PARAMETRIC STUDY OF LAMINAR NATURAL CONVECTION IN GLAZING ENCLOSURE

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ABSTRACT

This work includes a numerical investigation of steady two dimensional laminar natural convection heat transfer in glazed rectangular enclosure at different tilt angles ranged from (0^o to 90^o) for Rayleigh number (10^3-10^6), aspect ratios(4,5, and 8),and Prandtl number (0.7). The study considers the effect of different boundary conditions on the heat transfer within the enclosure. The continuity equation, Navier-Stokes equations and the energy equation are solved by developing a numerical model based on finite volume method using SIMPLE algorithm with hybrid scheme to compute the velocity vectors, temperature, pressure, and Nusselt No. (local and average). Results show an increase in heat transfer rate with increasing of Rayleigh number, and for low Rayleigh numbers, the conduction is the dominant heat transfer mode. At Ra= 10^3, the mean Nusselt number remains constant around unity for all cases. The tilt angle has the major effect on the heat transfer for Rayleigh number larger than 10^4. A comparison of the present results with previously published data is conducted, and good agreement is seen to indicate the effectiveness and flexibility of the developed numerical procedure.

الخلاصة

يتناول هذا البحث دراسة عددية لجريان الهواء وانتقال الحرارة بواسطة الحمل الحر الطباقي في حيز زجاجي مستطيل ذو زوايا ميل تتراوح بين(0°-90°) ولحالة مستقرة ثنائية البعد.الدراسة تغطي المدى لرقم رايلي من10^3 الى 10^6 ولنسب باعية (8,5,4) ولعدد برانتل (Pr=0.7). تم حل معادلات الاستمرارية , معادلات نافير- ستوكس ومعادلة الطاقة ببناء نموذج عددي بأستخدام طريقة الحجم المحدد وأتباع خطوات (SIMPLE) بالطريقة الهجينة لحساب قيم كل من , مركبات السرعة ودرجة الحرارة والضغط ورقم نسلت (الموضعي والمعدل). بينت النتائج ان معدل انتقال الحرارة يزداد مع زيادة رقم رايلي , وان انتقال الحرارة بالتوصيل يظهر بشكل واضح عند رقم رايلي منخفض, في حين ان رقم نسلت المتوسط يبقى ثابتا بقيمة واحدة عند رقم رايلي 10^3. وأن زاوية ميل الحيز لها التأثيرالأكبر على أنتقال الحرارة لقيم رايلي أكبر من 10^4. تمت مقارنة النتائج المستحصلة في هذه الدراسة مع المنشورة سابقا ووجد تقارب جيد جدا بينهما للدلالة عل فعالية ومرونة الحل العددي الذي تم بناءه.

KEYWORDS

natural convection; rectangular enclosure; uniform and non-uniform heating

INTRODUCTION

The number of heat transfer applications in which the natural convection is a dominant phenomenon within enclosures is large, and better understanding of this phenomenon has even increased the number of applications and has led to a number of sophisticated industrial and environmental designs. Natural convection flows in enclosures such as double pane systems have received a great interest in engineering applications. Windows have always been a significant part of building design, and provide many functions and play a significant role in daily building operations. **Hiroyuki Ozoe (1973)** , studied (numerically using FDM and experimentally) two dimensional natural circulations in an inclined, confined box heated on one side and cooled on the opposing side. The angle of inclination was varied from (0^o to 180^o) for four aspect ratios. The study assumed Pr=0.7 and Rayleigh number ranged from 10^3 to 10^6. The preferred mode of fluid circulation was observed to change with the angle of inclination and the aspect ratio. He found that a minimum in the heat flux occurred at the point of transition (merging of the circulation cells), while the maximum occurred as the angle was further increased**. Ivan Catton** (1973), studied numerically natural convection flow in a finite, two-dimensional rectangular slot arbitrarily oriented with respect to the gravity vector for aspect ratios (0.1 to 20), and tilt angles from -30^o to $+75^o$ and Rayleigh numbers up to $2*10^6$.It was found that the flow structure and heat transport were dependent on aspect ratio, angle of tilt and Rayleigh number. **Mayer, Mitchell, and El-Wakil**(1979) studied experimentally the effect of natural convection in moderate aspect ratio (0.25-4) enclosures using an interferometric technique. The slat enclosure is represented as an array of cells which are essentially contiguous enclosures each having a small aspect ratio. The Rayleigh number range tested was up to $7*10^4$ and the angles were from 45^o to 90^o.They showed that the convective heat transfer is a strong function of the aspect ratio (less than 4). Also slat angles less than 90^o (i.e., oriented downward) reduce convective heat transfer. **Elsherbiny et al**. (1982), studied experimentally the effect of thermal boundary conditions on natural convection in vertical and inclined air layers between two isothermal plates of different temperature depends on the side and end walls boundary conditions. These walls are usually assumed to be perfectly conducting or adiabatic. The study has qualitatively described the wall conduction, radiation and fluid convection interactions. **Wang** and **Hamed** (1980), investigated numerically steady two-dimensional natural convection in air filled, rectangular enclosure using finite volume approach. The effect of various configurations of bidirectional temperature gradients on mode transition of thermal convection inside the cavity has been investigated. Simulations have been carried out for Rayleigh numbers ($10^3<Ra<10^6$), aspect ratio=4 and angle of inclination from (0^o to 90^o) . They showed that it is quite possible to alter the resulted heat transfer rates by adjusting the thermal conditions; by varying cavity angle of inclination, or by modifying initial conditions of fluid inside the cavity.

In this work, laminar flow in inclined glazing enclosure is adopted for various heating cases to study the effect of different boundary conditions and different effective parameters on flow field and heat transfer within it.

MATHEMATICAL MODEL AND BOUNDARY CONDITIONS

Figure.(1) depicts the schematic of the rectangular enclosure considered here. The enclosure has a length of (L) along the x-axis and a width of (W) along the y-axis. The two side walls of the enclosure are insulated, the others perfectly conducting walls are considered isothermal and are maintained at constant temperatures T_h and T_c , respectively**.** The tilt angle γ , is the angle of inclination of the enclosure with respect to the horizontal.

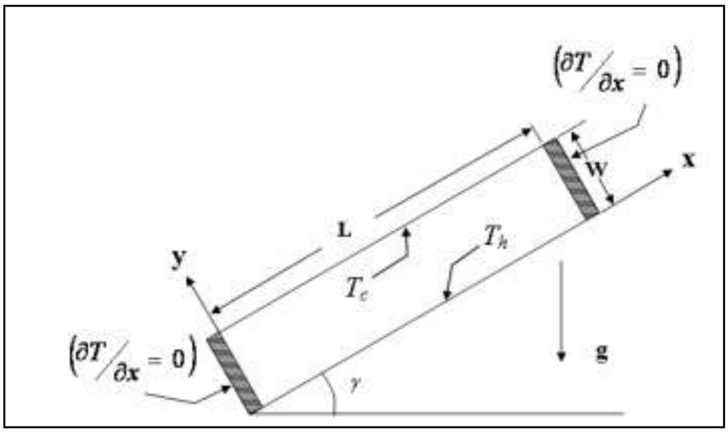


Fig. (1): Two-Dimensional Enclosure Geometry.

The following assumptions accompanied to the adopted mathematical model: Steady, two-dimensional natural convection flow inside the enclosure, assuming constant and employing the Boussinesq approximation for the gravity term. The governing equations for natural convection flow using conservation of mass, momentum and energy can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\left.\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\beta g(T-T_c)\sin\gamma \\ u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\beta g(T-T_c)\cos\gamma \end{aligned}\right\} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (3)$$

Using the following change of variables:

$$Y=\frac{y}{W},\ X=\frac{x}{W},\ V=\frac{vW}{\alpha},\ U=\frac{uW}{\alpha},\ \theta=\frac{T-T_c}{T_h-T_c},\ \Pr=\frac{\nu}{\alpha},\ P=\frac{pW^2}{\rho\alpha^2},\ Ra=\frac{\Pr g\beta(T_h-T_c)W^3}{\nu^2} \qquad (4)$$

The governing equations (1)-(3) reduced to the following set of dimensionless equations:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad (5)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-Ra\Pr\theta\sin(\gamma)$$

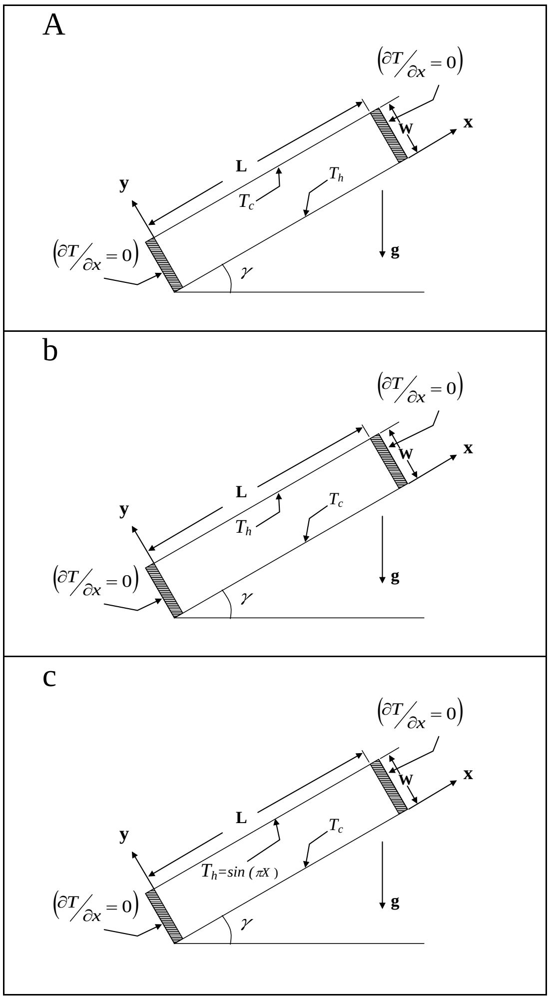


Fig.(2): Enclosure Geometry And Coordinate System For The Studying Cases

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-Ra\Pr\theta\cos(\gamma) \qquad (6)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (7)$$

The Boussinesq approximation is introduced in the present study, which treat the density as a constant variable in the continuity equation and in the inertia term of the momentum equation, but allow it to change with temperature in the buoyancy terms, such that:

$$\rho=\rho_\infty\left(1-\beta\left[T-T_\infty\right]\right) \qquad (8)$$

The no-slip boundary condition is applied at all four boundaries.

$U(X,0)=U(X,1)=U(0,Y)=U(\mathrm{I},Y)=0$ (where I = L/W)

$V(X,0)=V(X,1)=V(0,Y)=V(\mathrm{I},Y)=0$

Thermal boundary conditions adopted in the present work are related to the cases considered shown in Fig (2). These cases are:

I. Enclosure which has a lower wall heated and upper wall cooled with two insulated sidewalls as shown in Fig (2a). i.e.:

$\theta(X,0)=1$

$\theta(X,1)=0$ (Constant wall temperature)

$\frac{\partial\theta}{\partial X}(0,Y)=\frac{\partial\theta}{\partial X}(I,Y)=0$ (Insulated sidewalls)

II. Enclosure which has a lower wall cooled and upper wall heated with two insulated sidewalls as shown in Fig (2b). i.e.:

$\theta(X,0)=0$

$\theta(X,1)=1$ (Constant wall temperature)

$\frac{\partial\theta}{\partial X}(0,Y)=\frac{\partial\theta}{\partial X}(I,Y)=0$ (Insulated sidewalls)

III. Enclosure with upper wall is subjected to non-uniform temperature (sinusoidal) distribution in space coordinate while the lower wall is cooled and the sidewalls are kept insulated as shown in Fig (2c).

i.e.:

$\theta(X,0)=0$ (Constant wall temperature)

$\theta(X,1)=\sin(\pi X')$ (Sinusoidal temperature distribution and X'=Xw/L)

$\frac{\partial\theta}{\partial X}(0,Y)=\frac{\partial\theta}{\partial X}(1,Y)=0$ (Insulated sidewalls)

The Nusselt number was calculated using the following equation:

$$Nu_{ave}=\frac{1}{W}\int_0^W\frac{q}{k(T_h-T_c)}dx \qquad (9)$$

NUMERICAL ALGORITHM AND RESULTS VALIDITY

Eqs. (5) to (8) are solved using FVM and the SIMPLE algorithm. The procedure was regarded converged when maximum difference of all independent variables (U, V, P, and θ) and maximum change in calculated average Nusselt number between two successive iterations were less than 10^{-4} .

These governing differential equations can be rewritten in the general form as: **Patankar** (1980)

$$\frac{\partial}{\partial x}(\rho u\Phi)+\frac{\partial}{\partial y}[\rho v\Phi]=\frac{\partial}{\partial x}\left[\Gamma_{\Phi}\frac{\partial\Phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[\Gamma_{\Phi}\frac{\partial\Phi}{\partial y}\right]+S_{\Phi} \qquad (10)$$

Where the left hand side represents the convective terms, the right hand side represents the diffusion and source terms. The physical domain is divided into elements as shown in Fig.(3), and a Fortran 90 computer program is developed with the structure chart shown in Fig. (4).

RESULTS AND DISCUSSION

Verification of the Numerical Discussion

In order to check the accuracy of the results obtained using the present numerical algorithm, results were compared with those available in the literature. The computed and the reference Nusselt No. values of **Wang** (2005) and **Yao** (1999) along with the percentage deviation, are presented in table (1). **Wang** and **Yao** studied an enclosure at ($\gamma = 0^{\circ}$) with insulated upper and lower walls and isothermal side walls. Table (1) shows that the minimum and maximum deviations are (0.0007%) and (2.288%) respectively.

Where: Deviation % = $\dfrac{Nu_{ref} - Nu_{comp}}{Nu_{ref}} \times 100$

Table.1. Verification of the Mean Nusselt number

Ar	Pr	Ra	Nu			Deviation %
			Present Study	Wang	Huafu Yao	
4	0.7	1000	1.000003	1.00001	-	0.0007
4	0.7	3000	1.554243	1.57002	-	1.004
4	0.7	5000	2.000682	2.0022	-	0.075
4	0.7	10^4	2.507354	2.55	-	1.672
1	0.71	10^4	2.294327	-	2.243	2.288
1	0.71	10^5	4.520762	-	4.519	0.039
1	0.71	10^6	8.774747	-	8.9693	2.169

Glazing Enclosure Heated From Lower Surface - (Case I)

The effect of inclination angle is studied by varying (γ) from 0° to 90° on flow and temperature fields for Ra=10^4 are shown in Fig. (5) for (Ar = 4 and 8), respectively . At (Ar=4), the multi-cells begin to coalesce into one. A pronounced one-ce11 motion was observed between $30^{\circ} < \gamma < 90^{\circ}$ while multicellular form could be observed for (Ar=8) at

(γ=0). At an angle of 60° hot wall produces an acceleration to the flow along the top and bottom walls, causes the velocity vectors in the core region to stretch toward the top right and bottom left corner of the enclosure. The number of circulation cells for Ar=8 is larger than that for Ar=4 due to the effect of shallow effect of the enclosure. The circulation cells tends to merge and move to the upper end of the inclosure with increasing of the inclination angle as shown in Fig. (6) due to the gravity effect. Fig.(7) illustrates the velocity vectors and isotherm contours for Ra=10^3,10^4,10^5, and 10^6 for Aspect ratio of (4 and 8) at γ =0°. As Ra increased slightly, the upslope temperature gradient causes a change in the number of cells from four to two for (Ar=4), and from six cells to four cells for (Ar=8). Fig.(8) shows the relation between the Nu_{ave} vs. Ra for various aspect ratios and inclination angle. Results show that e Nu_{ave} increases as increasing the Ra No. and at Ra=1000 the value of Nu_{ave} was unity which indicate that the conduction and convection effect is the same. Also it can be shown that the specified values of inclination angle the circulation fluid cells merged and cause a decrease in Nu number, while a slight effect of Ar is shown.

Glazing Enclosure Heated From Upper Surface - (Case II)

The effect of the inclination angle is studied by varying γ from 0° to 90° for the case of Ra=10^4. The Isotherms and Velocity vectors at Aspect ratios (4, 5, and 8) was studied and found to be so similar in trend as that for case II , therefore only Ar= 4 is presented in Fig. (9) in this section to avoid repetition.. This figure shows that for (γ =0°), two fluid cells are formed, while they are merged for higher tilt angles, also a weak convection heat transfer exist tending the fluid to move towards the enclosure ends at the upper heated wall. This fluid movement increases with increasing of γ noting that the fluid moves upward near the heated side due to decreasing the fluid density, and moves downward near the colder wall to form one flow cell for (γ =30°, 60°, & 90°). Effect of Ra No. on the vertical velocity distribution and the isothermal contour along enclosure centerline is shown in Fig. (10) where the fluid flows is clock-wise unicellular, except for (Ar=4, Ra=10^5, γ=0) where twin in opposite direction circulation fluid cells are formed, also it shows that increasing of Rayleigh number leads to increase the circulation strength due to the increase in temperature difference. As Ra increases to 10^5 and 10^6 ,the thermal boundary layer gets intensive and thinner. Compressed isotherms extend longer along the y-axis and the isotherms are further distorted due to the stronger convection, this is also shown in Fig.(11).

Fig.(12) shows the effect inclination angle on the averaged Nu for (Ar= 4 and 8).It can be concluded that increasing γ leads to increasing the heat transfer and the effect of increasing the aspect ratio has larger effect, due to the confirmation effect to fluid circulations in shallow enclosure. Also it can be shown that for γ=0 all values of Nu_{avg} is equal to one i.e convection effect is the same as conduction.

Glazing Enclosure with Non-Uniform Heating of Upper Surface - (Case III)

The effect of the inclination angle is studied by varying γ from 0° to 90° and Ra=10^4 for the case(III) as shown in Fig.(13). The Isotherms and Velocity vectors at Aspect ratios (4, 5, and 8) was found to be so similar in trend, therefore only (Ar= 4) is presented in this section to avoid repetition. It can be seen from velocity plot at γ=0°, two circulating fluid cells are formed inside the enclosure. Further increase in the tilt angle to γ=30° and γ=60° leads to merge to a unicellular pattern with the tendency to increase the rotational velocity with the tilt angle. At the vertical position γ=90°, where the gravitational field is parallel to the boundary layer the circulation cell is squeezed towards the upper section of the enclosure.

The effect of Rayleigh number on the heat transfer and fluid motion is investigated for ranges of 10^3, 10^4, 10^5 , and 10^6 as shown in Fig.(14).

For small Ra, the fluid is almost stagnant and pure conduction through adjacent fluid layers dominates the heat transfer as it is confirmed by the temperature distribution. It also noted for horizontal position that the temperature at the top wall is non –uniform and a maximum temperature is located at the center. At $Ra=10^5$, the circulation pattern is qualitatively similar to the uniform upper surface heating as shown in Fig.(15). Fig.(16) shows the effect of Ra No. on the vertical velocity distribution along the enclosure centerline. Fig. (17) shows the combined effect of inclination angle and aspect ratio and Ra No. on the averaged Nu No. The same trend in case II is presented with the exception that the conduction is dominated at $\gamma=0$ than convection **.**

CONCLUSION

Steady, two-dimensional, natural convection in an air-filled rectangular enclosure has been numerically investigated using the finite –volume based SIMPLE numerical algorithm. Based on the obtained results, conclusions can be listed as follows. When $Ra \leq 10^3$, the inclination of the enclosure does not have any influence on $\overline{Nu}$ for all aspect ratios Ar =4, 5 and 8. And for $5*10^3 \leq Ra \leq 10^4$,the Nusselt number tends to increase when the cavity inclination increases and then drops to minimum value when transition mode occurs. Heat transfer is increased with the increasing of Rayleigh number for all parameters which affect on flow and temperature field. Inclination angle is the dominant parameter on fluid flow and heat transfer and has a big effect on the cellular fluid form.

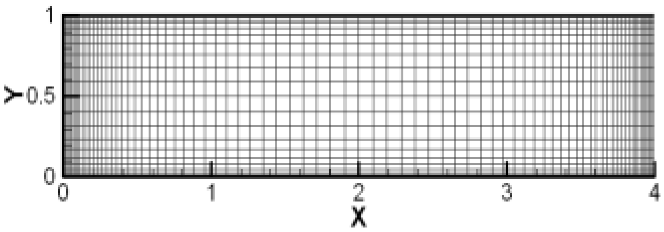


Fig.(3):The Computational Domain of the Enclosure

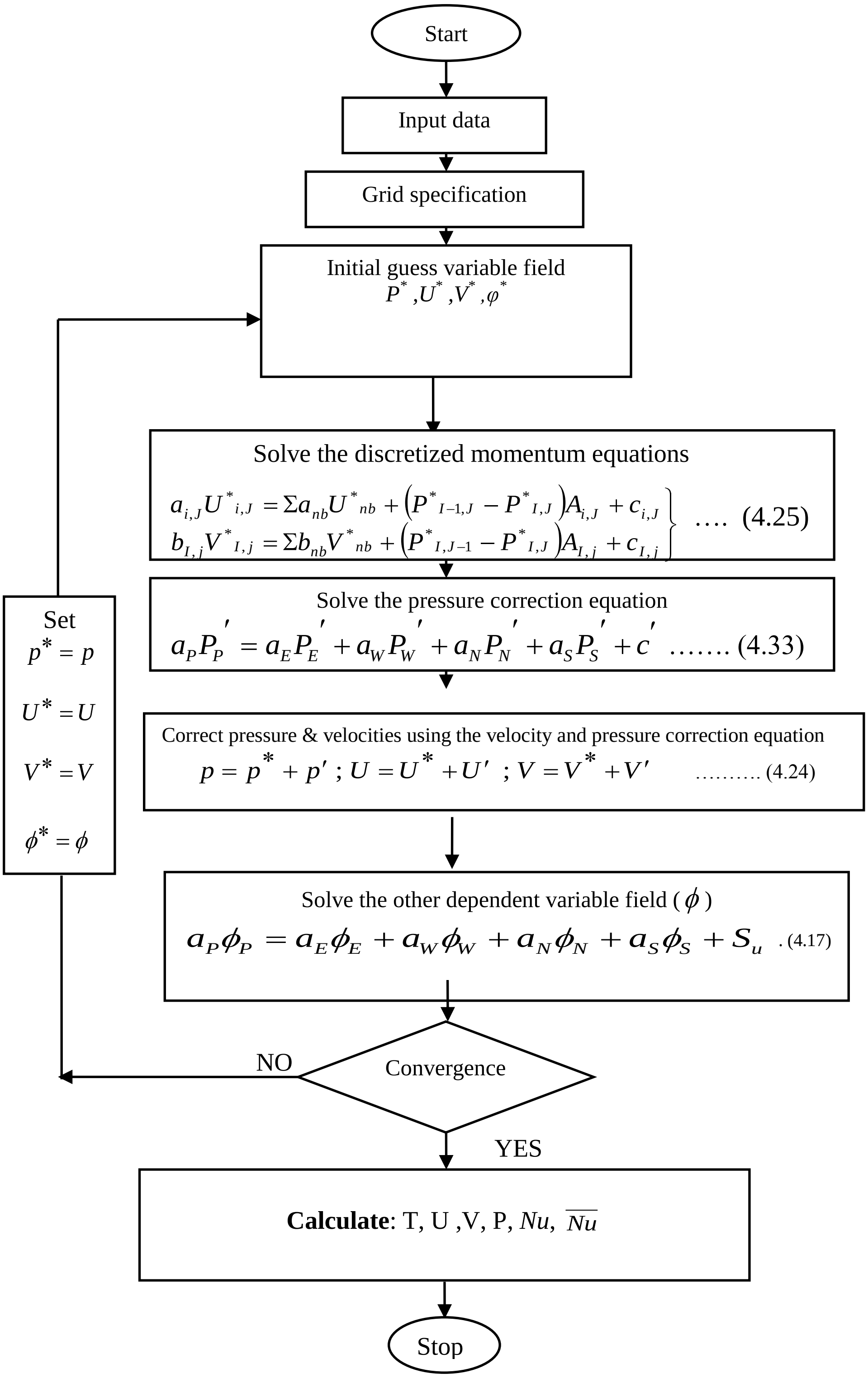
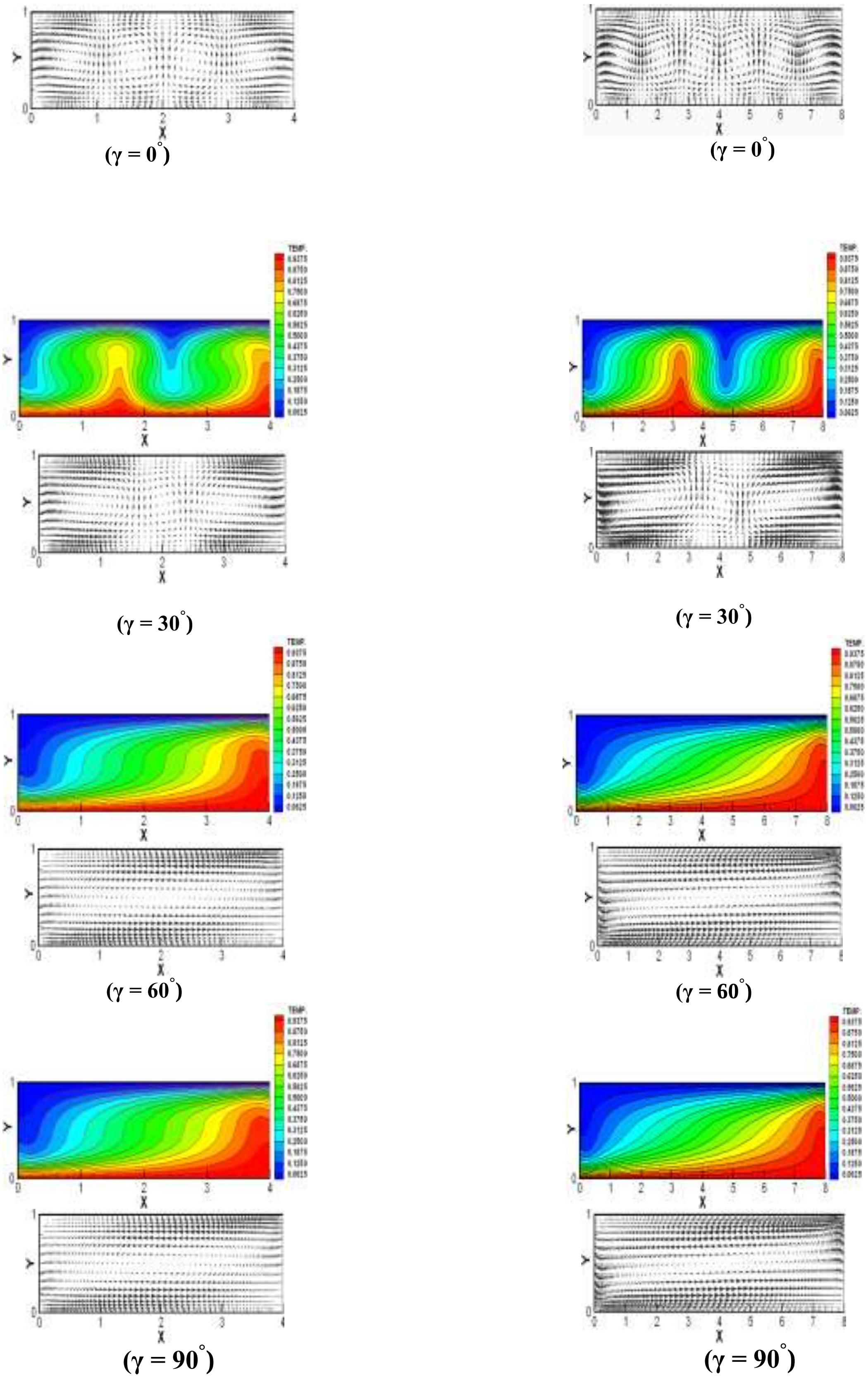
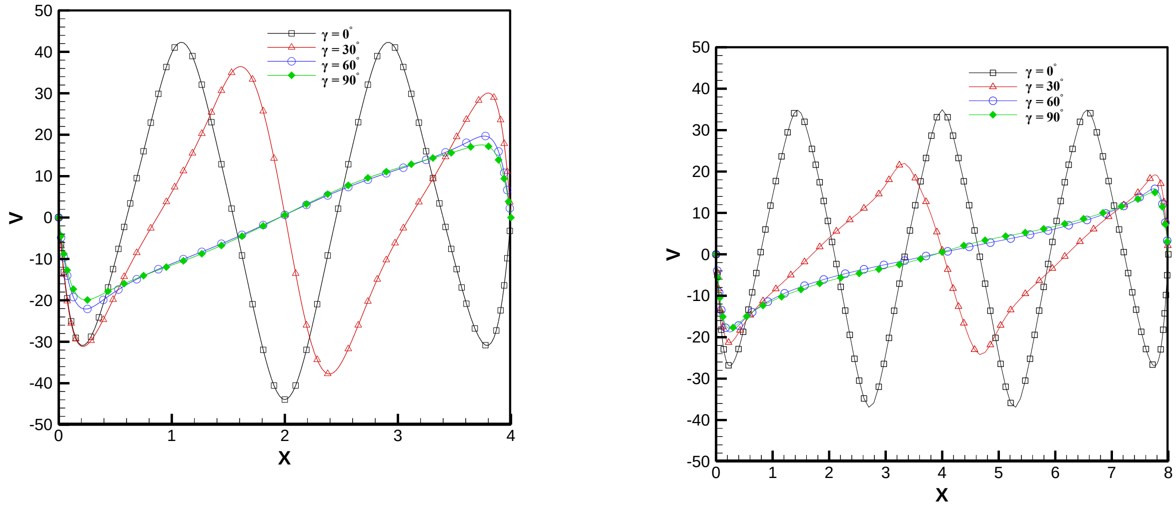


Fig.(4): Flow Chart of the Computer Program.



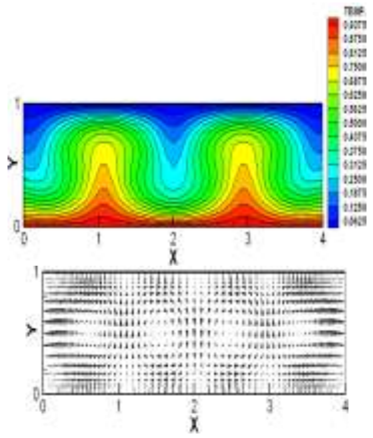
Ar=4 Ar=8

Fig (5) Effect of Inclination Angle on Temperature Contours and Velocity Vectors for (Case I),
[Ar= 4 and 8, Ra=10^4]

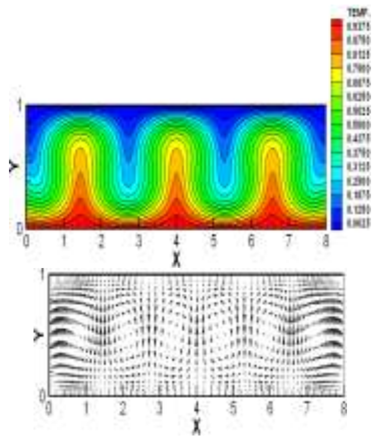


Ar=4 **Ar=8**

Fig (6): Effect of Inclination Angle on Vertical Velocity Distribution along Enclosure Centerline for (Case I), [Ar = 4 and 8, Ra =10^4]



(Ra = 10^3)



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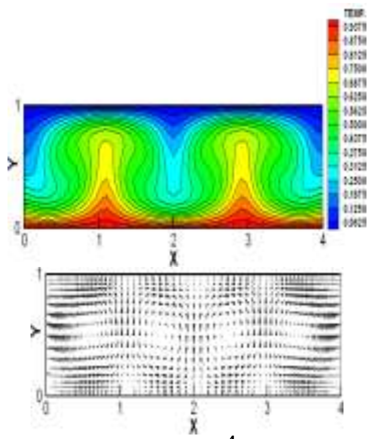

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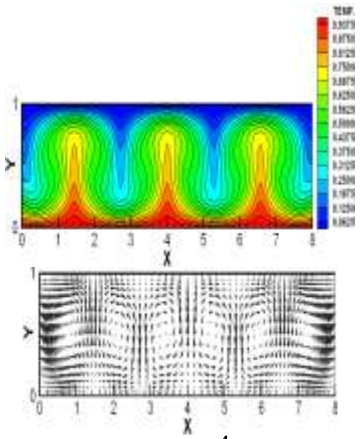
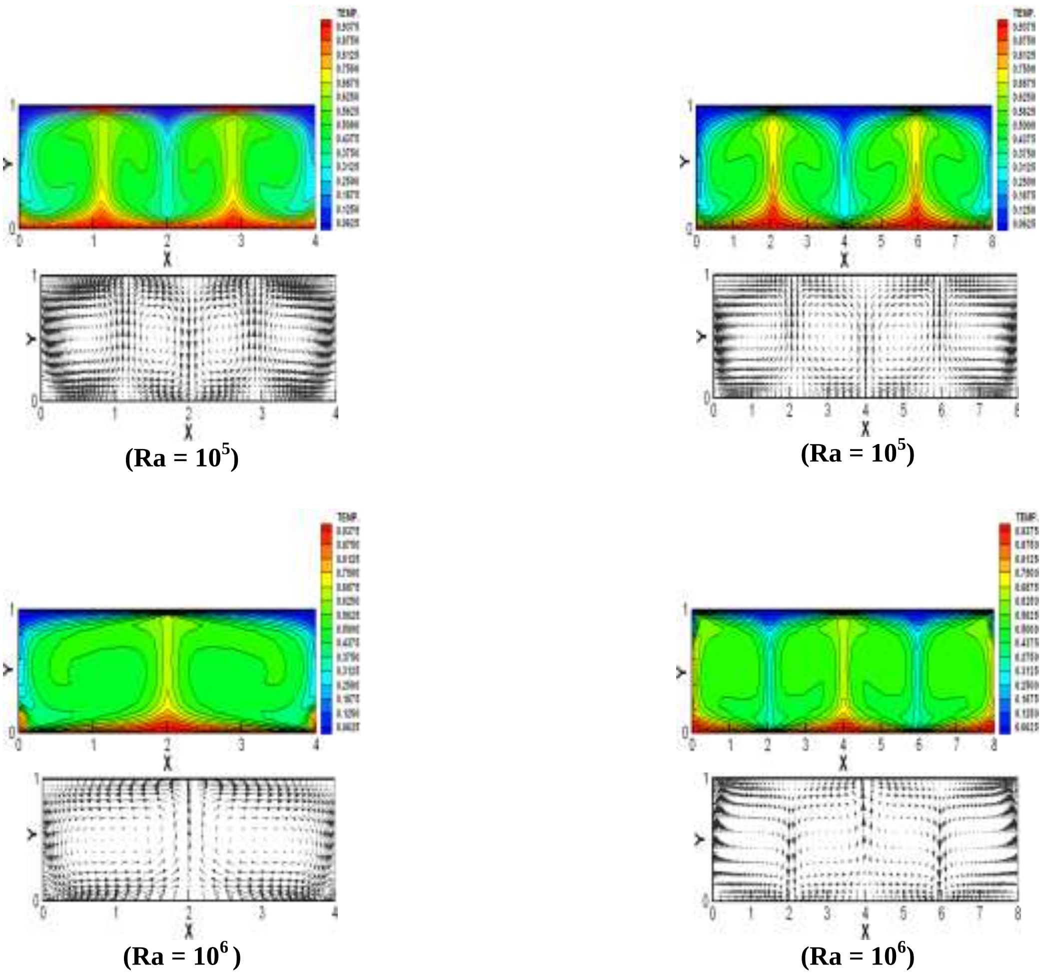
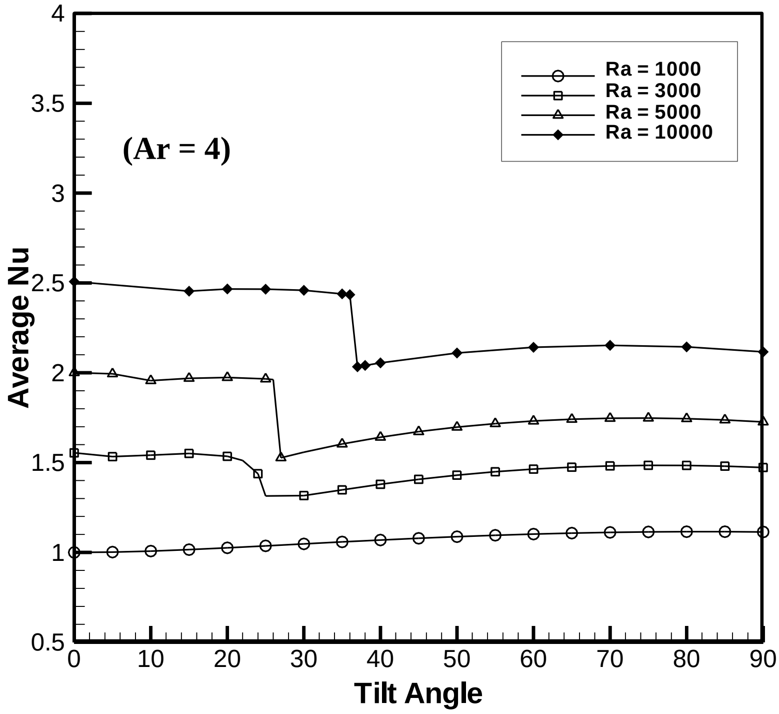


(Ra = 10^4)



Ar = 4 **Ar = 8**

Fig (7): Effect of Rayleigh Number on Temperature Contours and Velocity Vectors for (Case I),[Ar = 4 and 8, γ =0°]



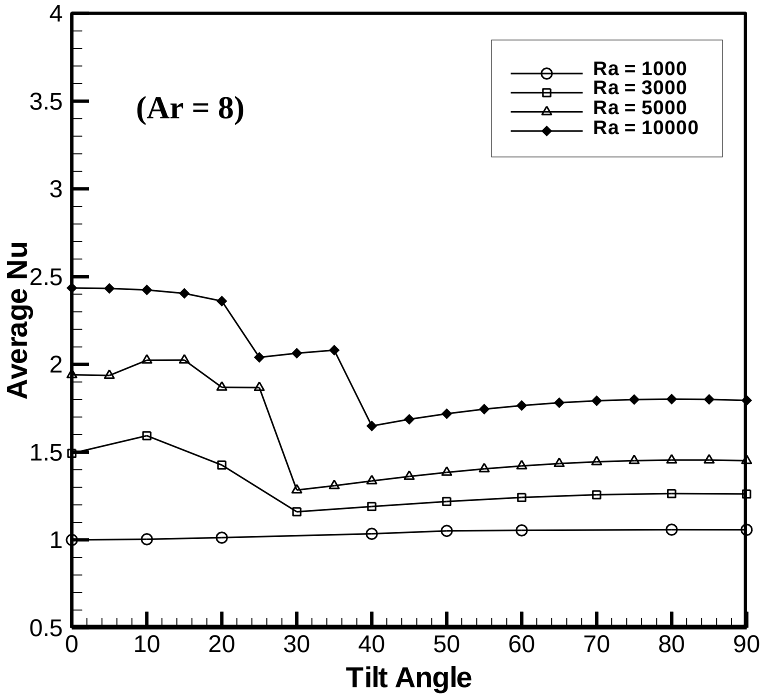
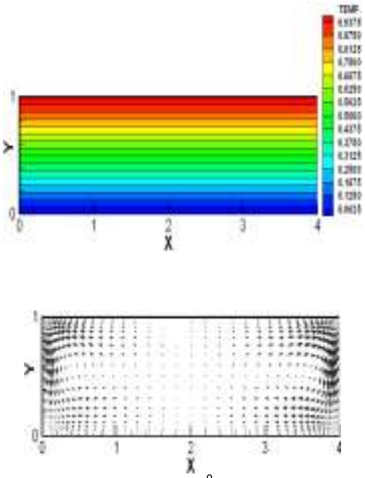
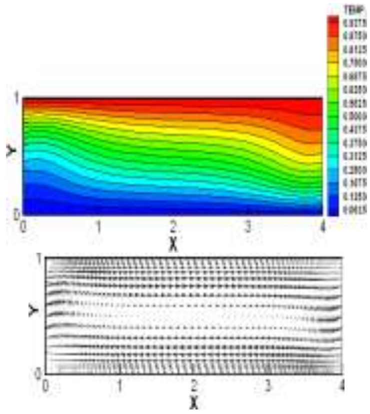


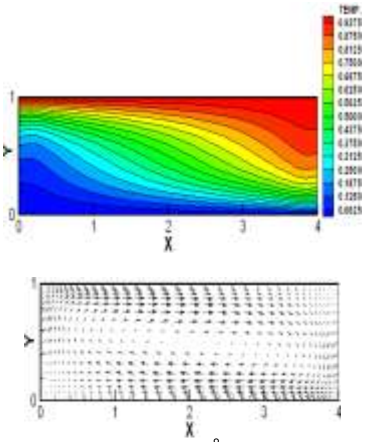
Fig (8); Effect of Aspect Ratio on Average Nusselt Number at Different Inclination Angles for (Case I).



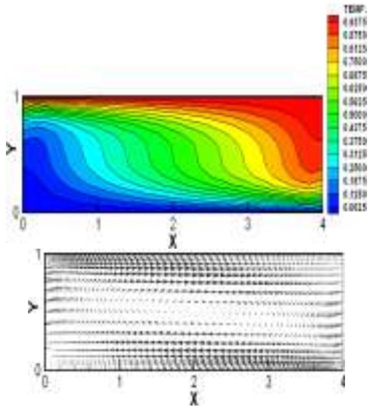
($\gamma = 0^{\circ}$)



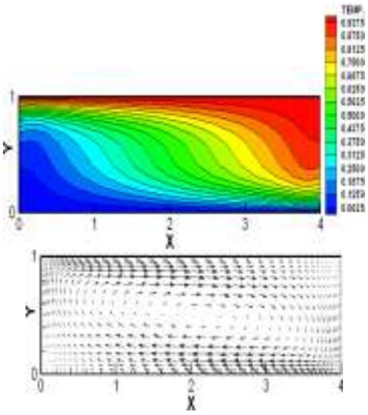
(Ra = 10^3)



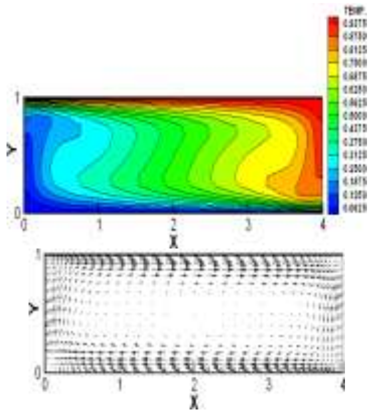
($\gamma = 30^{\circ}$)



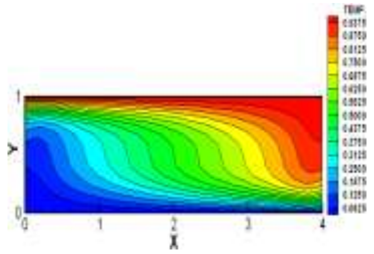
(Ra = 10^4)



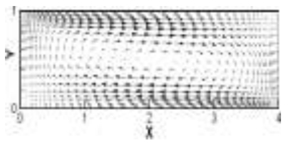
($\gamma = 60^{\circ}$)



(Ra = 10^5)



($\gamma = 90^{\circ}$)



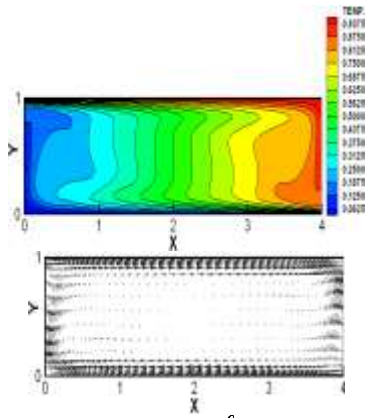
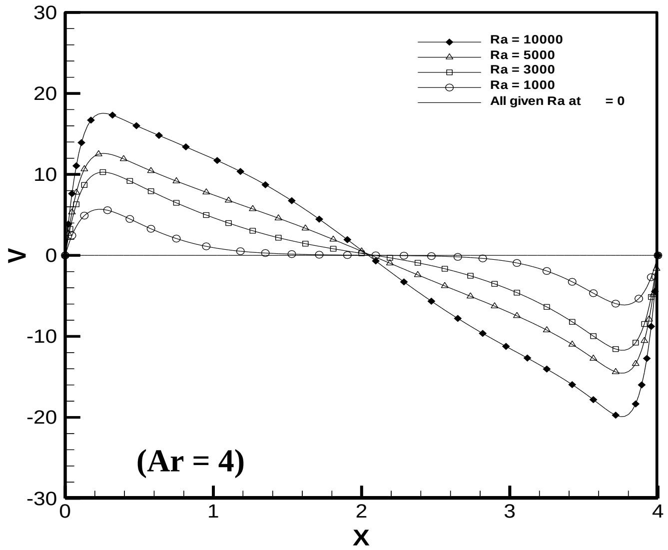
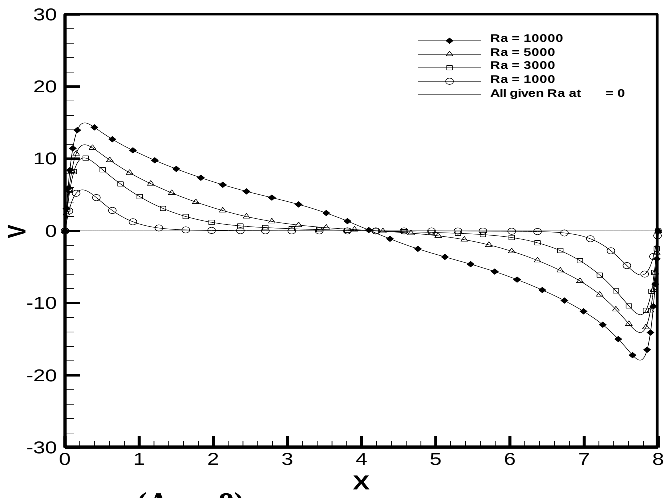


Fig (9): Effect of Inclination Angle on TemperatureContours and Velocity Vectors for (Case II), [Ar= 4 , Ra=10^4]

(Ra = 10^6)

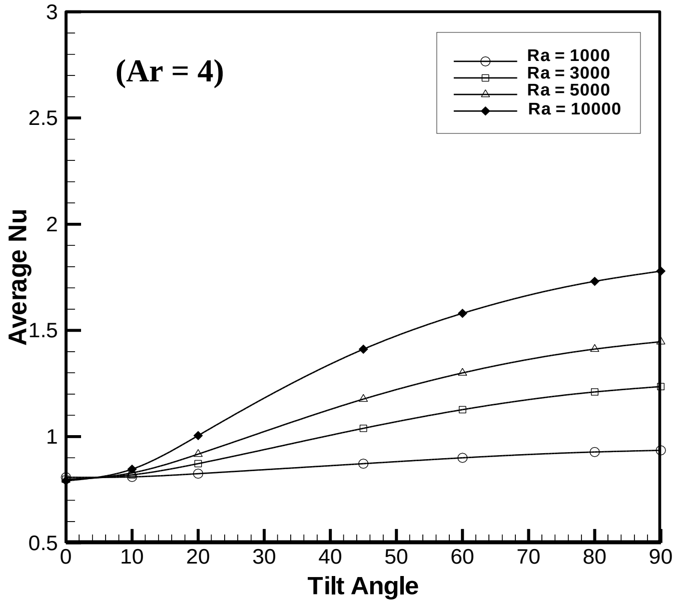
Fig (10): Effect of Rayleigh Number on Temperature Contours and Velocity Vectors for (CaseII), [Ar = 4, and $\gamma = 90^{\circ}$]





(Ar = 8)

Fig (11): Effect of Rayleigh Number on Vertical Velocity Distribution along Enclosure Centerline for (CaseII),[Ar = 4, and $\gamma = 90^{\circ}$]



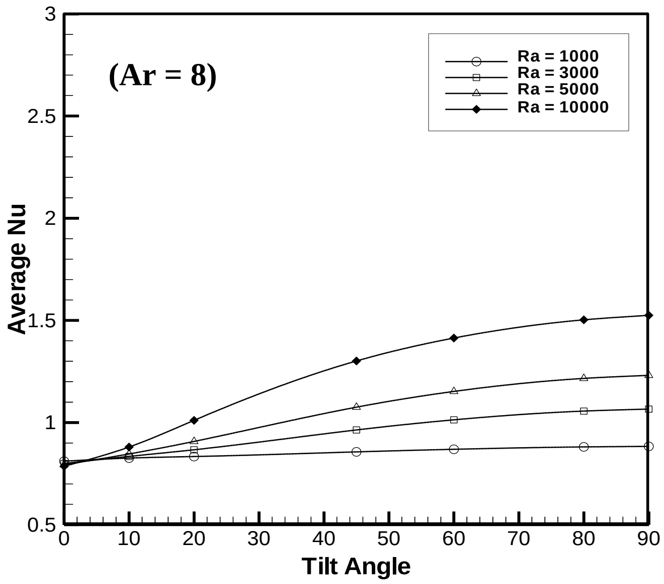
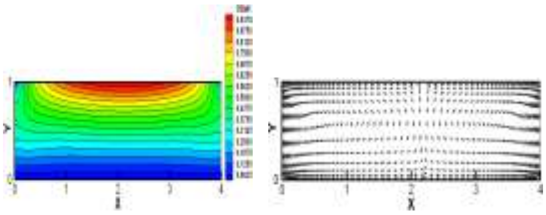
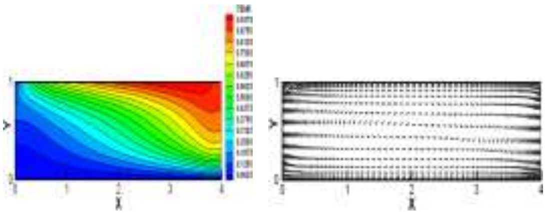


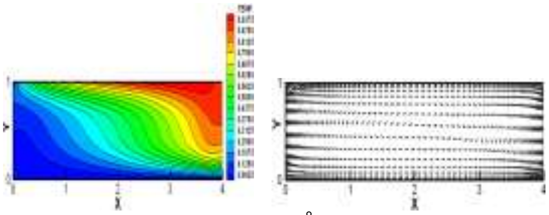
Fig (12) Effect of Aspect Ratio on Average Nusselt Number at Different Inclination Angles for (Case II).



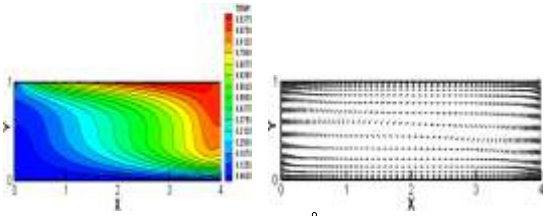
($\gamma = 0^{\circ}$)



($\gamma = 30^{\circ}$)

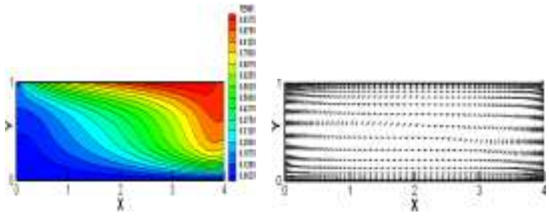


($\gamma = 60^{\circ}$)

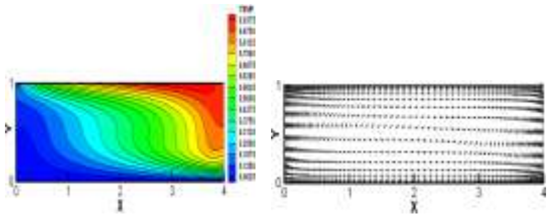


($\gamma = 90^{\circ}$)

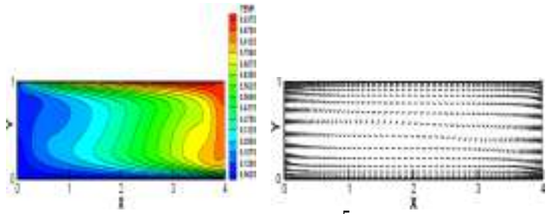
Fig (13): Effect of Inclination Angle on TemperatureContours and Velocity Vectors for (Case III),
[Ar= 4 , Ra=10^4]



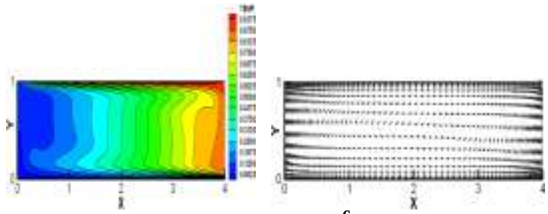
(Ra = 10^3)



(Ra = 10^4)



(Ra = 10^5)



(Ra = 10^6)

Fig (14) Effect of Rayleigh Number on Temperature Contours and Velocity Vectors for (CaseIII),
[Ar = 4, and $\gamma = 90^{\circ}$]

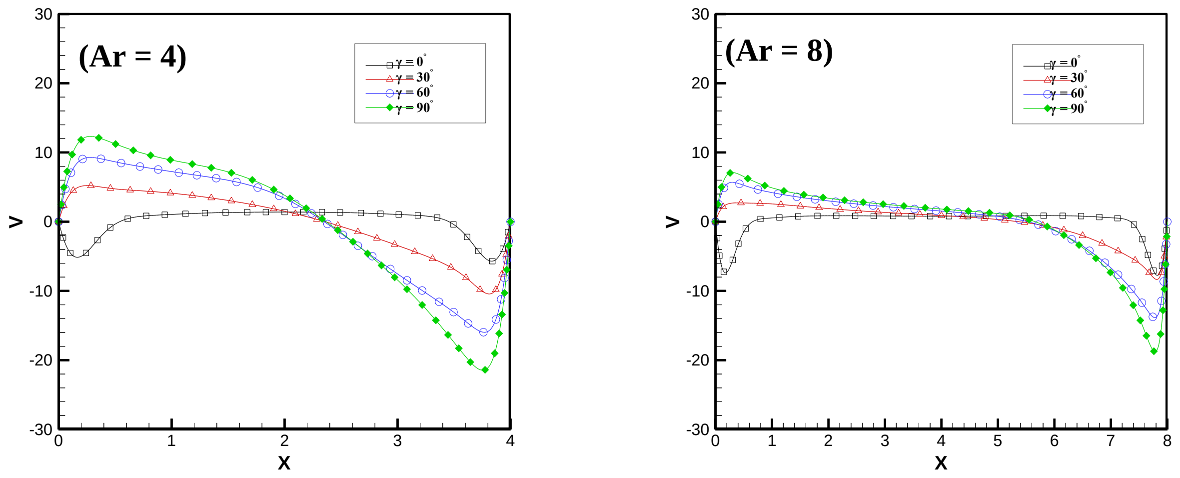


Fig.(15): Effect of Inclination Angle on Vertical Velocity Distribution along Enclosure Centerline for (Case III), [Ar = 4 and 8, Ra =10^4]

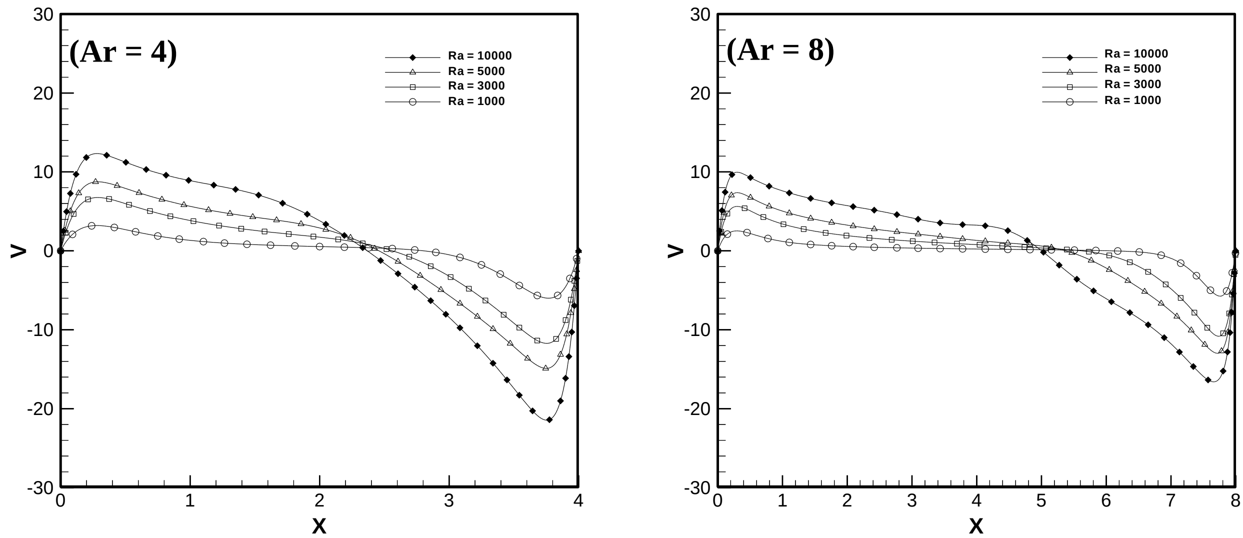


Fig (16): Effect of Rayleigh Number on Vertical Velocity Distribution along Enclosure Centerline for (CaseIII),
[Ar = 4, and γ = 90°]

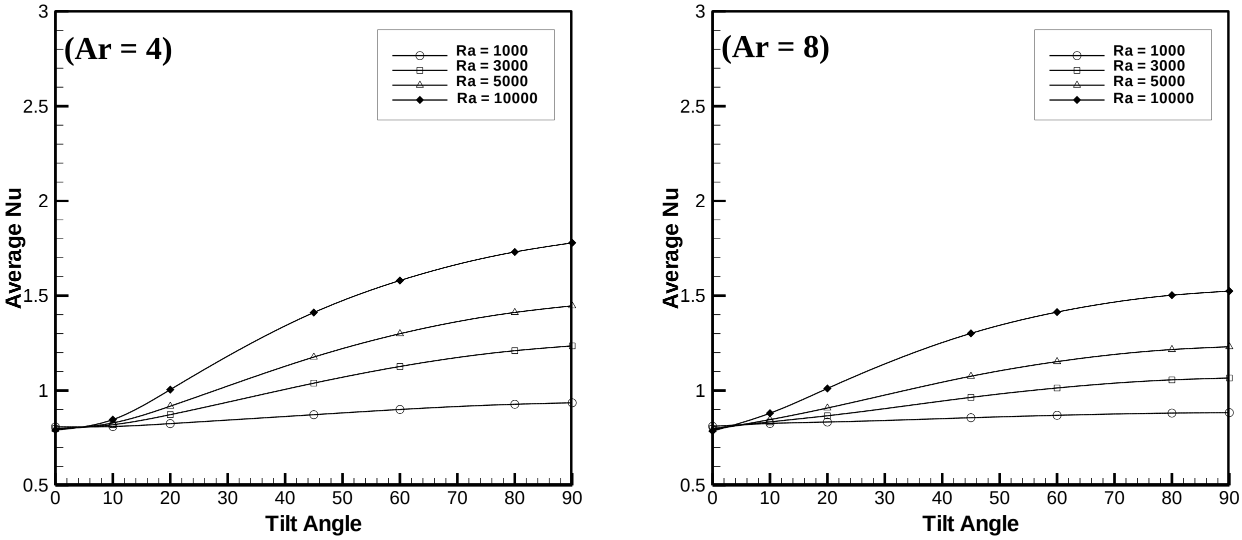


Fig (17) Effect of Aspect Ratio on Average Nusselt Number at Different Inclination Angles for (Case III).

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Fig (4-5) Effect of Inclination Angle on Temperature Contours and

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- **NOMENCLATURE**

LATIN SYMBOLS

A_r	L/W Dimensionless aspect ratio	S	Source term
g	Gravitational acceleration	u	Velocity component in x-direction m/s
h	Convective heat transfer coefficient W/m^2 ^{o}C	U	Dimensionless velocity in x-direction
K	Thermal conductivity W/m.^{o}C	v	Velocity component in y-direction
Nu_{ave}	Average Nusselt number	V	Dimensionless velocity in y-direction
P	Pressure (Pa)	T	Temperature ^{o}C
Pr	Prandtl number,□□□□□□□□□	W	Dimensionless width of enclosure
Q	Heat flux per unit area (W/m^2)	x,y	Cartesian coordinate
Ra	Rayleigh number	X,Y	Dimensionless Cartesian coordinate

GREEK SYMBOLS

α	Thermal diffusivity
β	Thermal expansion coefficient
Γ	Diffusion coefficient
ϕ	General dependent variable
γ	Inclination angle
ν	Kinematics viscosity
ρ	Density
θ	Dimensionless Temperature

Superscripts & Subscripts

-	Average
C	Cold
h	Hot