



SPLIT ASSIGNMENT WITH TRANSPORTATION MODEL FOR JOB-SHOP LOADING (CASE STUDY)

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ABSTRACT

The aim of this paper, is to analyze the assignment problem in industry where the constraint of allowing to allocate only one job to each machine is relaxed. Thus, splitting the job is permitted and processed by more than one machine. The problem is demonstrated with a real life case study. We solved the problem by splitting the jobs and converting lot of jobs as well as the actual hours of each machine to Standard Machine Hours (SMH). Transportation model is also suggested to overcome the problem, and the optimum solution is obtained by using POM software

KEYWORDS

Assignment problem, splitting, allocate, real life, transportation model.

INTRODUCTION

Loading or shop loading is required to assign specific jobs or teams to specific facilities. Loading is needed for machine shop, hospitals and offices. An important step in production scheduling is to load facilities, which mean taking the actual order and assigning them to designated facilities, loading answers the question which department is going to what work and loading assumes that the material requirement analysis has been done and that order has been properly placed for required material and for needed parts and sub-assemblies.

Specifically, loading assigns the work to divisions, departments, work centers, load centers, stations and people. Whatever names are used in a given organization, orders are assigned to those who will be responsible for performing the work. Loading releases jobs to facilities. In fact, average scheduling use standard hours based on forecasts to determine what resources should be assembled over the planning horizon. Loading takes place in the job shop when the real orders are on hand. If the aggregate scheduling job was done well, then the appropriate kinds and amounts of resources are available for loading. The master production schedule also makes resource assignments that can be modified if capacity is not adequate. A major objective of loading is to spread the load so that waiting time is minimized, the flow is smooth and rapid and congestion is avoided Nahimas, (1989). These objectives are constructed by the fact that not all machines can do all kinds of jobs.

Even though the job uses general purpose equipment, some machines and workers are better suited for specific job than other. Some machines cannot do particular jobs but other machines can do. Some are faster than others and tend to be overloaded. Also, in some situations, planning for actual shop assignments is irregular, where a particular job must be assigned to more machine specially if there is a high request for that job. Thus, let the production manager think about the splitting of jobs assignment and the scheduling objective will be how to smooth the load with balanced work assignments at machines in the flow shop.

In the literature, a great deal and much attention of researchers have been focused on solving the production scheduling problems.

Chan and Wong (2005) developed an assignment and scheduling model to study the impact of machine flexibility on production as a job lateness and machine utilization and presented a genetic algorithm based approach to solve generic machine assignment.

Also, Chan and Wong (2006) addressed the problem from the point of flexible job shop scheduling under resource constraints. They permitted an operation of each job to be processed by more than one machine. Caffrey and Hitchings (1995) considered scheduling of five jobs through a flow shop with five machines. Torres and Centeno (2008) considered a permutation flowshop problem with secondary resources with the objective of minimizing the number of tardy jobs. Recently, Konstantin et. al. (2005) focused on a dynamic generalization of the assignment problem where each task consists of a number of units to be performed by an agent or by a limited number of agents at a time.

ADDRESS THE PROBLEM

In this paper, we deal with the modification of the standard assignment problem where the constraint of allowing to assign only one job to each machine is relaxed. So the splitting the job is permitted to be processed by more than one machine. That means, the job can be divided into a parts, and on the other hand, the machine be allowed to do more jobs without violating the capacity constraint.

The research methodology is to assign operations to machine and determine the processing order of job on machines.

SYMBOLS USED

The following symbols are used throughout the paper:

t_j : Processing Time of Job j

n : Number of jobs to be assigned

m : Number of Machines in the flow shop

M_i : Machine number $i=1,2,\dots,m$

SM : Standard Machine

SMH: Standard Machine Hours

D_u : Demand in units for each kind of job



Pr : Production rate in units per SMH

D_{SMH} : Demand in aggregated standard hours

Pr_{ij} : Production rate in pieces per hour for machine i and job j

PROBLEM FORMULATION

The time minimizing assignment problem is the problem of finding an assignment of n jobs to m facilities, one to each, which minimizes the time for completing all the jobs subject to the assumption that all the jobs are commenced simultaneously Taha, (2007).

The general assignment problem with n jobs and m machines mathematically can be formulated as follows:

$$\text{Minimize Max } \{ t_{ij} | X_{ij} = 1 \} \quad (1)$$

Subject to

$$\sum_{j=1}^n X_{ij} \text{ for all } i \quad (2)$$

$$\sum_{i=1}^m X_{ij} \text{ for all } j \quad (3)$$

$$X_{ij} = 0 \text{ or } 1 \text{ for } i \text{ and } j.$$

Here, t_{ij} is the time required for completing the i th job if it is assigned to the j th machine and

$$X_{ij} = \begin{cases} 0 & \text{if the } i\text{th job is not assigned to the } j\text{th machine} \\ 1 & \text{if the } i\text{th job is assigned to the } j\text{th machine} \end{cases}$$

In the above formula, the important assumption made is that all the jobs are independent and can be commenced simultaneously.

RELAXING THE ASSUMPTION

The weakness of the classical assignment model is that no more jobs are allowed to one machine and also under certain circumstances, a weakness of the previous assignment model is that the splitting the jobs are not allowed. Moreover, the model assumed that the machine in job shops are identical and each one can do each job but with different time. To review the concept of relaxing the assumption, it can be seen from Table 1 which shows that how three jobs J1, J2 and J3 are assigned on machines M1, M2 and M3. In this case, machine M1 is assigned three jobs called J_{11} , J_{21} and J_{31} . Also, we can release another type of split prohibited by the classical assignment model is that no job can be divided with that model Starr (2004).

Table 1: Three machines and three jobs

		<i>J1</i>	<i>J2</i>	<i>J3</i>
Machines	<i>M1</i>	J_{11}	J_{21}	J_{31}
	<i>M2</i>			J_{32}
	<i>M3</i>		J_{23}	J_{33}

In the Table 1, job J3 has split into three parts J_{31} , J_{32} and J_{33} .

The above difficulty can be overcome by transportation methods. Although, it has certain restricted assumptions of its own. Particularly, when demand and supply must be in common terms standard units. A convenient common terminology in our case is to express supply and demand in standard hours.

To create standard hours, it is necessary to assume that strict proportionality exists between the productivity rates of the machines. The next section will review this concept.

STANDARD UNITS

Definitions

Standard units are used as the common dominator for aggregated units in the production scheduling problem where jobs are converted from pieces of work (order size) into standard hours on a designated machine called the standard machine. Forecast of specific items such as sweaters of different colors cotton, wool, and mixtures are converted into the standard machine hours required to make each type of sweater. This is referred to as aggregation of the mixed-model product line of the job shop. Aggregating planning is achieved by collecting and lumping all the items to be produced together. The idea is to strip away the specifics while retaining the aggregate properties of the product. The purpose is to match aggregate capacity against aggregate demand. This results in generalized determination of workforce requirements.

Computation of Standard Hours

Standard hours computation is better to know through An example to show how actual hours of work can be changed into standard units of required production capacity.

In Table 2 the standard unit is selected as Standard Machine Hours (SMH) based on using a Standard Machine (SM) for the standard of comparison. The total production capacity consists of four machines. The four machines called M1, M2, M3 and M4, work at different rate. It should be noted that the rate differentials apply across all of the jobs that these machines are assigned. That is M2 is fastest for all of the jobs, and by the same amount in comparison with the other machines. For this reason, it is convenient to choose M2 as the Standard Machine (SM). It will be assigned an index of 100%. Then, all of the other machines will have fractional (SM) index.



Table 2: Supply of actual and SMH hours available per week

<i>Machines</i>	<i>SMH Index (%)</i>	<i>SMH per week</i>	<i>Actual Hours</i>
M1	70%	56	80
M2	100%	77	77
M3	60%	90	150
M4	50%	40	80
		263	387

As shown in the above table, M4 is half as fast as M2. The equation for the SMH index is derived in more general terms, as follows:

$$SMH\ index\ of\ M(ij) = \frac{Production\ output\ of\ (M_{ij})}{Production\ output\ of\ M(SM_j)}$$

For each machine, when SMH index is multiplied by the actual machine hours available per week, the standard machine hours (SMH) available per week are obtained.

Each machine is ranked by an index number that when multiplied by the actual machine hours available per week yields the standard machine hours (SMH) available per week. Thus, Table 2 provides the actual machine hours that are available per week converted by means of the SMH index. There is a total of 263 standard machine hours (SMH) available per week. In fact, there are 387 actual machine hours available per week has no significance for the resolution of the problem. Actual hours must first be converted to standard hours for application to the jobs that need to be done. This will be understood by examining the job requirements. In this department, the following represents a forecast of demand for next week for each job:

Jobs	A	B	C	D
Units	900	480	600	200

On the other hand, the productivity rate (the production output rate) of the standard machine for each of the jobs listed above is as follows:

Jobs	A	B	C	D
Productivity rate of standard machine	9	8	4	8

The computations are guided by the following equation (4) where unit dimensions are shown in parentheses. The dimension of units is the numerator and denominator is cancelled out. The remaining dimension is Standard Machine Hours (SMH). Thus, demand in units has been transformed to demand in standard machine hours for each job.

Input (D) as a unit → Output (D) as SMH

$$D_u \div \text{Pr} \left(\frac{\text{Units}}{\text{SMH}} \right) = D_{SMH} \quad (4)$$

where D_u : Demand in units for each kind of job

Pr : Production rate in units for SMH

D_{SMH} :Demand in aggregated standard hours.

According to the Equation (4), actual units of demand for Jobs A through D can be converted into standard machine hours (SMH) as follows:

D(SMH)

A: $900/9 = 100$ SM hours

B: $480/8 = 60$ SM hours

C: $600/4 = 150$ SM hours

D: $200/8 = 25$ SM hours

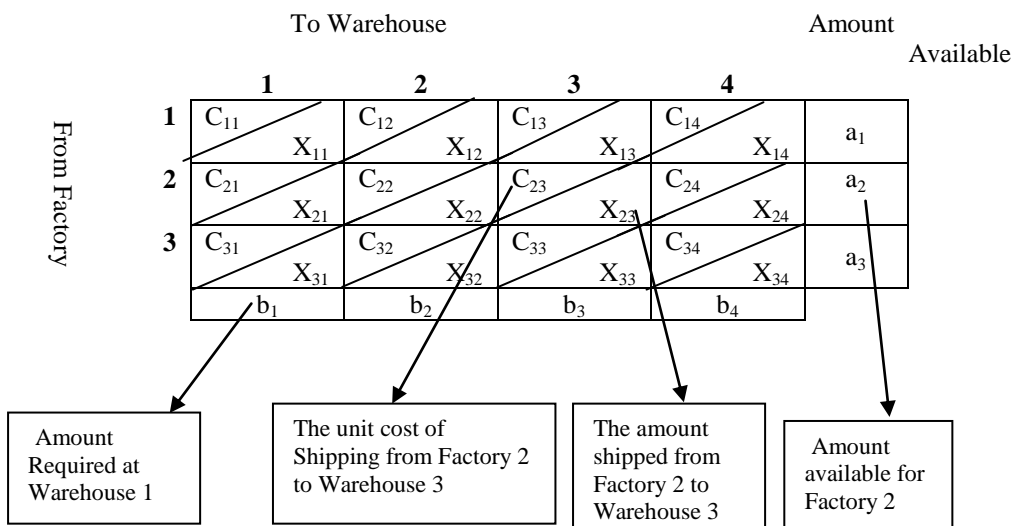
Total = 335 SM hours

Now, both supply (available capacity of machines) and demand (request for the job units) have been converted to the common terms of standard machine hours (SMH). Then this can be solved to find the loading arrangement that uses the resources available in terms of standard machine hours. In this case, the transportation model can be used to get the solution. On the other hand, the above loading problem can be dealt with linear programming. It does not have the constraint of proportionality.

Transportation Technique

Basically, the general transportation model deal with the distribution of goods from one location to another. The general transportation problem can be described in the form of a matrix with shipments from factories to warehouses with factories represented by the rows and warehouses by the columns of the matrix. In Figure 1, there are three factories and warehouses. Each cell in matrix represents a route from a particular factory to a particular warehouse. Listed on the right hand side of the matrix are the amounts available at each factory. Listed on the bottom of the matrix are the amounts required at each warehouse.

Figure 1: Transportation Model (Gillett 1985).



The objective of the transportation problem is to find the shipping routes from factories to warehouses which will minimize the total cost of transportation Russel, (2006). In each cell of the matrix the unit cost of shipping one unit through the cell or route is shown. The total cost of transportation is then the sum of the amounts shipped through each cell multiplied by the unit cost of shipping through that cell.

Mathematically, in this model, if we let

X_{ij} = amount shipped from factory i to warehouse j

C_{ij} = unit cost of shipping from factory i to warehouse j

Thus, the total transportation cost is: (Lockyer and Bestwick 1982).

$$C = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \quad (5)$$

where m is the number of factories and n is the number of warehouses.

We wish to find the values of X_{ij} which will minimize the value of C subject to the constraints

$$\sum_{i=1}^m X_{ij} = b_j \quad (6)$$

The total shipped to each warehouse j must equal the amount required at the warehouse.

$$\sum_{j=1}^n X_{ij} = a_i \quad (7)$$

The total amount shipped from factory I must equal the amount available at the factory.

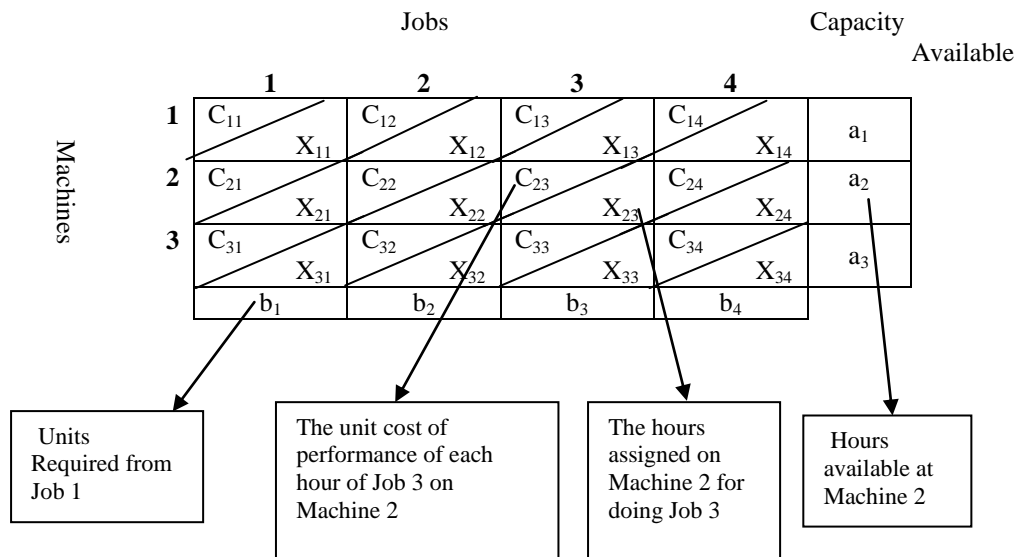
$$X_{ij} \geq 0$$

All shipments must be nonnegative.

To solve a transportation problem, we first find an initial solution (values of X_{ij}), and then improve the initial solution by reducing the cost through successive iterations until the minimum-cost is found. The transportation method is an iterative method, which reduces the solution on each successive iteration Nahimas, (1989).

Consequently, the above model can be modified and used to find the loading arrangement that uses the resources available to minimize the total cost. Then the assignment can be generated using the standard techniques employed to solve the transportation model. Figure 2 represents the equivalent matrix for the assignment model.

Figure 2: Equivalent Matrix for Assignment Model



Softwares are readily available to solve the model involving many variables. i.e. POM software package Natarajan, (2007). To solve the problem, we first find the initial solution and then improve the initial solution by reducing the cost. One way to find an initial solution is to use the North-West-Corner rule or Lowest Cost rule or the Vogel Approximation Method (VAM) Taha (2007). The next step is the evaluation of the solution for optimality of the initial solution. This can be carried out by using the Stepping Stone Method. Finally, the above problem can be solved alternatively by using the linear programming model.

IMPLEMENTATION

To implement the formula and to achieve the objective of this study, a real case study has been taken from the furniture industry. The problem was how to allocate 12 jobs on seven saw machines in cutting department. Any job related to one kind of product are named J1 upto J2 which can be assigned to any saw machine. The specification of each job is defined and the efficiency of each machine has also been defined according to the data collected from the production department. The processing time to complete each task is different. However, one of these machines (number 4) is more efficient than the others, which has been chosen to be the standard machine (SM). Table 3 shows relative productivity and the equivalent SMH index for each machine. Note that Machine 7 is half as fast as Machine 4.

Similarly, Machine 1 is 86% as fast as the standard machine (SM).

Table 3: Relative productivity and the equivalent SM

Machines	SMH Index (%)	Relative Productivity
M1	86%	86% as fast as the SM
M2	75%	75% as fast as the SM
M3	66%	66% as fast as the SM
M4(SM)	100%	Standard Machine
M5	55%	55% as fast as the SM
M6	80%	80% as fast as the SM
M7	50%	50% half as fast as the SM



The restrictive assumption is that the SMH index for each machine applies to all 12 jobs. It is called the presumption of productivity proportionality. Thus, Machine 4 is fastest for all of the jobs by the same amount, in comparison with the other machines. Table 4 provides the actual machine hours that are available per week. These hours are converted by means of the SMH index into standard machine hours available per week as shown in the fourth column. The actual machine hours available per week is 442. Table 4 concludes that total SMH is 324.75. Table 5 provides production rates Pr in pieces per hour for each machine and job. The entries on the matrix are labeled Pr(ij) where i=machine and j=job.

The production rate of the standard machine for the jth job is Pr(SM,j) see the fourth row (Machine 4) in Table 5. Table 6 contains the number of units ordered per job.

Table 4: Actual machine hours per week

Machines	SMH Index (%)	Actual Hours	SMH per week
M1	86%	50	53
M2	75%	60	45
M3	66%	75	49.5
M4	100%	78	78
M5	55%	35	19.25
M6	80%	60	48
M7	50%	84	42
Total		442	324.75

Table 5: Production rate in pieces per hour for each machine and job (Pr_{ij})

Machines	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12
M1	5.16	3.44	2.58	6.88	5.16	6.02	10.32	3.87	10.32	5.16	2.58	6.02
M2	4.5	3	2.25	6	4.5	5.25	9	3.375	9	4.5	2.25	5.25
M3	3.96	2.64	1.98	5.28	3.96	4.62	7.92	2.97	7.92	3.96	1.98	4.62
M4	6	4	3	8	6	7	12	4.5	12	6	3	7
M5	3.3	2.2	1.65	4.4	3.3	3.85	6.6	2.475	6.6	3.3	1.65	3.85
M6	4.8	3.2	2.4	6.4	4.8	5.6	9.6	3.6	9.6	4.8	2.4	5.6
M7	3	2	1.5	4	3	3.5	6	2.25	6	3	1.5	3.5

Table 6: Number of units ordered per job

Jobs	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12
No. of units required	120	108	141	145	150	200	570	100	245	310	120	74
Standard hours of demand	20	27	47	18.13	25	28.6	47.5	22.2	20.4	51.7	40	10.6

As shown in the Table 6, the requests initially stated as the number of units ordered per job. They vary greatly because there are many different types of orders. These quantities have been turned into standard machine hours to match the amount of supply available from machine of type M4.

To accomplish the transformation of all dimensions to standard hours, start by dividing the number of units requested for J1 by the production rate of the standard machine for Ji. This yields demand in standard hours instead of demand in units. The results are shown in the second row of the Table 6.

Now, both capacity available (supply) and the requests for the jobs (demand) have been converted to the common terms of standard machine hours (SMH). There are 324.75 machine hours available per week and 358.13 hours required to complete the orders. Therefore, 33.38 standard machine hours of demand will not be satisfied.

A dummy machine is created to pickup the slack. It is called MD (for machine dummy). By definition, MD has zero output for all jobs. Some part of whichever job is assigned to the dummy will not be done. The balanced capacity available $S(i)$ and request $D(j)$ with the number in standard hours including the output rate per standard hours in the upper right hand corner of each assignment cell is given in Table 7.

As with the assignment model, cost is considered with the transportation model. The cost is calculated for each job j , which is the cost per standard hours of each machine i . The cost per piece (C_{ij}) is independent of the machine used, given in Table 8.

Table 8: Cost C_{ij} per piece (Riyal)

Jobs	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12
Cost per piece (C_{ij})	200	470	550	700	320	150	480	780	100	950	400	100

The product output rates per standard hours have been multiplied by cost per piece to reflect the profit per standard hours ($C_{ij} \cdot P_{ij}$). These are shown in the small boxes in the upper right hand corner of each assignment cell. This information reflects in Table 9 which display the matrix adjusts for cost.

8. Generating the Assignment

After the information is arranged into transportation matrix (Table 9), the solution for assignment can be obtained. Although manual solution of transportation problems is fairly straightforward, computer solutions are generally preferred. The solution is generated by using POM software package. To solve the problem, first we find an initial solution and then improve the initial solution by reducing the cost. One way to find an initial solution is to use the North-West Corner rule or Lowest Cost rule or the Vogel Approximation Method (VAM). The next step in the transportation method is evaluation of the solution for optimality, given a start solution. By using the POM software, the optimal loading schedule is shown in Table 10 in standard machine hours with the minimum total cost (578177.80)

$$\text{Min (Cost) } Z = \left[\begin{array}{l} 20 \times 1032 + 1.4 \times 1419 + 20.4 \times 1032 + 1.2 \times 1032 + \\ 20 \times 1410 + 25 \times 1440 + 4.6 \times 1089 + 18.13 \times 3696 + \\ 22.2 \times 2317 + 4.57 \times 3762 + 28.6 \times 1050 + 38.8 \times 1200 + \\ 10.6 \times 700 + 19.25 \times 3135 + 7 \times 1504 + 41 \times 1320 + \\ 14.12 \times 2880 + 33.38 \times 0 + 27.88 \times 2850 \end{array} \right]$$



There are eight rows and twelve columns which means that $M+N-1=19$ assignments should be made for transportation model. The requisite nineteen are in place. The optimal has been found in terms of standard hours must now be converted back into actual machine hours, and actual job units. Divide the standard hours assignments by the SMH index for each machine. This is shown in Table 11 and Table 12. Table 11 represents the actual assignment in terms of actual units and Table 12 represents the actual assignment in terms of actual hours.

The load is established for each job by consulting Table 10, Table 11, and Table 12 and the calculations as follows:

For Job J1

$20 \text{ SMH} \div \text{SMH index of } 86\% \text{ equal } 23.3 \text{ actual hours assigned on Machine 1 for Job J1.}$
This yields $23.3 \times 5.16 = 120$ units for J1.

For Job J2

$20 \text{ SMH} \div \text{SMH index of } 75\% \text{ equal } 26.3 \text{ actual hours assigned on Machine 2 for Job J2.}$
This yields $26.3 \times 3 = 80$

$7 \text{ SMH} \div \text{SMH index of } 80\% \text{ equal } 8.75 \text{ actual hours assigned on Machine 6 for Job J2.}$
This yields $8.75 \times 3.2 = 28$

Total $80 + 28 = 108$ units as required per J2.

For Job J3

$1.4 \text{ SMH} \div \text{SMH index of } 86\% \text{ equal } 1.63 \text{ actual hours assigned on Machine 1 for Job J3.}$
This yields $1.63 \times 2.58 = 4.2$

$4.6 \text{ SMH} \div \text{SMH index of } 66\% \text{ equal } 7 \text{ actual hours assigned on Machine 3 for Job J3. This}$
yields $7 \times 1.98 = 13.8$

$41 \text{ SMH} \div \text{SMH index of } 80\% \text{ equal } 51.25 \text{ actual hours assigned on Machine 6 for Job J3.}$
This yields $51.25 \times 2.4 = 123$

Total $4.2 + 13.8 + 123 = 141$ units as required per J3.

For Job J4

$18.13 \text{ SMH} \div \text{SMH index of } 66\% \text{ equal } 27.5 \text{ actual hours assigned on Machine 3 for Job J4.}$
This yields $27.5 \times 5.28 = 145$

For Job J5

$25 \text{ SMH} \div \text{SMH index of } 75\% \text{ equal } 33.33 \text{ actual hours assigned on Machine 2 for Job J5.}$
This yields $33.33 \times 4.5 = 150$

For Job J6

$28.6 \text{ SMH} \div \text{SMH index of } 100\% \text{ equal } 28.6 \text{ actual hours assigned on Machine 4 for Job J6.}$
This yields $28.6 \times 7 = 200$

For Job J7

14.12 SMH ÷ SMH index of 50% equal 28.24 actual hours assigned on Machine 7 for Job J7. This yields $28.24 \times 6 = 169.4$

33.38 SMH is assigned to the dummy machine (MD) so Job J7 will be partially completed. This is a shortage of $(570 - 169.4) = 400.6$ units for Job J7.

For Job J8

22.2 SMH ÷ SMH index of 66% equal 33.6 actual hours assigned on Machine 3 for Job J8. This yields $33.6 \times 2.97 = 100$

For Job J9

20.4 SMH ÷ SMH index of 86% equal 23.7 actual hours assigned on Machine 1 for Job J9. This yields $23.7 \times 10.32 = 245$

For Job J10

4.57 SMH ÷ SMH index of 66% equal 6.9 actual hours assigned on Machine 3 for Job J10. This yields $6.9 \times 3.96 = 27.2$

19.25 SMH ÷ SMH index of 55% equal 35 actual hours assigned on Machine 5 for Job J10. This yields $35 \times 3.3 = 115.5$

27.88 SMH ÷ SMH index of 50% equal 55.8 actual hours assigned on Machine 7 for Job J10. This yields $55.8 \times 3 = 167.3$

Total $27.2 + 115.5 + 167.3 = 310$ units as required per J10.

For Job J11

1.2 SMH ÷ SMH index of 86% equal 1.4 actual hours assigned on Machine 1 for Job J11. This yields $1.4 \times 2.58 = 3.6$

38.8 SMH ÷ SMH index of 100% equal 38.8 actual hours assigned on Machine 4 for Job J11. This yields $38.8 \times 3 = 116.4$

Total $3.6 + 116.4 = 120$ units as required per J11.

For Job J12

10.6 SMH ÷ SMH index of 100% equal 10.6 actual hours assigned on Machine 4 for Job J12. This yields $10.6 \times 7 = 74$.



CONCLUSION

- In this research, problem of splitting the jobs among many machines and allow for each machine to perform a lot of jobs is tackled. This problem is mobilized by using standard machine hours (SMH) to convert lot of jobs as well as the actual hours of each machine to Standard Machine Hours (SMH).
- A real case study demonstrates how a production manager can use this model in real life. The problem was how to allocate 12 jobs on seven machines; one of these machines (No 4) has been chosen to be the standard machine (SM).
- The actual machine hours that are available per week were (442). These hours then converted by means of the SMH to 324.75.
- Later on, we suggest any available quantitative techniques, particularly, transportation model which is more convenient to deal when the supply and demand are in common terms which is in our case as SMH. In this case a dummy machine is created to pickup the slack standard machine hours 33.38.
- The optimal solution which has been found in terms of standard hours also converted back into actual machine hours, and actual job units.
- The solution for assignment has been obtained. Therefore splitting the jobs are permitted and processed by more one machine. Thus it can be seen from the results, machine M1 is assigned four jobs: J1, J3, J9, and J11. While job J2 is split and assigned to machines M2, and M6 as well as other jobs.
- Finally, the optimal loading schedule has been generated in standard machine hours with the minimum total cost (578177.80).

RECOMMENDATION

Based on the empirical findings of this study, following several points can be talked later on which are not discussed in this paper.

- The transportation model have been solved for total cost minimization, it is as simple to use profits and solve for total profit maximization. Total time minimization, productivity maximization, and other goals can be chosen as well.
- The formulation which has been used in this study assumes that costs per piece are the same on machines. If not, it would be necessary to use the cost(C_{ij}) for each cell in the matrix.
- A criterion that must be satisfied for using the transportation approach to shop loading is that reasonable proportionality exists between machines output rates. When the SMH index does not apply, a heuristic modification of the transportation method can be tried. If one machine is especially efficient for J1 and another machine is best for J2, the heuristic could make those assignments more profitable, or less costly. In other words, some assignments would be made a priori based on clear advantages.
- An issue we have not addressed is uncertainty of processing time in practice. It is possible and even likely that the exact completion time of one or more jobs may not be predictable in advance. It is of interest to know whether or not there are some results concerning the optimal assignment when processing time is uncertain

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