) ليستعمل للعتبة المقوسة كذلك المستقيمة مع مقارنة الترددات المستخرجة مع باحثين اخرين. طبق نظام المنطق الضبابي(Fuzzy logic system) على مرحلتين لحساب كمية الضرر وموقعه،يتم الاستدلال على الضرر من خلال عمل تخفيض في جساءة الهيكل وبثلاث مستويات (20%،40%،60%). في المرحلة الاولى نوع الضرر المستخرج من نظام المنطق الضبابي عبارة عن اربع مستويات من الضرر(غير متضرر،خفيف،متوسط و عالي)، اما في المرحلة الثانية فالقيم المستخرجة من النظام تحدد موقع الضرر في اي عنصر من العتبة وباستعمال طريقتان (centroid and middle of maximum).
النتائج المستخرجة اوضحت كفاءة طريقة الفرق في التردد في تحديد كمية وموقع الضرر وبدقة بحدود (99.5%) بالنسبة للضرر الخفيف والمتوسط وبحدود (100%) بالنسبة للضرر العالي. وبالنتيجة فأن نظام المنطق الضبابي أنجز وبشكل جيد في تحديد كمية وموقع الضرر في العتبة المقوسة.

Keywords: curved beam; fuzzy logic; damage detection

1. INTRODUCTION

It is easily accepted that when damage occurs, a structure would suffer a decrease in stiffness. And as a consequence, there was a decrease in natural frequencies of vibration. For a beam structure a loss in stiffness would imply an increase in curvature of the elastica which can be used for damage detection **[1]**. In the most general terms damage can be defined as changes introduced into a system that adversely affect the current or future performance of that system. Implicit in this definition is the concept that damage is not meaningful without a comparison between two different states of the system, one of which is assumed to represent the initial, and often undamaged, state. Structural damage identification based on change dynamic characteristics provides a global way to evaluate the structural condition. These methods are based on the premise that modal parameters (i.e., natural frequencies, mode shapes, modal damping ratios, etc.) are a function of the physical properties of the structure (stiffness, damping, mass and boundary conditions). The approach is based on the fact that natural frequencies are sensitive indicators of structural integrity. Thus, an analysis of periodical frequency measurements can be used to monitor structural condition. Since frequency measurements can be cheaply acquired, the approach could provide an inexpensive structural assessment technique. **Ju and Mimovich [2]** used changes in modal frequencies to locate damage occurring at sections of a beam to within 3% of the length. It was found that the accuracy of the damage localization was improved to less than 1% of the length when the built-in end of the experimental beam was represented by a torsion spring. **Cawley and Adams [3]** study the sensitivity concept and it is based on the premise that the ratio of frequency changes in two modes is a function of the location of the damage only, if changes in stiffness are independent of frequency. To locate the defect, theoretical frequency shifts, due to damage at selected positions on the structure, are calculated and compared with measured values. **Uzgider and Sanli [4]** proposed a damage location method which uses measured natural frequencies to identify stiffness parameters. The natural frequencies of the selected modes are then used to identify the stiffness parameters. The relative magnitudes of the differences between the identified parameters and prior estimates are used to indicate the presence of structural damage. **H. R. Öz and M. T. Das [5]** studied the in- plane vibrations of cracked circular curved beams, the beam is an Euler-Bernoulli beam. Only bending and extension effects are included, the curvature was in a single plane. An in-plane vibration is analyzed using FEM. In the analysis, elongation, bending and rotary inertia effects are included, four degrees of freedom for in-plane vibrations is assumed. Increasing the crack depth decreases the frequencies. At the recent years many researchers used artificial intelligence likes "neural network, genetic algorithm, fuzzy logic" to detect damage for structure. Fuzzy logic systems have been widely used in engineering applications; because of the flexibility they offer designers and their ability to handle uncertainty and has a natural way of dealing with paradoxes **[6]** another important feature is that fuzzy behavior was shown to produce good results, even in cases with incompletely defined dependencies **[7].** Finally, an important advantage of a fuzzy system over a “classic” expert system is that a fuzzy system usually has significantly fewer rules **[8],** so we take fuzzy logic in this study as the all below researchers use it. **RANJAN GANGULI [9]** study the rotor blade which modeled as an elastic beam undergoing transverse (flap) and in plane (lag) bending, axial and torsion deformations. A finite element model of the rotor blade is used to calculate the change in blade frequencies (both rotating and no rotating) because of structural damage. The measurement deviations due to damage are then fuzzified and mapped to a set of faults using a fuzzy logic system. **Prashant M and Ranjan [10]** propose a genetic fuzzy system used to find the location and extent of damage. A finite element model of a cantilever beam is used to calculate the change in beam frequencies. The genetic fuzzy logic system in this study is proposed as a method for automatic rule generation in fuzzy systems for structural damage detection. **M. Chandrashekhar, Ranjan Ganguli [11]** a fuzzy logic system (FLS) with a new sliding window defuzzifier is developed for damage detection. The effect of changes in the damage evaluation parameter (frequency) due to uncertainty in material properties is explored and the results of the probabilistic analysis are used to develop a robust FLS for damage detection. The FLS also accurately classifies the undamaged condition in presence of the mentioned uncertainties reducing the possibility of false alarms. From an algorithmic standpoint, this paper connects the disparate areas of probability and fuzzy logic to alleviate uncertainty issues in damage detection.

In this study, it had been used a fuzzy logic system for damage detection and location in (in and out-of-plane) curved beam based on frequency difference, the method apply in two stages; the first stage used to detect the damage extent along beam and the second used to detect the damage at any element of beam.

2. INTRODUCTION TO FUZZY LOGIC

Fuzzy logic deals with reasoning with inexact or fuzzy concepts **[12]**. Fuzzy logic has two different meanings. In a narrow sense, fuzzy logic is a logical system, which is an extension of multivalued logic. However, in a wider sense fuzzy logic (FL) is almost synonymous with the theory of fuzzy sets, a theory which relates to classes of objects with unsharp boundaries in which membership is a matter of degree **[13].**

2.1 Membership Function

A membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The input space is sometimes referred to as the universe of discourse. The simplest membership functions are formed using straight lines. In this work three types of membership function were used (triangular, trapezoidal and gaussian).

2.2 Fuzzy Rules

Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic. Usually the knowledge involved in fuzzy reasoning is expressed as rules in the form:

If x is A Then y is B

Where x and y are fuzzy variables and A and B are fuzzy values defined by fuzzy sets. The if-part of the rule "x is A" is called the antecedent or premise, while the then-part of the rule "y is B" is called the consequent or conclusion. Statements in the antecedent (or consequent) parts of the rules may well involve fuzzy logical connectives such as 'AND' and 'OR'.

2.3 Fuzzy Inference System

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The fuzzy inference system was consisting of parts as shown in Fig.1 **[13]**.

- **Fuzzification**. Convert crisp set to fuzzy set.
- **Rule evaluation**. Consist of two parts; fuzzy operators and conditional statement.
- **Aggregation.** Combine all output fuzzy sets from rule evaluation to a single fuzzy set.
- **Defuzzification.** Reduction of fuzzy set to singleton. There are five defuzzification methods as shown in Fig.2. In this study takes two defuzzification methods (Centroid and Middle of Maximum (Mom)).

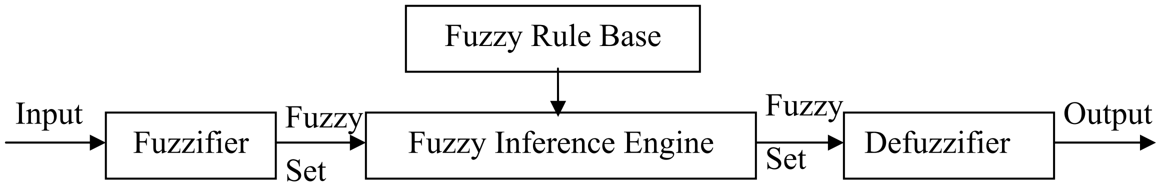

Fig.1. Schematic representation of a fuzzy inference system

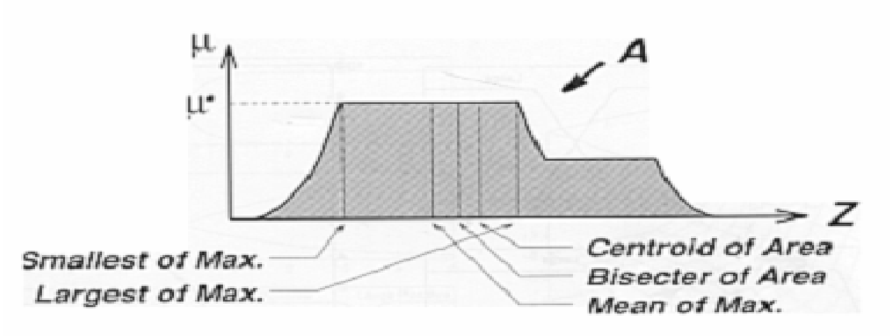

Fig.2. Defuzzification methods

3. MODELING THE DAMAGED BEAM

In this study the equation of motion for simply curved beam acquired from Kang and Yoo's theory of thin- walled curved beams **[14]** to drive the element stiffness and mass matrices respectively. The curved beam element is shown in Fig.3 in curvilinear coordinate system. Each node of the curved beam element possesses seven degrees of freedom including the warping degree of freedom. Using Hamilton's principle, the dynamic equilibrium can be expressed in the variation form as following **[15]**.

$$\int_{t1}^{t2} (\delta T + \delta U + \delta \forall) dt = 0 \qquad (1)$$

Where δT is the variation kinetic energy, δU is the variation strain energy, and $\delta \forall$ is the variation potential energy loss due to applied loads. The symbol (δ) means the first variation. For the linear elastic body, the variation of strain energy stored in the body is

$$\delta U = \int_V \tau_{ij} \, \delta \varepsilon_{ij} dV \qquad (2)$$

Where τ_{ij} refers to the components of the stress tensor and ε_{ij} to those of the strain tensor. The variation in kinetic energy of a thin-walled curved beam is

$$\delta T = \int_V \rho \frac{\partial^2 u_i}{\partial t^2} \delta u_i dV \qquad (3)$$

Where ρ is the mass density, u_i is the displacement components of the curved beam, and t is time. The variation potential energy loss due to applied loads with body forces neglected is

$$\delta \forall = -\int_l q_i \, \delta u_i dz \qquad (4)$$

Where q_i stands for distributed loads applied on the line of shear center and l is the length of the element. Substituting the strain-displacement relationship and the stress resultant–displacement relationship into eqs. (1), (2), (3) and (4) and carrying out the conventional procedure of the calculus of variation, the following set of equations of motion is obtained **[15].**

$$\rho\left(A\ddot{u}_o - I_y \ddot{u}_o'' - \frac{2I_y}{R}\ddot{w}_o'\right) - \frac{EA}{R}\left(w_o' - \frac{u_o}{R}\right) + EI_y\left(u_o^{IV} + \frac{2}{R^2}u_o'' + \frac{1}{R^3}w_o'\right) = q_x - m_y'$$

(5a)

$$\rho\left[A\ddot{v}_o\left(I_x+\frac{I_\omega-2K_{xy\omega}}{R^2}\right)\ddot{v}_o''-\frac{I_y}{R}\ddot{\beta}-\frac{I_\omega-K_{xy\omega}}{R}\ddot{\beta}''\right]+EI_x\left(v_o^{IV}-\frac{1}{R}\beta''\right)+\frac{EK_{xy\omega}}{R}\left(\beta^{IV}+\frac{2}{R}v_o^{IV}-\frac{\beta''}{R^2}\right)+$$
$$\frac{EI_\omega}{R}\left(\beta^{IV}+\frac{v_o^{IV}}{R}\right)-\frac{GK_T}{R}\left(\beta''+\frac{v_o''}{R}\right)=q_y+m_x' \quad (5b)$$

$$\rho\left[\left(A+\frac{3I_y}{R^2}\right)\ddot{w}_o+\frac{2I_y}{R}\ddot{u}_o\right]+E\left(\frac{I_y}{R^2}-A\right)\left(w_o''-\frac{1}{R}u_o'\right)=q_z \quad (5c)$$

$$\rho\left[\left(I_x+I_y\right)\ddot{\beta}-I_\omega\ddot{\beta}''-\frac{I_y}{R}\ddot{v}_o-\frac{I_\omega-K_{xy\omega}}{R}\ddot{v}_o''\right]-\frac{EI_x}{R}\left(v_o''-\frac{1}{R}\beta\right)+\frac{EK_{XY\omega}}{R}\left(v_o^{IV}-\frac{2}{R}\beta''-\frac{1}{R^2}v_o''\right)+$$
$$EI_\omega\left(\beta^{IV}+\frac{1}{R}v_o^{IV}\right)-GK_T\left(\beta''+\frac{1}{R}v_o''\right)=m_z+m_\omega' \quad (5d)$$

Where the reference displacements u_o, v_o, w_o, and β are displacements of the centroid in the x, y, and z directions and a rotation of the cross-section about z-axis, respectively. Displacement components u_o and w_o are associated with in-plane of curvature displacement field while displacement components v_o and β are referenced with out-of plane of curvature displacement field. The linear equations of motion given in equations (5a), (5b), (5c), and (5d) are partially, if not completely, uncoupled. It is observed that u_o and w_o appear only in eqs. (5a) and (5c) whereas v_o and β are present only in eqs. (5b) and (5d), which means that two displacement fields related with in-plane of curvature and out-of-plane of curvature, respectively are fully separated each other.

Only equations that are most related to the modeling of curved beams by straight-beam elements will be presented herein by consider a curved beam as comprising an infinitesimal straight beam. This assumption is consistent with those used by **[16]**. Theory of curved members developed can be reduced to that of straight beam simply by letting the radius of curvature approaches to infinity in eqs. (5a), (5b), (5c), and (5d) **[15].**

$$\rho A\ddot{u}_o-\rho I_y\ddot{u}_o''+EI_yu_o^{IV}=q_x-m_y' \quad (6a)$$

$$\rho A\ddot{v}_o-\rho I_x\ddot{v}_o''+EI_xv_o^{IV}=q_y-m_x' \quad (6b)$$

$$\rho A\ddot{w}_o-EAw_o'=q_z \quad (6c)$$

$$\rho\left(I_x+I_y\right)\ddot{\beta}-\rho I_\omega\ddot{\beta}''+EI_\omega\beta^{IV}-GK_T\beta''=m_z'+m_\omega' \quad (6d)$$

Where every displacement fields, u_o, v_o, w_o, and β, are not coupled with one another hence can be formulated separately. In the present study, the third order Hermit polynomials are employed as shape functions about u_o, v_o, and β. The axial displacement w_o is represented by a linear function.

A linear stiffness matrix and a consistent mass matrix are developed so that various analyses such as linear and free vibration analyses can be performed. Using shape functions, the dynamic equilibrium given in eq. (1) yields a set of simultaneous equations

$$\delta T+\delta U+\delta V=\delta d^T[\mathrm{M}\ddot{d}+\mathrm{K}d-\mathrm{f}]=0 \quad (7)$$

From which one obtains by letting f equal zero.

$$\mathrm{M}\ddot{d}+\mathrm{K}d=0 \quad (8)$$

Where K, M, d, and f are the linear stiffness matrix, the consistent mass matrix, the nodal displacement vector, and the applied force vector of a global structural system, respectively. The nodal forces and the corresponding nodal displacements are shown in Fig.3 in the positive senses. The nodal forces are seven components $(F_z, M_x, M_y, B, T_T, V_x, \text{and } V_y)$.The corresponding nodal displacements are $(w_o, \gamma, -v_o', -\tau, \beta, u_o, and\ v_o)$ where γ and τ are defined as

$$\gamma=u_o' \quad (9a)$$

$$\tau=\beta' \quad (9b)$$

w_o, u_o, and γ describe the in-plane displacements whereas v_o, $-v_o'$, β, and $-\tau$ are the out-of-plane displacements. These two parts of displacement fields are not coupled with each other and can be formulated separately. Then, the displacement fields can be expressed in terms of nodal displacements as following **[15].**

$$\begin{Bmatrix} u_o \\ v_o \\ w_o \\ \beta \end{Bmatrix} = \begin{bmatrix} N_u & & & \\ & N_v & & \\ & & N_w & \\ & & & N_\beta \end{bmatrix} \begin{Bmatrix} d^u \\ d^v \\ d^w \\ d^\beta \end{Bmatrix} \quad (10)$$

Where the shapes function, N is defined as.

$$N_u = [1-3\xi^2+2\xi^3 \quad \iota(\xi-2\xi^2+\xi^3) \quad 3\xi^2-2\xi^3 \quad \iota(-\xi^2+\xi^3)] \quad (11a)$$

$$N_v = N_\beta = [1-3\xi^2+2\xi^3 \quad \iota(-\xi+2\xi^2-\xi^3) \quad 3\xi^2-2\xi^3 \quad \iota(-\xi^2+\xi^3)] \quad (11b)$$

$$N_w = [1-\xi \quad \xi] \quad (11c)$$

Where $\xi = z/\iota$

Where the nodal displacement, d is represented

$$d^u = [u_{oi} \quad \gamma_i \quad u_{oj} \quad \gamma_j]^T \quad (12a)$$

$$d^v = [v_{oi} \quad -v'_{oi} \quad v_{oj} \quad -v'_{oj}]^T \quad (12b)$$

$$d^w = [w_{oi} \quad w_{oj}]^T \quad (12c)$$

$$d^\beta = [\beta_i \quad -\tau_i \quad \beta_j \quad -\tau_j]^T \quad (12d)$$

From the variation of strain energy presented in eq. (2) and the shape function in eqs. (11a), (11b), and (11c) the element stiffness matrix for curved beam is derived as shown **[15].**

$$[k_u] = \begin{bmatrix} EI_yK_a & 0 & 0 & 0 \\ 0 & EI_xK_b & 0 & 0 \\ 0 & 0 & EAK_c & 0 \\ 0 & 0 & 0 & EI_\omega K_d + GK_TK_e \end{bmatrix} \quad (13)$$

From the variation kinetic energy presented in eq. (3) and following the similar procedure as used for the element stiffness matrix for curved beam formulation, the mass matrix is derived **[15].**

$$[m_u] = \rho \begin{bmatrix} AM_a + I_yM_e & & & \\ & AM_b + I_xM_b & & \\ & & AM_c & \\ & & & (I_x+I_y)M_a + I_\omega M_f \end{bmatrix} \quad (14)$$

Where $[k_u]$ $[m_u]$ represented undamaged stiffness and mass matrices respectively, but for damage cases it had been taken reduction for stiffness matrix at any element of curved beam.

4. FREQUENCY DIFFERENCE

The difference between the frequency of the damaged and undamaged for simply curved beam is that used as the system indicator for damage and is referred to as a ‘‘summation deltas’’ since the reduction in stiffness for a damaged simply supported curved beam decreases the frequency. The summation deltas is expressed as follow.

$$\sum\Delta\omega = \sum_{i=1}^{m}(\omega_{ui} - \omega_{di}) \quad (15)$$

Where
$\sum\Delta\omega$: summation delta.
i: mode number (i=1,2,3,….,m)
ω_{ui}: Frequency undamaged.
ω_{di}: Frequency damage.

A finite element approach is used to calculate the natural frequencies of the simply curved beam. Each beam finite element has seven degrees of freedom. Damage is modeled as a reduction in element stiffness of (20, 40 and 60%) respectively. These damage sizes are classified as ‘‘slight damage’’, ‘‘moderate damage’’ and ‘‘severe damage’’, respectively. Damage sizes below ‘‘slight damage’’ are classified as undamaged. Damage sizes greater than ‘‘severe damage’’ are classified as ‘‘catastrophic damage’’.

5. FORMULATION OF FUZZY LOGIC SYSTEM

5.1. Input and Output

Inputs to the FLS are summation deltas, at the first stage; it has been taken the least number of modes which realize minimum interference between damage extent values, so the chosen number of modes is fifteen for in-plane and twenty-six for out-of-plane with use eq. (15), and outputs are structural damage faults (SLD,MOD and SVD). We have three summation deltas represented by y and five fault quantity represented by x. The objective is to find a functional mapping between y and x. mathematically this can be represented as:

$$x= f(y) \quad (16)$$

Where x= {damage extent} and $y = \{\sum\Delta\omega_1, \sum\Delta\omega_2, \sum\Delta\omega_3\}$.

At second stage represents the FLS input is frequency difference for eight modes for in-plane and nine modes for out-of-plane, and the damage element indicator as output, here x represented element number x={damage at element} and y represented frequency difference y=$\{\Delta\omega_1, \ldots, \Delta\omega_9\}$.

5.2 Fuzzification

Here the structural damages are crisp numbers. To get a degree of resolution of the extent of damage, each of this damage extent is allowed several levels of damage and split into linguistic variables. For example, at first stage consider ‘‘beam" as a linguistic variable. Then it can be decomposed into a set of terms T(beam) = {Undamaged,Slight Damage,Moderate Damage, Severe Damage, Catastrophic Damage}.where each term in T(beam) is characterized by a fuzzy set in the universe of discourse U(beam)={0, 70} as shown in Fig.4. The summation deltas ∑Δω also treated as fuzzy variables. For example, at first stage consider ∑Δω as a linguistic variable. It can be decomposed into a set of terms T(∑Δω)={Negligible, Low,Medium, High, Very High}.where each term in T(∑Δω) is characterized by a fuzzy set in the universe of discourse U (∑Δω)= {0,max} as shown in Fig.5 for in-plane and Fig.6 for out-of-plane. The other two summation deltas are defined using the same set of terms. Summation deltas larger than covered by the universe of discourse will represent an extensive structural damage indicative of a catastrophic failure. Fuzzy sets with Trapezoidal membership functions are used for the first stage input variables and triangular membership functions are used for the first stage output variables. Tables (1) and (2) give the linguistic measure and rules associated with each fuzzy set at first stage. The values mentioned in the Tables (1) and (2) were indicate by substituting summation deltas ∑Δω for each element along curved beam in three damaged extent and then using fuzzy logic tool box to construct membership functions and rules.

The second stage FLS is the same manner as previous, fuzzy sets with gaussian membership functions are used for the second stage input variables. These fuzzy sets can be defined using the following equation **[13]**.

$$\mu(x) = e^{-0.5(x-\frac{m_0}{\sigma})^2} \quad (17)$$

Where m_0 is the midpoint of the fuzzy set and σ is standard deviation associated with the variable. Fig. 7 and Fig.8 represent the input fuzzy sets for in and out-of-plane respectively at second stage which consist of ten gaussian membership functions (N=negligible, VVL=very very low, VL=very low, L=low, LM=low medium, M=medium, MH=medium high, H=high, VH=very high and VVH=very very high) where each term in T(Δω) is characterized by a fuzzy set in the universe of discourse U(Δω) ={-68.75,550}, and output represented by thirteen triangular membership functions for in-plane and fifteen triangular membership functions for out-of-plane which represent element number as shown in Fig. 9 and Fig.10 respectively where each term in T(damage at element) is characterized by a fuzzy set in the universe of discourse U(damage at element) ={0,1,2,…,15}

5.3 Rules Generation

Rules for the fuzzy system are obtained by fuzzification of the numerical values obtained from the finite element analysis using the following procedure.

1. A set of summation deltas and frequency difference corresponding to a given structural fault is input to the FLS and the degrees of membership of

the elements of ($\sum\Delta\omega$) and ($\Delta\omega$) are obtained.

2. Therefore, each summation deltas has five degrees of memberships for first stage and each frequency difference has ten degrees of memberships for second stage.
3. Each summation deltas and frequency difference is then assigned to the fuzzy set with the maximum degree of membership.
4. One rule is obtained for each fault by relating the summation deltas and frequency difference with a fault.

These rules can be read as follows for the first stage:

IF $\Sigma\Delta\omega$ Is Low THEN Slight Damage.

IF $\Sigma\Delta\omega$ Is Medium THEN Moderate Damage.

IF $\Sigma\Delta\omega$ Is High THEN Severe Damage.

Then for example at second stage the rule for in-plane of "Severe Damage" fault is shown below,

IF

$\Delta\omega_1$ Is Very Very Low AND

$\Delta\omega_2$ Is Very Very Low AND

$\Delta\omega_3$ Is Low AND

$\Delta\omega_4$ Is Low AND

$\Delta\omega_5$ Is Very Very Low AND

$\Delta\omega_6$ Is Low AND

$\Delta\omega_7$ Is Medium AND

$\Delta\omega_8$ Is Medium High

Then damage at element No.5.

The same procedure applies to slight and moderate damage except the rules are different. These rules are tabulated in Tables (3) and (4) (these rules were indicate to damage extent 20%, 40% and 60% at second stage and results represented damage location at any element along the curved beam)

6. NUMERICAL RESULTS

In the present work we consider a simply (in and out-of-plane) curved beam for illustrating the fuzzy logic system for the damage detection problem; the fuzzy rules are automatically generated. Dimensions and material properties for the simply supported in and out-of-plane curved beam are shown in Tables (5) and (6) respectively

The simply in-plane curved beam is divided into 25 finite elements of equal length and out-of-plane is divided into 30 finite elements. The selection of number of element is justified in Fig. 11 and Fig.12 respectively to minimize FEM modeling error, in this figure the ratio of the eighth mode which is the highest mode used in the numerical results for in-plane and nine for out-of-plane with the first mode are shown. From the graph, it appears that 25 elements for in-plane and 30 elements for out-of-plane give a good resolution. The undamaged beam is uniform. Therefore, the frequency predictions from the FEM model of undamaged beam are validated by comparing with other researchers as shown in Tables (7) and (8) respectively

The generated rules are selected for first stage of summation deltas; by taking the appropriate modes here in this study take fifteen modes for in-plane and twenty-six modes for out-of-plane (the selected modes chosen according to minimize interference between damage extent for three cases) . At the second stage of frequency difference will be taken the eight input deltas for in-plane and nine input deltas for out-of-plane represent the highest natural frequencies used (these values were taking as mention in Fig. 11 and Fig. 12 which declare the difference between). The linguistic forms will remain constant for different structure but the numerical values of midpoints and standard deviation will change.

7. RESULTS AND DISCUSSION

After construct the fuzzy engine and check all damage elements under the test it could be calculated the error between proposed and predict values for all elements using two defuzzification methods (Centroid and Middle of Maximum) to compare and evaluate results product from fuzzy logic system. We chose these two methods because they had been already approval that they represent damage with the least error percentage, and this didn't mean we compare between them.

For in-plane curved beam as shown in the Fig.13, from charts it could be seen the middle of maximum defuzzification method is the best for detect slight (D=20%) and moderate (D=40%) damage but at severe (D=60%) damage the centroid defuzzification method is the best.

For out-of-plane curved beam as shown in the Fig.14, from charts it could be seen the middle of maximum defuzzification method is the best for detect slight (D=20%) and moderate (D=40%) damage but at severe (D=60%) damage the centroid defuzzification method is the best. Each value in Fig. 13 and Fig. 14 represented damage location because input and output values to fuzzy engine represent crisp set, referring that this study has been used theoretical results (FEM) as input data instead of experimental data.

The accuracy for detection damage at in and out-of-plane curved beam tabulated in Table (9) it could be seen the accuracy approach to (99.4%) for slight damage, (99.52%) for moderate damage and (100%) for severe damage.

8. CONCLUSIONS

The present study declares that a fuzzy logic depends on one of the well-known methods for damage detection as a base to generate rules, but here we use frequency difference as a new application as a base for fuzzy logic system. The results show that the fuzzy logic system presented in this paper provided a reliable and accurate outcome in recognition of different damage extent and appear more efficient when using summation for frequency difference once taking the fifteen modes for in-plane and twenty-six modes for out-of-plane at first stage, and taking the eight modes for in-plane and nine modes for out-of-plane at second stage. This method were used to detect damage at one location in beam but when we want to locate multi damage in beam we need to apply anther methods together like "strain energy, sensitivity, etc". The other advantage of this method is used to detect damage at low extent (less than 20%) with a good accuracy.

NOTATION

A Sectional area (m^2)
B_i Bimoment (N .m)
E Young modulus (N/m^2)
G Shear modulus (N/m^2)
I_y Area moment of inertia about y-axis (m^4)
I_x Area moment of inertia about x-axis (m^4)
I_ω Warping moment of inertia (m^6)
J Area polar moment of inertia (m^4)
K_T St Venant constant of a straight member(m^4)
l Length of the finite element (m, cm)
m_x, m_y, m_z Uniform distributed moments about x-, y-, and z-axis
m_ω Uniform distributed bimoment
M_x, M_y Moment about x- and y-axis (N.m)
q_x, q_y, q_z Uniform distributed forces about x-, y-, and z-directions
R Radius of initial curvature (m)
T Kinetic energy (N.m)
U Strain energy (N.m)
u_o, v_o Displacement components of the shear center in x- and y- directions, respectively
V Volume of body (m^3)
V_x, V_y Transverse shear forces (N)
w_o Average longitudinal displacement of cross-section

GREEK LETTERS

ρ Mass density (Kg/m^3)
β Rotation of the cross-section about z-axis
θ Subtended angle (degree)
ε_{ij} Components of strain tensor
δ Variation
γ, τ Nodal displacements
τ_{ij} Components of stress tensor

REFERENCES

[1] O. S. Salawu, "Detection of structural damage through changes in frequency: a review", Engineering Structures, Vol. 19, 1997.

[2] Ju, F. D. and Mimovich, M. "Modal frequency method in diagnosis of fracture damage in structures" in Proceedings, 4th International Modal Analysis Conference, Los Angeles, 2, pp 1168-1174, 1986.

[3] Cawley, P. and Adams, R. D. "The location of defects in structures from measurements of natural frequencies", Journal of Strain Analysis, 4 (2), pp 49-57 (1979).

[4] Uzgider, Z., Sanli, A. K., Piroglu, F. and Caglayan, D. B. "Identification of railway bridges using locomotive-induced vibrations "in Bridge Management 2 (edited J. E. Harding, G. A. R. Parke and M.J. Ryall), Thomas Telford, London, pp 833-841, 1993.

[5] H. R. Öz and M. T. Das, "In-plane vibrations of circular curved beams with a transverse open crack", Mathematical and Computational Applications, Vol. 11, No. 1, pp. 1-10,(2006).

[6] Hajek, P., Paris, J. and Shepherdson, J. (2000) "The liar paradox and fuzzy logic", J. Symb. Logic 65, 339–346.

[7] Raju, K. and Majumdar, "Fuzzy functional dependencies and lossless join decomposition of fuzzy relational database systems", ACM Trans. Database Syst. 13(32), 129–166, (1988).

[8] Juuso, Esko K. "Intelligent systems design with linguistic equations", Dortmund, Germany. (Dortmund University), pp 177–196, (1999).

[9] RANJAN GANGULI, "A Fuzzy Logic System for Ground Based Structural Health Monitoring of a Helicopter Rotor using Modal Data". Journal of intelligent material system and structures, Vol. 12, (2002).

[10] Prashant M and Ranjan, "Genetic fuzzy system for damage detection in beams and helicopter rotor blades". Journal of computer methods in applied mechanics and engineering .Vol. 192, Pages 2031–2057, (2003).

[11] M. Chandrashekhar, Ranjan Ganguli," Uncertainty handling in structural damage detection using fuzzy logic and probabilistic simulation" Journal of Mechanical Systems and Signal Processing, pp 1-21(2007).

[12] M. Ganesh, "Introduction to Fuzzy Sets and Fuzzy Logic". Prentice-Hall of India Private Limited, New Delhi (2006).

[13] Fuzzy Logic Toolbox 2 User’s Guide, The Math Works, Inc, 1995–2007.

[14] Kang YJ, Yoo CH."Thin-walled curved beams. I: formulation of nonlinear equations". ASCE. Journal of Engineering Structures, Vol. 120, Pages 2072–2101 (1994).

[15] Ki-Young Yoona, Young-Jong Kangb, Young-Joon Choic, and Nam-Hoi Parkb. "Free vibration analysis of horizontally curved steel I-girder bridges". Journal of Thin-Walled Structures. Vol. 43, Pages 679–699(2005).

[16] Shyh Rong Kuo and Yeong Bin Yang," New theory on buckling of curved beams". ASCE. Journal of Engineering Mechanics, Vol. 117, Pages 1698–1717 (1991).

[17] Ki-Young Yoon, Nam Hoi Park, Young Joon Choi, and Young-Jong Kang "Natural frequencies of thin-walled curved beams" Journal of Finite Elements in Analysis and Design. Vol. 42, Pages 1176 – 1186(2006).

Faults	Summation Deltas (ΣΔω)	Numeric value
SLD	Low	351 - 687
MOD	Moderate	900 - 1657
SVD	High	1900 - 3137

Table 1 Rules for fuzzy system in-plane curved beam (first stage)

Faults	Summation Deltas (ΣΔω)	Numeric value
SLD	Low	162.5 - 356
MOD	Moderate	427 – 848.5
SVD	High	925 - 1580

Table 2 Rules for fuzzy system out-of-plane curved beam (first stage)

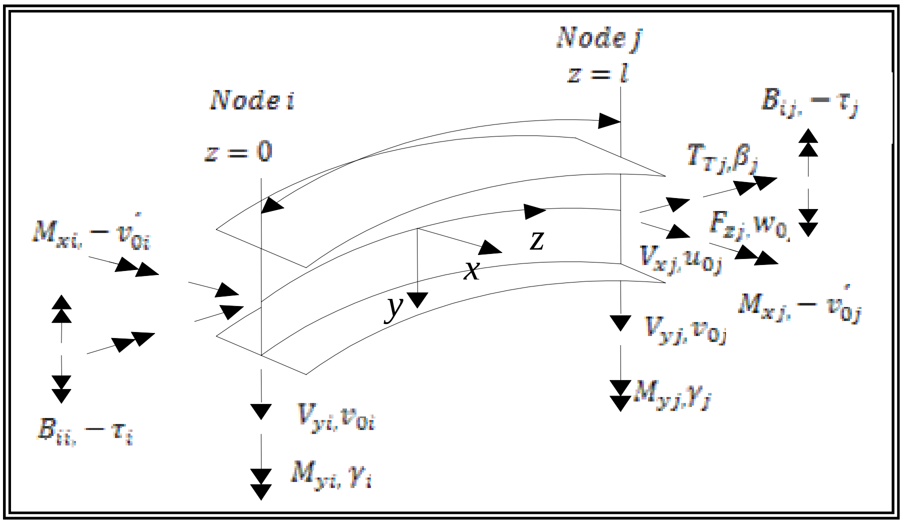

Fig.3. Curved beam element

.

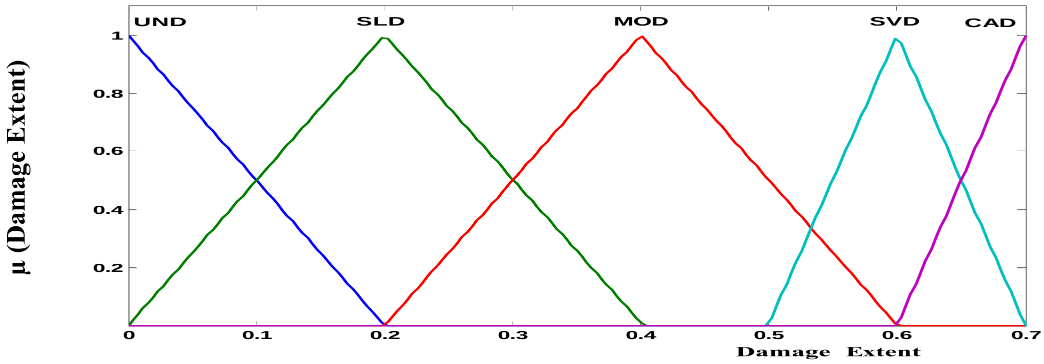

Fig.4. Input fuzzy sets representing damage levels (first stage)

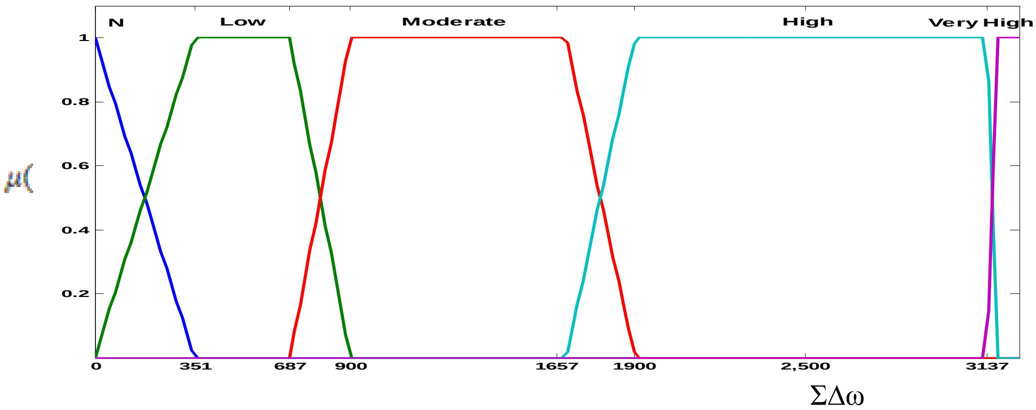

Fig.5. Input Fuzzy sets representing summation deltas for in-plane curved beam (first stage)

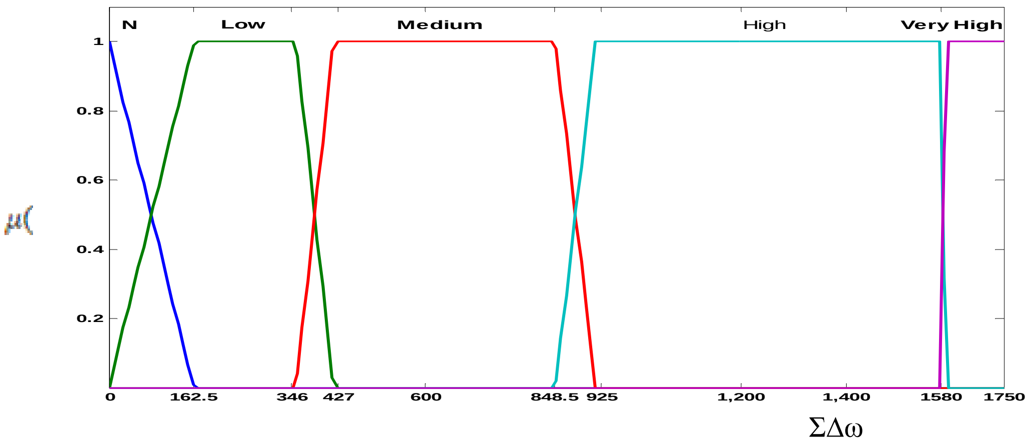

Fig.6. Input Fuzzy sets representing summation deltas for out-of-plane curved beam (first stage)

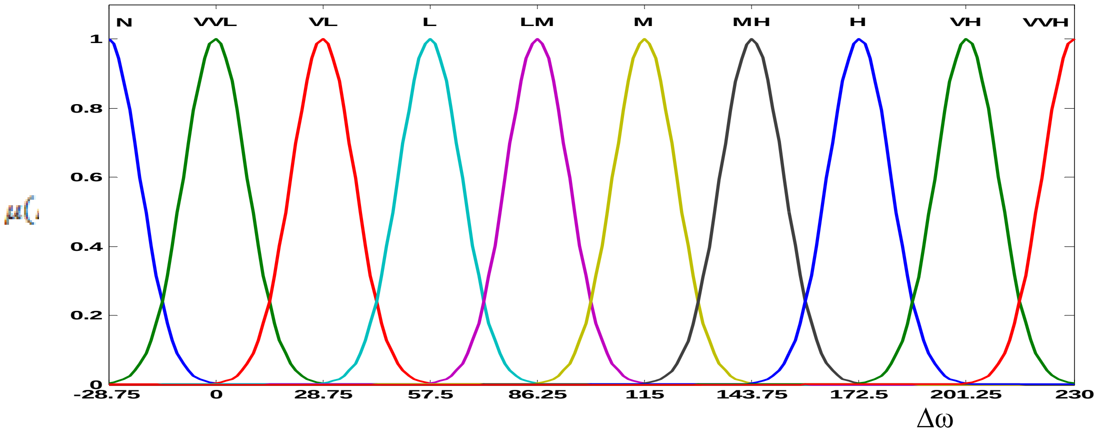

Fig.7. Input fuzzy sets for in-plane curved beam (second stage)

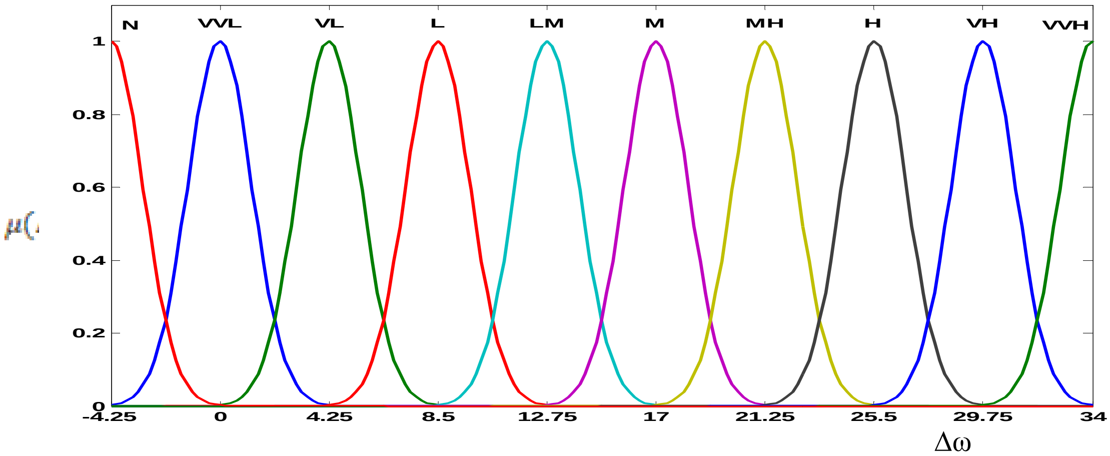

Fig.8. Input fuzzy sets for out-of-plane curved beam (second stage)

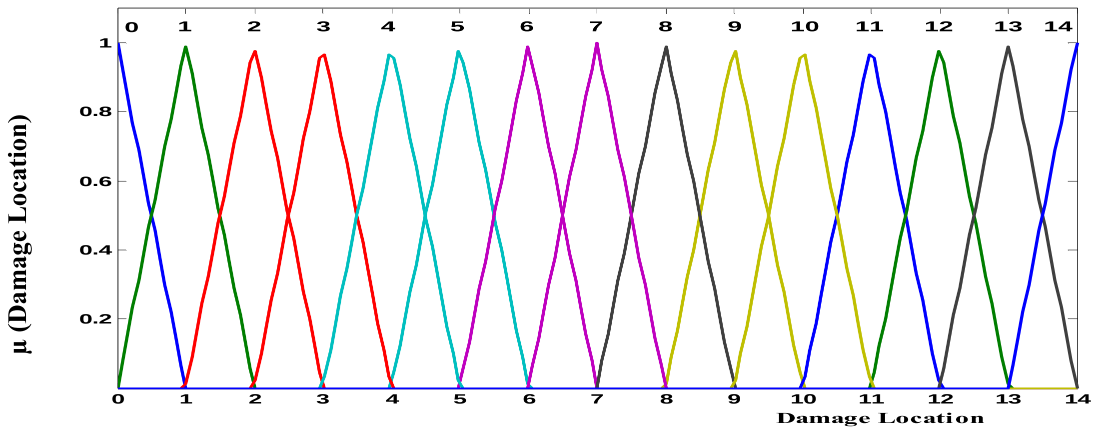

Fig.9. Output fuzzy sets for in-plane curved beam (second stage)

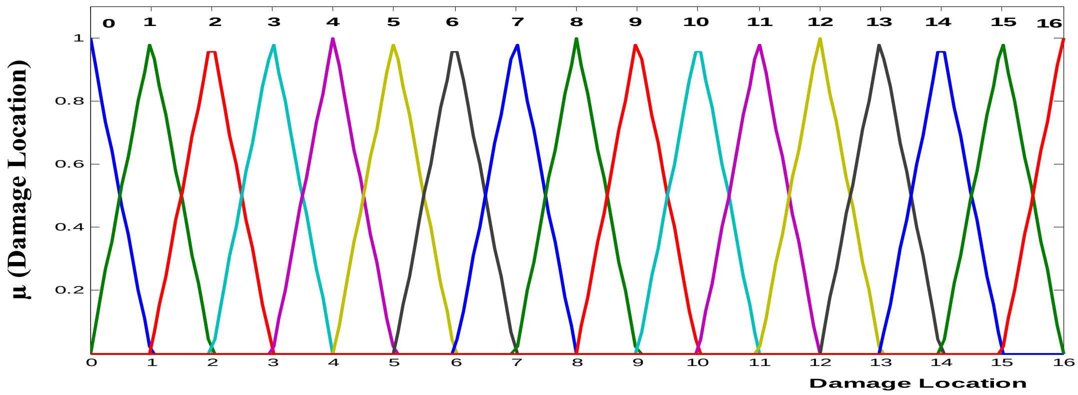

Fig.10. Output fuzzy sets for out-of-plane curved beam (second stage)

Table 3 Rules for fuzzy system of in-plane curved beam damaged cases (second stage)

Damaged at element No.	0	1	2	3	4	5	6	7	8	9	10	11	12	13
D=0.2														
Δω1	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω2	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω3	N	VVL	VVL	VL	VL	VVL	VVL	VVL	VVL	VL	VL	VVL	VVL	VVL
Δω4	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω5	N	VVL	VL	VL	VL	VVL	VVL	VL	VL	VL	VVL	VL	VVL	VL
Δω6	N	VL	VL	VL	VVL	VVL	VL	VL	VVL	VVL	VL	VVL	VVL	VVL
Δω7	N	VL	VL	VL	VL	VL	VL	VL	VL	VVL	VVL	VVL	VVL	VVL
Δω8	N	VVL	L	VL	VVL	VL	L	VL	VVL	L	VL	VVL	VL	L
D=0.4														
Δω1	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω2	N	VVL	VVL	VL	VL	VL	VL	VVL	VVL	VVL	VVL	VVL	VL	VL
Δω3	N	VVL	VL	VL	VL	VL	VVL	VVL	VL	VL	VL	VL	VVL	VVL
Δω4	N	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VL
Δω5	N	VVL	L	L	L	VVL	VVL	L	L	L	VVL	VVL	L	L
Δω6	N	L	LM	LM	VL	VL	L	LM	VL	VVL	L	L	VL	VVL
Δω7	N	LM	M	M	LM	L	L	L	L	VL	VL	VL	VVL	VVL
Δω8	N	VL	M	LM	VVL	LM	M	VL	VL	M	LM	VVL	L	M
D=0.6														
Δω1	N	VVL	VVL	VVL	VVL	VL	VL	VL	VL	VVL	VVL	VVL	VVL	VVL
Δω2	N	VVL	VL	VL	L	VL	VL	VL	VVL	VVL	VVL	VL	VL	L
Δω3	N	VVL	L	LM	LM	L	VVL	VVL	VL	LM	LM	L	VL	VVL
Δω4	N	L	L	L	L	L	L	L	L	L	L	L	L	L
Δω5	N	VL	M	MH	LM	VL	VL	LM	M	LM	VL	VL	LM	M
Δω6	N	M	H	MH	LM	L	MH	MH	L	VL	LM	MH	L	VVL
Δω7	N	H	VVH	VH	H	M	MH	MH	LM	L	L	L	VL	VVL
Δω8	N	LM	VVH	MH	VL	MH	VH	LM	L	VH	H	VL	M	VVH

Table 4 Rules for fuzzy system of out-of-plane curved beam damaged cases (second stage)

Damaged at element No.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D= 20%																
Δω1	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω2	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω3	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω4	N	VVL	VVL	VVL	VL	VL	VVL	VVL	VVL	VVL	VVL	VL	VL	VVL	VVL	VVL
Δω5	N	VVL	VL	VL	VL	VL	VVL	VVL	VL	VL	VL	VL	VVL	VVL	VL	VL
Δω6	N	VVL	VVL	VVL	VVL	VVL	VVL	VL	VL	VL	VL	VL	VL	VL	VL	VL
Δω7	N	VVL	VVL	VVL	VL	VL	VL	VL	VL	VL	VL	VL	VL	VVL	VVL	VVL
Δω8	N	VVL	VVL	VL	VL	L	L	VL	VL	VVL	VVL	VVL	VVL	VL	VL	L
Δω9	N	VVL	VL	L	VL	VVL	VL	VL	L	VL	VVL	VVL	VL	L	VL	VVL
D= 40%																
Δω1	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω2	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω3	N	VVL	VVL	VVL	VL	VL	VL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VL	VL
Δω4	N	VVL	VL	VL	VL	VL	VL	VVL	VVL	VVL	VL	VL	VL	VL	VL	VVL
Δω5	N	VL	L	LM	LM	L	VL	VL	L	LM	LM	L	L	L	L	LM
Δω6	N	VVL	VVL	VVL	VL	VL	VL	L	L	L	LM	LM	L	L	LM	LM
Δω7	N	VVL	VVL	VL	L	L	LM	LM	LM	LM	L	L	VL	VL	VVL	VVL
Δω8	N	VVL	VL	L	LM	M	M	LM	L	VL	VVL	VVL	VL	L	LM	M
Δω9	N	VL	LM	M	LM	VL	VL	LM	M	LM	VL	VL	LM	M	LM	VL
D= 40%																
Δω1	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω2	N	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL	VVL
Δω3	N	VVL	VVL	VL	VL	VL	VL	VL	VL	VVL	VVL	VVL	VVL	VL	VL	VL
Δω4	N	VL	VL	LM	LM	LM	L	VL	VVL	VL	L	LM	LM	L	VL	VVL
Δω5	N	VL	LM	MH	MH	LM	LM	MH	H	VH	VH	VH	VH	VH	VH	VVH
Δω6	N	VVL	VVL	VL	L	LM	L	L	M	MH	MH	M	L	L	M	H
Δω7	N	VVL	VL	L	M	MH	MH	MH	MH	M	M	LM	L	VL	VL	VVL
Δω8	N	VVL	L	MH	VH	VVH	VH	MH	LM	L	VVL	VVL	L	M	H	VH
Δω9	N	L	VH	VVH	MH	VL	L	H	VVH	MH	VL	L	H	VVH	MH	L

.

Table 5 Material properties of the in-plane curved beam

Area of cross section (A)	$4.071 \times 10^{-3}\ m^2$
Radius of the arch (R)	2.438 m
Mass density (ρ)	$7855\ {kg}/{m^3}$
Subtended angle (θ)	97°
Modules of Elasticity (E)	206.8 GPa
Modules of Rigidity (G)	77.9 GPa
Moment of inertia (I)	$6.456 \times 10^{-6} m^4$

.

Table 6 Material properties of the out-of- plane curved beam

Area of cross section (A)	9.292×10^{-3} m^2
Length (L)	10.16 m
Mass density (ρ)	7855 kg/m^3
Subtended angle (θ)	89°
Modules of Elasticity (E)	200 GPa
Modules of Rigidity (G)	77.2 GPa
Moment of inertia (Ix)	1.134×10^{-4}m^4
Moment of inertia (Iy)	3.886×10^{-5}m^4
Warping moment of inertia (Iω)	5.559×10^{-7}m^6
Venant constant (K_T)	5.386×10^{-7}m^4

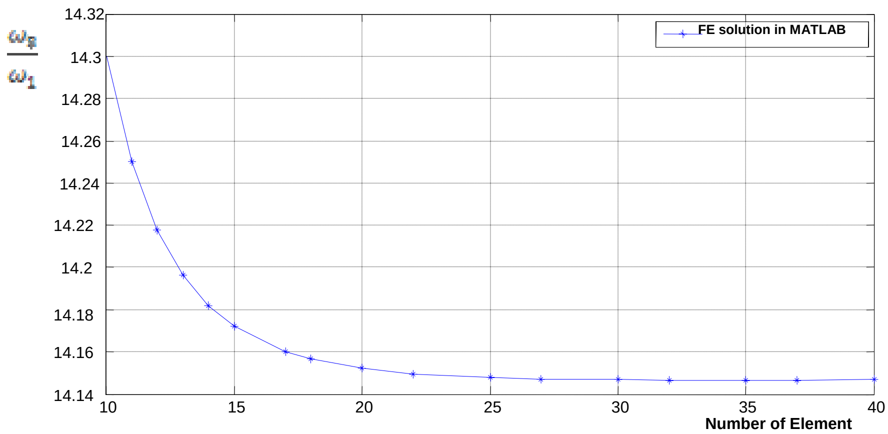

Fig.11. Convergence test for in-plane curved beam

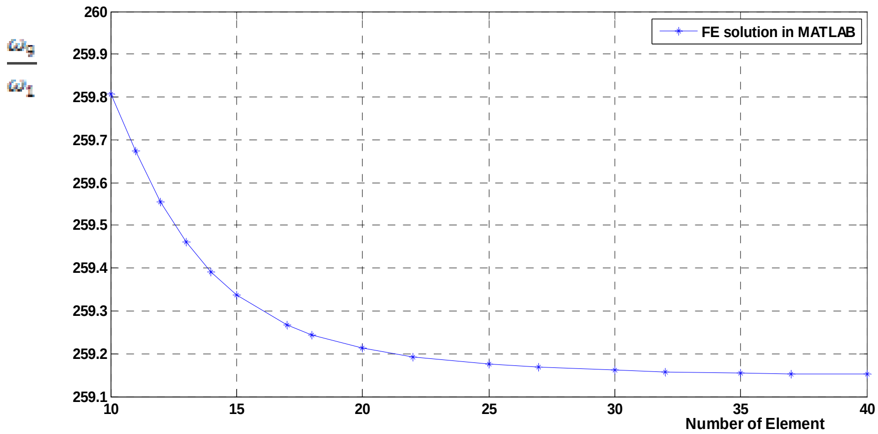

Fig.12. Convergence test for out-of-plane curved beam

Table 7 Comparisons of modal frequencies for in-plane curved

Mode No.	Natural Frequency(Hz)		Error (%)
	[Ki. Young et al] Results[17]	Present Numerical Results	
1	63.18	63.22	0.0633
2	148.21	148.38	0.1147
3	286.05	286.19	0.0489

Table 8 First natural frequencies for the simply supported out-of- plane curved beam

Subtended Angle (degree)	Natural Frequency (rad/sec)			Error (%)
	Analytical Results[17]	Numerical Results[17]	Present Numerical Results	
0	53.3000	53.3000	53.266	0.06379
10	31.8648	31.8669	31.863	0.0056
20	19.9616	19.9614	19.9592	0.01202
30	13.9944	13.9931	13.9915	0.0207
40	10.5386	10.5372	10.5343	0.0408
50	8.2946	8.2888	8.28753	0.08523
60	6.7121	6.7012	6.70043	0.1739
70	5.5270	5.5090	5.50836	0.33725
80	4.5991	4.5707	4.57020	0.62838
90	3.8479	3.8048	3.87485	0.70038

Table 9 Accuracy for in and out-of-plane curved beam (second stage)

Damage Extent (D)	Accuracy (%)			
	In-plane		Out-of-plane	
	Centroid	Mom	Centroid	Mom
SLD	71.01	99.40	88.8	99.27
MOD	99.42	99.52	99.34	99.47
SVD	100	99.33	99.74	99,15

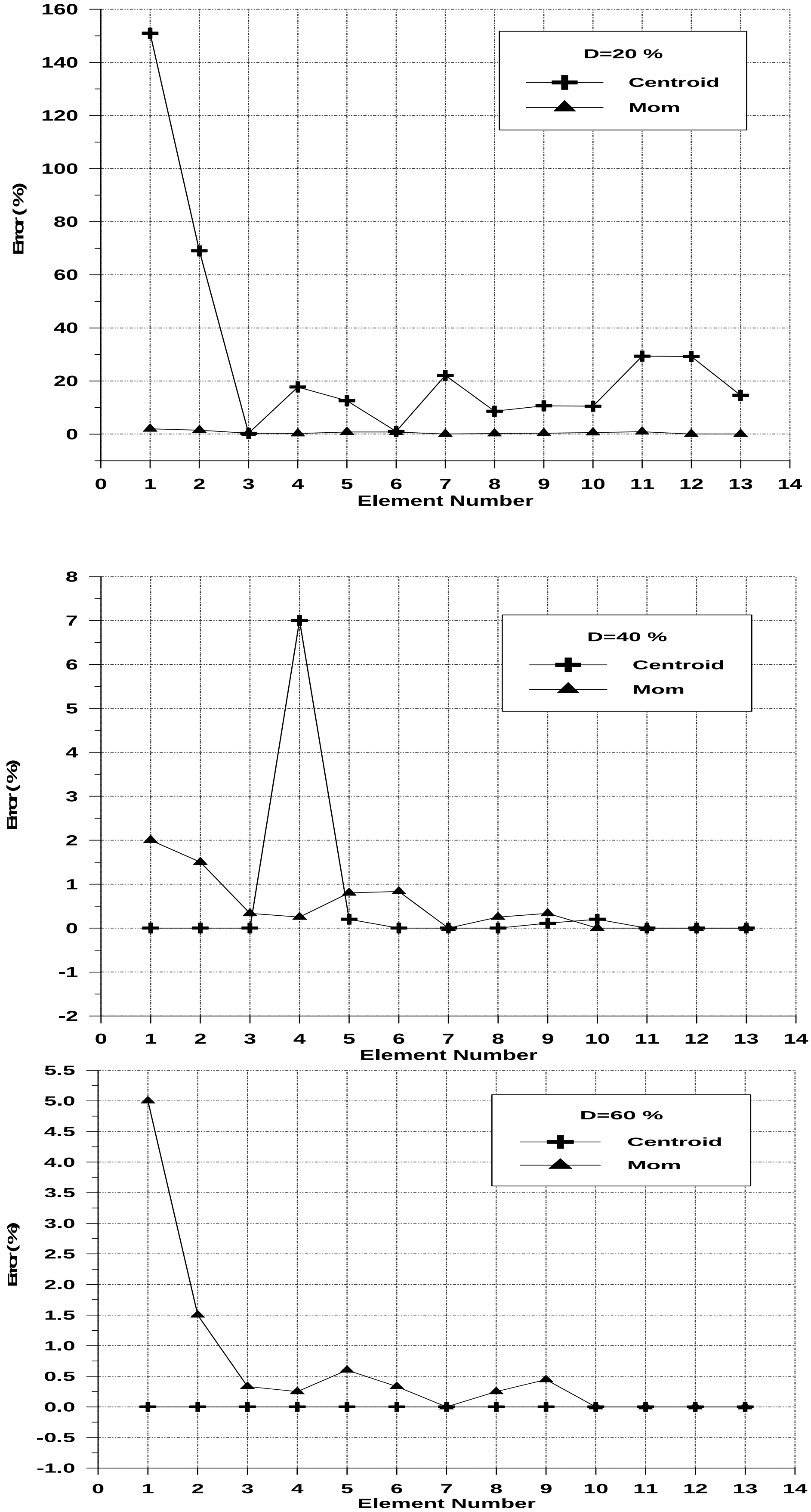

Fig.13. Error between proposed and predict damage at each element for in-plane curved beam

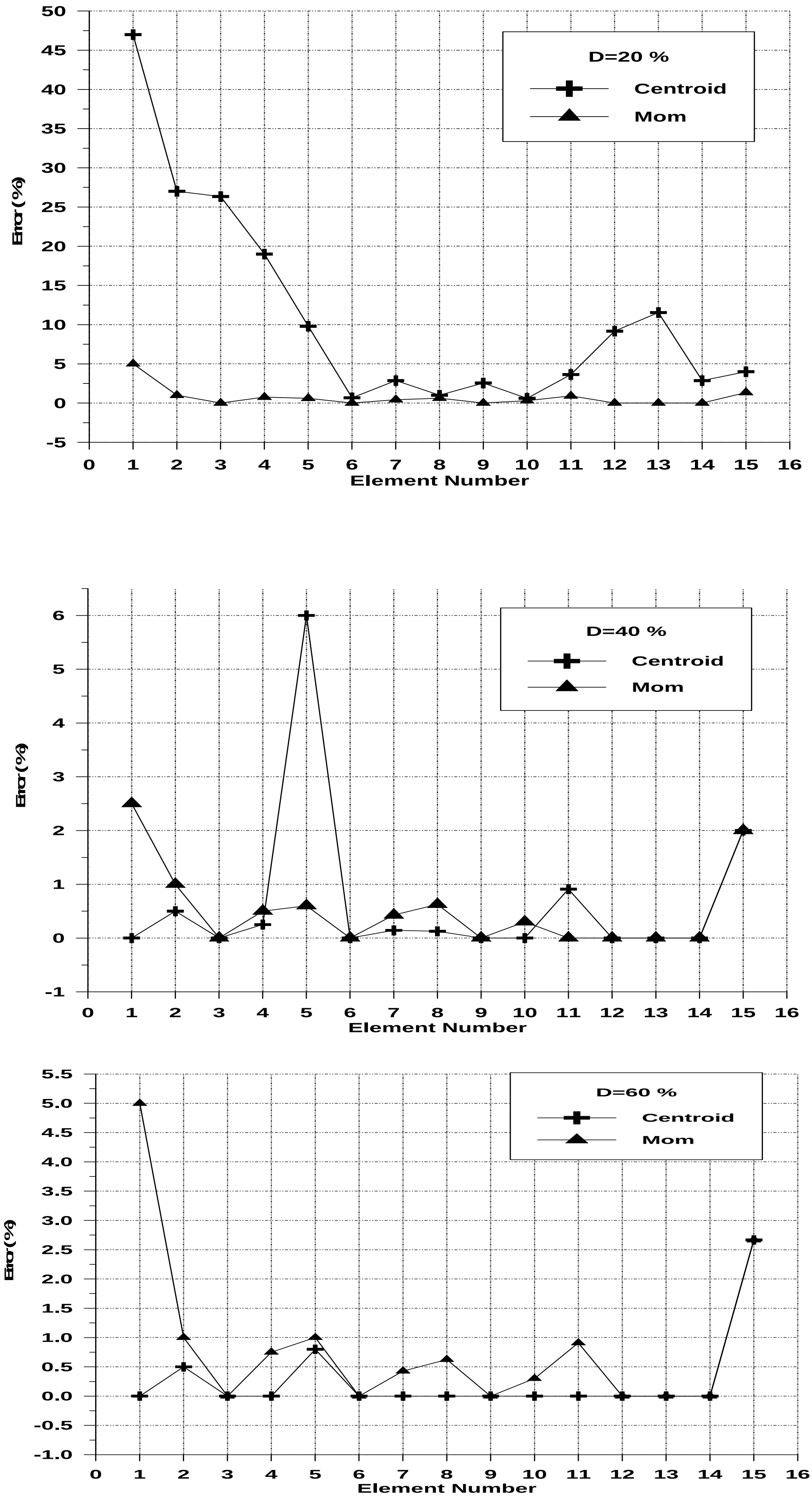

Fig.14. Error between proposed and predict damage at each element for out-of-plane curved beam