Vibration analysis of angle-ply laminates composite plate under thermo-mechanical effect

Dr. Adnan Naji Jameel
professor
College of Engineering- University of Baghdad
Email:adnanaji2004@yahoo.com

Rasha Mohammed Hussien
Assistant lecturer
College of Engineering- University Technology
Email:mechanicalflower99@yahoo.com

ABSRACT

The paper presents mainly the dynamic response of an angle ply composite laminated plates subjected to thermo-mechanical loading. The response are analyzed by analytically using Newmark direct integration method with Navier solution, numerically by ANSYS. The experimental investigation is to fabricate the laminates and to find mechanical and thermal properties of glass-polyester such as longitudinal, transverse young modulus, shear modulus, longitudinal and transverse thermal expansion. Present of temperature could increase dynamic response of plate also depending on lamination angle, type of mechanical load and the value of temperature.

Keywords: composite laminated plate, dynamic response, thermo-mechanical loading

تحليل الاهتزاز للصفائح المركبة تحت تأثير احمال ميكانيكية حرارية

أ.د.عدنان ناجي جميل
استاذ
كلية الهندسة\ جامعة بغداد

م.م. رشا محمد حسين
مدرس مساعد
قسم هندسة الميكانيكية\ جامعة بغداد

الخلاصة هذه الدراسة ركزت بشكل رئيسي على الاستجابة الديناميكية للصفائح المركبة ذات زاوية معينة معرضة الى تاثير درجة الحرارة. ويتم تحليل الاستجابة الديناميكية عن طريق التحليل النظري باستخدام طريقة نيومارك مع حل نافيير ، التحليل العددي باستخدام برنامج .ANSYSفي الجانب العملي تم تصنيع الصفائح المركبة المصنوعة من الالياف الزجاجية والبوليستر لايجاد الخواص الحرارية والميكانيكية مثل معامل يونك الطولي والعرضي و معامل القص والتمدد الحراري الطولي والعرضي . وكان الاستنتاج الرئيسي ان وجود درجة الحرارة يمكن ان يزيد الاستجابة الديناميكية اعتمادا على زاوية التصفيح ونوع الحمل الميكانيكي ودرجة الحرارة.

الكلمات الرئيسية: صفائح مركبة, الاستجابة الديناميكية,الحمل حراري الميكانيكية.

1. INTRODUCTION

During the last decades, needs for *composite materials* consist of two or more types of materials mixed together homogenously have appeared to produce desirable properties , the constituents are combined at a macroscopic level, one constituent is called the reinforcing phase called fiber and the one in which is embedded is called the matrix. Reddy J.N.]

The analysis of structural vibration is necessary in order to calculate the natural frequencies of a structure, and the response to the expected excitation. In this way it can be determined whether a particular structure will fulfil its intended function and, in addition, the results of the dynamic loadings acting on a structure can be predicted, such as the dynamic stresses, fatigue life and noise levels. Ref. [Beards C.E]

Many researches had studied free vibration analysis and vibration of plate under mechanical or thermal or thermo-mechanical loading**.**

Chorng-Fuh Liu and Chih-Hsing Huang,1996 performed a vibration analysis of laminated composite plates subjected to temperature change. The first order shear deformation theory of a plate is employed. The resulting finite element formulation leads to general nonlinear and coupled simulation equations and calculate the frequencies of vibration of a symmetric cross-ply plate. **Hui-Shen Shen, et.al**, **2003,** studied the dynamic response of laminated plates subjected to thermo-mechanical loading and resting on a two-parameter elastic foundation. The formulation is based on higher order shear deformable plate theory and includes the thermal effect. Effects of foundation stiffness, thickness ratio, and temperature change on the dynamic response are discussed. **Kullasup P. et al., 2010,** analysed free vibration of symmetrically laminated composite plates with various boundary conditions by Kantorovich method. The beam function is used as an initial trial function in the repeated calculation, which is employed to calculate the natural frequency. **Suresh K. J. et al., 2011,** developed an analytical procedure is to evaluate the free vibration characteristics of laminated composite plates based on higher order shear deformation with zig-zag function. Slope discontinuities improved by Zig-zag function at the interfaces of laminated composite plates. The solutions are obtained using Navier's method. **Junaid Kameran Ahmed et al., 2013,** presented a static and dynamic analysis of Graphite /Epoxy composite plates. In this work the behavior of laminated composite plates under transverse loading using an eight-node diso-parametric quadratic element based on First Order Shear Deformation Theory was studied,. **Pushpendra k. kushwaha1 and jyoti vimal, 2014,** the natural frequencies and mode shapes are compared for different boundary condition. Comparisons are made with the result for thin and thick composite laminated plate. Numerical results have been computed for the effect of number of layers, thickness ratio of plate, different boundary conditions, different aspect ratio, and different angle of fiber orientation of laminated composite plate.

The point of originality of the present work is how to derive the analytical solution of dynamic response for composite laminated plates by classical laminated plate theory for the first time under thermo-mechanical loading, by applied different type of loading on the symmetric and anti- symmetric angle ply composite laminated plates using Newmark direct integration method with Navier solution. Thermal and mechanical properties for composite plate made from (glass-polyester) with fiber volume fraction (0.3) are determined experimentally.

Also Finite element coded by ANSYS14.0 used to find natural frequency of composite laminate plate.

2. ANALYTICAL SOLUTION (CLASSICAL LAMINATE PLATE THEORY)

2.1 Displacement

Classical lamination theory (CLPT) based on the Kirchhoff hypothesis based on assuming the straight line perpendicular to the mid surface before deformation remains straight after deformation which means neglecting shear strains and transverse normal strain and stress in the analysis of laminated composite plates. Ref.[Reddy J.N.]

$$u(x,y,t)=u_o(x,y,t)-z\frac{\partial w_0}{\partial x} \qquad \textbf{(1. a)}$$

$$v(x,y,t)=v_o(x,y,t)-z\frac{\partial w_0}{\partial y} \qquad \textbf{(1. b)}$$

$$w(x,y,t)=w_o(x,y) \qquad \textbf{(1. c)}$$

Where $\frac{\partial w_0}{\partial x}$, $\frac{\partial w_0}{\partial y}$ denote the rotations about y and x axis respectively.

u_o,v_o and w_o denote the displacement components along (x, y, z) directions respectively of a point on the mid-plane (i.e.…z=0).

2.2 Stress and Strain

The total strains can be written as follows

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \end{Bmatrix} + z * \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} - \alpha_{xx}\Delta T \\ \frac{\partial v_0}{\partial y} - \alpha_{yy}\Delta T \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - \alpha_{xy}\Delta T \end{Bmatrix} + z \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2 * \frac{\partial^2 w_0}{\partial x\,\partial y} \end{Bmatrix} \qquad \textbf{(2.a)}$$

Where ($\varepsilon_{xx}^{(0)}$, $\varepsilon_{yy}^{(0)}$, $\gamma_{xy}^{(0)}$) are the membrane strains and ($\varepsilon_{xx}^{(1)}$, $\varepsilon_{yy}^{(1)}$, $\gamma_{xy}^{(1)}$) are the flexural (bending) strains, known as the curvatures α_{xx} , α_{yy} and $\alpha_{xy}$are thermal expansion coefficients defined

$$\alpha_{xx} = \alpha_{11}(\cos\theta)^2 + \alpha_{22}(\sin\theta)^2 \qquad \textbf{(2.b)}$$

$$\alpha_{yy} = \alpha_{11}(\sin\theta)^2 + \alpha_{22}(\cos\theta)^2 \qquad \textbf{(2.c)}$$

$$2\alpha_{xy} = 2(\alpha_{11} - \alpha_{22})\sin\theta\cos\theta \qquad \textbf{(2.d)}$$

α_{11} and α_{22} Are longitudinal and transverse thermal expansions respectively. And θ is the lamination angle.

The change in temperature defined

$$\Delta T = \textit{applied temperature} - \textit{reference temperature} \qquad \textbf{(2.e)}$$

Where reference temperature $T_{ref} = 25C^{\circ}$ [Reddy J.N.]. The transformed stress-strain relations of an orthotropic lamina in a plane state of stress are; for $\bar{Q}_{ij}$ see [Reddy J.N.]

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k \begin{Bmatrix}\varepsilon_{xx} - \alpha_{xx}\Delta T\\ \varepsilon_{yy} - \alpha_{yy}\Delta T\\ \gamma_{xy} - 2\alpha_{xy}\Delta T\end{Bmatrix} \qquad \textbf{(3)}$$

The resultant of inplane force *Nxx* ,*Nyy* and *Nxy* and moments *Mxx*, *Myy* and *Mxy* acting on a laminate can be obtained from integration of the stress in each layer or lamina through the laminate thickness. Knowing the stressed in terms of the displacements, the inplane force resultants *Nxx*, *Nyy*, *Nxy*, *Mxx*, *Myy* and *Mxy* can be obtained.

The inplane force resultants are defined as

$$\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}_k dz \qquad \textbf{(4.a)}$$

Where σ_x , σ_y and σ_{xy} are normal and shear stress.

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xy}^{0} \\ \varepsilon_{xy}^{0} \\ \gamma_{xy}^{0} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{1} \\ \varepsilon_{yy}^{1} \\ \gamma_{xy}^{1} \end{Bmatrix} - \begin{Bmatrix} N_{xx}^{t} \\ N_{yy}^{t} \\ N_{xy}^{t} \end{Bmatrix} \quad \textbf{(4.b)}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_k zdz \quad \textbf{(5.a)}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xy}^{0} \\ \varepsilon_{xy}^{0} \\ \gamma_{xy}^{0} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^{1} \\ \varepsilon_{yy}^{1} \\ \gamma_{xy}^{1} \end{Bmatrix} - \begin{Bmatrix} M_{xx}^{t} \\ M_{yy}^{t} \\ M_{xy}^{t} \end{Bmatrix} \quad \textbf{(5.b)}$$

Here, A_{ij} are the extensional stiffness, B_{ij} the coupling stiffness, and D_{ij} the bending stiffness.

$$A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (z_{k+1} - z_k) \quad \textbf{(6.a)}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\bar{Q}_{ij})_k ({z^2}_{k+1} - {z^2}_k) \quad \textbf{(6.b)}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\bar{Q}_{ij})_k ({z^3}_{k+1} - {z^3}_k) \quad \textbf{(6.c)}$$

And Where$\{N^t\}$ and $\{M^t\}$ are thermal stress and bending results, respectively

$$\begin{Bmatrix} N_{xx}^{t}, M_{xx}^{t} \\ N_{yy}^{t}, M_{yy}^{t} \\ N_{xy}^{t}, M_{xy}^{t} \end{Bmatrix} = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 2\alpha_{xy} \end{Bmatrix} (1, z)\, \Delta T dz \quad \textbf{(6.d)}$$

2.3 Equation of Motion

The equations of motion are obtained by setting the coefficient of δu_0 , δv_0 , δw_0 to zero separately

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w}{\partial x \partial t^2} \quad \textbf{(7.a)}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \frac{\partial^2 v}{\partial t^2} - I_1 \frac{\partial^3 w}{\partial y \partial t^2} \quad \textbf{(7.b)}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \widehat{N}_{xx}\frac{\partial^2 w}{\partial x^2} + \widehat{N}_{yy}\frac{\partial^2 w}{\partial y^2} + \widehat{N}_{xy}\frac{\partial^2 w}{\partial x \partial y} = I_0 \frac{\partial^2 w}{\partial t^2} + I_1\left(\frac{\partial^3 u}{\partial x \partial t^2} + \frac{\partial^3 v}{\partial y \partial t^2}\right) - I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2}\right) - q(x,y,t) \quad \textbf{(7.c)}$$

Where

$$(I_0, I_1 . I_2) = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \rho^{(k)} (1, z, z^2) dz \quad \textbf{(8)}$$

$\rho^{(k)}$ being the material density of k^{th} layer and q(x,y,t) is a dynamic force subjected on a system. $\widehat{N}_{xx}, \widehat{N}_{yy}\ and \widehat{N}_{xy}$ equal to zero because there were no buckling.

These equations of motion (7 a-c) can be expressed in terms of displacements **(**δu_0 , δv_0 , δw_0) by substituting the forces results from Eqs. (4 ,5,8) into Eq. (**7.a**) to**(7.c)** and get partial differential equations,

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} + \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{Bmatrix} \ddot{u}_0 \\ \ddot{v}_0 \\ \ddot{w}_0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q \end{Bmatrix} + \begin{Bmatrix} f_1^t \\ f_2^t \\ f_3^t \end{Bmatrix} \quad \textbf{(9.a)}$$

$$c_{11} = A_{11} d_x^2 + 2A_{16} d_x d_y + A_{66} d_y^2 \quad \textbf{(9.b)}$$

$$c_{12} = A_{16} d_x^2 + (A_{12} + A_{66}) d_x d_y + A_{26} d_y^2 \quad \textbf{(9.c)}$$

$$c_{13} = -\left[B_{11} d_x^3 + 3B_{16} d_x^2 d_y + (B_{12} + 2B_{66}) d_x d_y^2 + B_{26} d_y^3\right] \quad \textbf{(9.d)}$$

$$c_{22} = A_{66}d_x^2 + 2A_{26}d_xd_y + A_{22}d_y^2 \quad \textbf{(9.e)}$$

$$c_{23} = -\left[B_{16}d_x^3 + (B_{12} + 2B_{66})d_x^2d_y + 3B_{26}d_xd_y^2 + B_{22}d_y^3\right] \quad \textbf{(9.f)}$$

$$c_{33} = -D_{11}d_x^4 - 4D_{16}d_x^3d_y - 2(D_{12} + 2D_{66})d_x^2d_y^2 - 4D_{26}d_x - D_{22}d_y^4 - \left(A_{11}\alpha_{xx} + A_{12}\alpha_{yy} + 2A_{16}\alpha_{xy}\right)\Delta T d_x^2 - \left(A_{16}\alpha_{xx} + A_{26}\alpha_{yy} + 4A_{66}\alpha_{xy}\right)\Delta T d_xd_y - \left(A_{12}\alpha_{xx} + A_{22}\alpha_{yy} + 2A_{26}\alpha_{xy}\right)\Delta T d_y^2 \quad \textbf{(9.g)}$$

$$f_1^t = \frac{\partial N_{xx}^t}{\partial x} + \frac{\partial N_{xy}^t}{\partial y} \quad \textbf{(9.h)}$$

$$\boldsymbol{f_2^t} = \frac{\partial N_{xy}^t}{\partial x} + \frac{\partial N_{yy}^t}{\partial y} \quad \textbf{(9.i)}$$

$$\boldsymbol{f_3^t} = -\left(\frac{\partial^2 M_{xx}^t}{\partial x^2} + 2\frac{\partial^2 M_{xy}^t}{\partial y \partial x} + \frac{\partial^2 M_{yy}^t}{\partial y^2}\right) \quad \textbf{(9.j)}$$

And the coefficients m_{ij} is defined by

$$m_{11} = -I_0d_t^2\ , m_{13} = I_1d_xd_t^2, m_{22} = -I_0d_t^2; m_{23} = I_1d_yd_t^2\ , m_{33} = I_0d_t^2 - I_2d_t^2\left(d_x^2 + d_y^2\right) \quad \textbf{(9.k)}$$

To solve equation (9-a) used Navier solution with state space approach.

For angle-ply rectangular laminates with edges y=0 and y=b simply supported and the other two edges x=0 and x=a simply supported. Assume the following representation of the displacement **[Reddy .J.N.]**

$$u_0(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} U_{mn}(t)\ sin\alpha x\ cos\beta y \quad \textbf{(10.a)}$$

$$v_0(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} V_{mn}(t)\ cos\alpha x\ sin\beta y \quad \textbf{(10.b)}$$

$$w_0(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} W_{mn}(t)\ sin\alpha x\ sin\beta y \quad \textbf{(10.c)}$$

Where: $\alpha = \frac{m\pi}{a}$; $\beta = \frac{n\pi}{a}$

m = No. of the mode in x-direction (*m*=1,2,3)

n = No. of the mode in y-direction (*n*=1,2,3)

U_{mn} , V_{mn} , W_{mn} are coefficients to be determined; and

$$\Delta T(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} T_{mn}(t)\sin\alpha x\sin\beta y \quad \textbf{(11.a)}$$

$T_{mn}(t) = \frac{4}{ab}\int_0^a\int_0^b \Delta T(x,y,t)\sin\alpha x \sin\beta y\, dxdy$By **(11.b)**

substituting Eqs. (10 and 11) in partial differential Eq. (9.a) and the result

$$\begin{bmatrix}\hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13}\\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23}\\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33}\end{bmatrix}\begin{Bmatrix}u_{mn}\\ v_{mn}\\ w_{mn}\end{Bmatrix} + \begin{bmatrix}\widehat{m}_{11} & 0 & 0\\ 0 & \widehat{m}_{22} & 0\\ 0 & 0 & \widehat{m}_{33}\end{bmatrix}\begin{Bmatrix}\ddot{u}_{mn}\\ \ddot{v}_{mn}\\ \ddot{w}_{mn}\end{Bmatrix} = \begin{Bmatrix}0\\ 0\\ Q_{mn}\end{Bmatrix} + \begin{Bmatrix}\alpha N_{mn}^1\\ \beta N_{mn}^2\\ -2\alpha\beta M_{mn}^6\end{Bmatrix}$$
(12.a)

Where

$\hat{c}_{11} = (A_{11}\alpha^2 + A_{66}\beta^2)$
(12.b)

$\hat{c}_{12} = (A_{12} + A_{66})\alpha\beta$
(12.c)

$\hat{c}_{13} = -(3B_{16}\alpha^2 + B_{26}\beta^2)\beta$ **(12.d)**

$\hat{c}_{22} = (A_{66}\alpha^2 + A_{22}\beta^2)$ **(12.e)**

$\hat{c}_{23} = -(B_{16}\alpha^2 + 3B_{26}\beta^2)\alpha$ **(12.f)**

$\hat{c}_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4$ **(12.g)**

$\widehat{m}_{11} = \widehat{m}_{22} = I_0$
(12.h)

$\widehat{m}_{33} = \left(I_0 + I_2(\alpha^2 + \beta^2)\right)$ **(12.i)**

The dynamic load subjected on the system,

$q(x,y,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} Q_{mn}(x,y,t)\sin\alpha x\sin\beta y$
(13.a)

$$Q_{mn}(x,y,t) = \frac{4}{ab}\int_0^b\int_0^a q(x,y).\sin\frac{m\pi x}{a}.\sin\frac{n\pi y}{b}.dx.dy.f(t)$$
(13.b)

$$Q_{mn}(x,y,t) = \bar{q}(x,y).f(t)$$ **(13.c)**

Many types of $q(x,y)$ loading can be considered as [Muhannad L. S. Al-Waily,2004]:

1. Uniformly distributed load $q_0$at area of plate (a*b). By substituting the load into Eq. (13.b), gives:

$\bar{q}(x,y) = \frac{16}{mn\pi^2} q_0$ **(14.a)**

2. Point load P_o at $x=a_1$ and $y=b_1$. By substituting the load into Eq.(13.b), gives:

$\bar{q}(x,y) = \frac{4}{ab} P_0 sin\frac{m\pi a_1}{a} sin\frac{n\pi b_1}{b}$

(14.b)

3. Sinusoidal distributed loading, $q(x,y) = q_0 sin\frac{\pi x}{a} sin\frac{\pi y}{b}$. By substituting the load into Eq. (13.b), gives:

$\bar{q}(x,y) = q_0$

(14.c)

4. Uniformly distributed load q_o at central area (A*B). By substituting the load into Eq. (13.b), gives:

$\bar{q}(x,y) = \frac{16}{nm\pi^2} q_0 sin\frac{m\pi A}{2a} sin\frac{n\pi B}{2b}$

(14.d)

And many types of $f(t)$ can be considered as [Khdeir A. A. & Reddy J. N,1988]

1. Sine pulse loading

$$f(t) = \begin{Bmatrix} \sin\frac{\pi t}{t_1} & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix} \qquad \textbf{(15.a)}$$

2. Step pulse loading

$$f(t) = \begin{Bmatrix} 1 & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix} \qquad \textbf{(15.b)}$$

3. Ramp pulse loading

$$f(t) = \begin{Bmatrix} \mathrm{t}/t_1 & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix} \qquad \textbf{(15.c)}$$

2.4 Solution of Dynamic Equilibrium Equations

The equations of equilibrium governing the linear dynamic response of a system can be written as the following formula

$MA\ddot{U} + KSU = R$

(16)

Where:

$$KS=\begin{bmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} \\ \hat{c}_{12} & \hat{c}_{22} & \hat{c}_{23} \\ \hat{c}_{13} & \hat{c}_{23} & \hat{c}_{33} \end{bmatrix} \quad MA=\begin{bmatrix} \widehat{m}_{11} & 0 & 0 \\ 0 & \widehat{m}_{22} & 0 \\ 0 & 0 & \widehat{m}_{33} \end{bmatrix} \quad \ddot{U} = \begin{Bmatrix} \ddot{u}_{mn} \\ \ddot{v}_{mn} \\ \ddot{w}_{mn} \end{Bmatrix}$$

$$U = \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{mn} \end{Bmatrix} \qquad R = \begin{Bmatrix} 0 \\ 0 \\ q \end{Bmatrix} + \begin{Bmatrix} f_1^t \\ f_2^t \\ f_3^t \end{Bmatrix}$$

MA and KS: are the mass and stiffness matrices.

R: is the vector of externally applied loads.

U and $\ddot{U}$: are the displacement and acceleration vectors

In the Newmark direct integration method, the first time derivative $\{\dot{U}\}$ and the solution $\{U\}$ are approximated at (n+1) time step (i.e. at time t = t_{n+1} = (n+1)Δt by the following expression [Rao V. Dukkipati,2010].

$$\{\dot{U}\}_{n+1} = \{\dot{U}\}_n + \left[(1-\bar{\alpha})\{\ddot{U}\}_n + \bar{\alpha}\{\ddot{U}\}_{n+1}\right]\Delta t$$
(17)

$$\{U\}_{n+1} = \{U\}_n + \{\dot{U}\}_n \Delta t + \left[\left(\frac{1}{2}-\bar{\beta}\right)\{\ddot{U}\}_n + \bar{\beta}\{\ddot{U}\}_{n+1}\right](\Delta t)^2$$
(18)

Where:

$\bar{\alpha}$ And $\bar{\beta}$: are parameters that control the accuracy and stability of the scheme, and the subscript n indicates that the solution evaluated at n^{th} time step (i.e. at time, t = t_n). The choice $\bar{\alpha} = 0.5$ and $\bar{\beta} = 0.25$ is known to give an unconditionally stable Scheme (average acceleration method), [Rao V. Dukkipati,2010].

3. NUMERICAL ANALYSIS

3.1 Element Selection and Modeling

An element called shell281 as shown in **Fig.1** is selected which is suitable for analyzing thin to moderately thick shell structures. The element has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z axes. It may be used for layered applications for modeling composite shells. It is include the effects of transverse shear deformation. The accuracy in modeling composite shells is governed by the first order shear deformation theory. The shell section allows for layered shell definition, options are available for specifying the thickness, material, orientation through the thickness of the layers. But to insert the temperature effect in calculations must be to adding degree of freedom (T). Then, the degrees of freedom change from (6 to7) in each node.

3.2 Verification Case Studies

In the present study, Series of preselected cases are modeled to verify the accuracy of the method of analysis. The case study discussed here for dynamic response without temperature change is a comparison of the present work with the numerical solution of **[Reddy .J.N, 1982]** for a laminated plate **Fig. 2**,

Close comparison between the two sets of results is evident, for a/h=5(maximum central non dimension deflection of present work for CLPT with Newmark direct integration method is= 23.5(error 2.174%), for present F.E.M ANSYS maximum central non dimension deflection is=23.65(error 2.8%). while for above reference = 23.

For thermo-mechanical transient response of simply supported laminated plates, the curves of central deflection as a function of time for a (0/90/0) symmetric cross-ply laminated plate subjected to suddenly applied dynamic loading are plotted and compared in **Fig. 3** ,with **[Hui-Shen Shen ,2003]**. Close comparison between the two sets of results is evident, (maximum central deflection of present work for CLPT with Newmark direct integration method = 2.35 cm (error2.08%), and for present F.E.M. ANSYS program is= 2.2375cm (error6.77%), while for above reference = 2.4 cm. **Fig.3**.

4. EXPERIMENTAL WORK

In the present work, three- purposes were investigated. First, to outline the general steps to design and fabricate the rectangular test models from fiber (E-glass) and polyester resin to form laminate composite materials. Second, the manufactured models are then used to evaluate the mechanical properties (E_1, E_2, G_{12}) with temperature change of unidirectional composite material. Third, evaluate coefficient of thermal expansion (CTE) of the composite plate.

4.1 Thermo-Mechanical Analyzer

Thermo-mechanical Analysis (TMA) determines dimensional changes of solids and liquids materials as a function of temperature and/or time under a defined mechanical force.
Irrespective of the selected type of deformation (expansion, compression, penetration, tension or bending), every change of length in the sample is communicated to a highly sensitive inductive displacement transducer (LVDT) via a push rod and transformed into a digital signal. The push rod and corresponding sample holders of fused silica or aluminum oxide can be quickly and easily interchanged to optimize the system to the respective application. **Figs.4 and 5.**
The dimension of sample is (5*20*4) mm. the thermal properties which obtain from this test shown in **Table 1.**

5. RESULTS AND DISCUSSION

The present study focused mainly on the dynamic response behavior of composite laminated plates subjected to mechanical and thermo-mechanical loads of finite duration uniform (step, sine and ramp) and sinusoidal (step, sine and ramp) on the top surface of the plate for three cases of temperature (without temperature effect, T=50°C and T=100°C) . The step loading $q(x,y,t) = \bar{q}(x,y)$, ramp loading $q(x,y,t) = \bar{q}(x,y)t/t1$ and sinusoid loading$q(x,y,t) = \bar{q}(x,y)sin\pi t/t1$. For uniform distributed load $\bar{q}(x,y) = \frac{16}{nm\pi^2}$ and for sinusoidal distributed $\bar{q}(x,y) = q_o$. The amplitude of force is $q_o = 100\text{N}/mm^2$ and the time of load applied on plate ist_1 =0.05sec. The dynamic response of central deflection of composite plate discussed for different parameters such as load condition, lamination angle , temperature change , symmetric or anti symmetric angle ply for simply supported composite plate analytically by CLPT with Newmark direct integration method and numerical result by ANSYS.

(5-1) Effect of Load Condition

Figs. 6 to **9** represent the variation of central transverse deflection with time (dynamic response) for four layer anti-symmetric and symmetric cross-ply and angle ply simply supported laminated plates under sinusoidal $(P(x,y) = qo\ sin(\pi\, x/a) sin(\pi\, y/b))$ and uniform$(P(x,y) = qo\ )$ variation loading, (step $q(x,y,t) = P(x,y)$,ramp loading $q(x,y,t) = P(x,y)\ t/t1$ and sinusoid loading q(x,y,t)=P(x,y) sinπt/t_1) for qo=100N/m^2 ,t_1=0.05 sec) without any temperature change solved analytically by CLPT with Newmark direct integration method and (F.E.M) by ANSYS program. The deflection due to step loading higher in magnitude than the other loads with percentage reach to 91.96%, 97.4% from sine and ramp load, respectively, because the step load subjected suddenly with constant value with the time.

Very good verification between CLPT with Newmark and FEM by ANSYS maximum error is12.9%. Maximum response for step load always occurs in the time of applying load (i.e. in the time less than t_1 after that the response became in negative sign and positive sign alternatively. for ramp load, the response increasing linearly with time until it reached to t_1 at this point the maximum response occurs, then the response became in negative sign and positive sign alternatively. For sine load the response behavior have the sine shape and the maximum response at $t_1/2$.

(5-2) Effect of Temperature Change with Varies Load Condition

Figs. 10 to 13 show the numerical result by ANSYS for dynamic response of central deflection of symmetric and anti-symmetric angle ply simply supported composite plate step uniform and step sinusoidal load and different condition of temperature effect i.e. (T=25°C, T=50°C, T=100°C). The deflection increases with percentage reaches to (58.47%) when temperature became 50°C and when the temperature reach to 100°C the response increases with higher percentage reaches to (200%) with respect to response without change in temperature for laminated plates for step uniform dynamic load.

The reason behind that is there where two loads (mechanical and thermal) each load causes the deflections (thermal and mechanical deflections) summation is the deflection of plate under thermo-mechanical loading. When the temperature increases the deflection increased with high percentage. The uniform load is higher than sinusoidal load for all load condition.

(5-3) Effect of Lamination Angle

Fig. 14 shows the effect of angle (θ) on central deflection for four layer symmetric angle-ply laminated plates, simply supported, subjected to sine uniform loading with applied temperature equal to 50°C, solved analytically by Newmark and numerical by ANSYS .From the results, the central deflection of laminated plate decreases with increasing the angle (θ) from 10 to 40 with percentage reach to 24.8%. Then increase the central deflection when θ increase from 40 to70 with percentage reaches to 9.2%. The maximum deflection with time for each case is when lamination angle is 10.

6. CONCLUSION

This study considers the vibration analysis of symmetric and anti-symmetric angle-ply composite laminate plate. From the present study, the following conclusions can be made:

1-The Young and shear modulus decrease when temperature increases with high percentages reach to96.3% when temperature changes from (20 °C to 100°C) for longitudinal young modulus, for transverse young modulus is96.53% and for shear modulus is 91.1%. The longitudinal and transverse coefficient of thermal expansion also decrease when temperature increase with percentage 80% and73.7% respectively for the same temperature.

2- The response due to step loading higher in magnitude than the other loads with percentage reach to 91.96%, 97.4% from sine and ramp load, respectively.

3- The response increase with maximum percentage reaches to (58.47%) when temperature became 50°C and when the temperature reach to 100°C the response increase with higher percentage reaches to (200%) with respect to response without change in temperature
4- It was seen that the different fiber orientation angles affected on dynamic response. The central deflection of laminated plate decreases with increasing the angle (θ) from 10 to 40 with percentage reach to 24.8%. Then increase the central deflection when θ increase from 40 to70 with percentage reaches to 9.2%. Thus, the maximum deflection with time is when lamination angle is 10 for four layer symmetric angle-ply laminated plates, simply supported, subjected to sine uniform loading with applied temperature equal to 50°C.

REFERENCES

- Beards C.E , 1996, *Structural Vibration: Analysis and Damping* 3ed, New York.
- Chorng-Fuh Liu and Chih-Hsing Huang, 1996, *Free Vibration of Composite Laminated Plates Subjected to Temperature Changes,* Computers & Structures, Vol. 60, pp. 95-10.
- Hussein A. Ammar ،Dr. Hassan H. Mahdi & Nessren H. Ahmed, 2011, *Numerical and Experimental Investigation of Mechanical and Thermal Buckling Loads of Composite Laminated Plates* Eng. & Tech. Journal, Vol. 29, No.9.
- Hui-Shen Shen, J. J. Zheng and X. L. Huang, 2003,"*Dynamic Response of Shear Deformable Laminated Plates Under Thermomechanical Loading and Resting on Elastic Foundations*", Composite Structures, Vol. 60, pp. 57-66.
- Junaid Kameran Ahmed, V.C. Agarwal, P.Pal, Vikas Srivastav, 2013, *Static and Dynamic Analysis of Composite Laminated Plate*, International Journal of Innovative Technology and Exploring Engineering (IJITEE) ISSN: 2278-3075.
- Khdeir A. A. & Reddy J. N, 1988, *Dynamic Response of Antisymmetric Angle-Ply Laminated Plates Subjected to Arbitrary Loading*, J. Sound and Vibration, Vol.126, No.3, p.p. 437-445.
- Kullasup Phongsrisuk, Prasong Ingsuwan, Wetchayan Rangsri and Wiwat Klongpanich, 2010, *Free vibration analysis of symmetrically laminated composite rectangular plates using extended Kantorovich method*, Maejo International Journal of Science and Technology ISSN 1905-7873, 4(03), 512-532.
- Muhannad L. S. Al-Waily, 2004, *Analysis of Stiffened and Unstiffened Composite Plates Subjected Time Dependent Loading,* M.sc, Thesis, Kufa University. College of Engineering.

- Pushpendra k. kushwaha1 and jyoti vimal, 2014, *Study of Vibration Analysis of Laminated Composite Plates Using FEM, International Journal of Advanced Mechanical Engineering*. ISSN 2250-3234.
- Rao V. Dukkipati, 2010 , *Matlab an Introduction with Applications*, New age international publishers. ISBN(13):978-81-224-2920-6.
- Reddy J. N. 1982, *On the Solution to Forced Motions of Rectangular Composite Plates”; J. Applied Mechanics*, Vol. 49, p.p. 403-408, June.
- Reddy J.N., 2004, *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis,* 2ed; CRC Press.
- Suresh Kumar J., Dharma Raju T. and Viaya Kumar Reddy K. 2011, *Vibration Analysis of Composite Laminated Plates Using Higher Order Shear Deformation Theory with Zig-Zag Function*”, Indian journal of science and technology vol.4 ,no.8,ISSN:0974-6846.
- "*Theory, Analysis, and Element Manuals"* ANSYS 13 Program.

NOMENCLATURE

Symbol	Description	Unit
a, b	Dimension of plate in x and y coordinate	m
A_{ij} , B_{ij} ,D_{ij}	Extensional stiffness, the coupling stiffness, and the bending stiffness	-
E_1, E_2 , E_3	Elastic modulus of composite material	GPa
G_{12}, G_{23} , G_{13}	Shear modulus of composite material	GPa
h	Thickness	m
I_0, I_1, I_2	Mass moment of inertia	$kg.m^2$
[MA]	Mass matrix	kg
M_{xx} , M_{yy} ,M_{xy}	Moment resultant per unit length	N.m/m
N	Total number of plate layers	-
Nxx, Nyy , Nxy	The resultant of in-plane force per unit length	N/m
$N_{xx}^t, N_{yy}^t, N_{xy}^t$	The resultant of in-plane force per unit length with thermal effect	N/m
$\widehat{N}xx, \widehat{N}yy , \widehat{N}xy$	Applied edge force	N/m
q(x,y,t)	Dynamic force subjected on a system	N/m^2
$\bar{Q}_{ij}^{(k)}$	Transformed lamina stiffness	N/m
R	Vector of externally applied loads	N
t	Time	min or s
Δt	Time Interval	min or s
t_1	The end time of load	sec
T	Temperature	C^0
ΔT	Temperature increment	C^0

T_{ref}	Reference temperature	C^0
$U, \dot{U}, \ddot{U}$	Displacement , velocity and acceleration vectors	m, m/s, m/ s^2
u_o , v_o , w_o	Displacement components along (x,y,z) directions respectively	m
U_{mn} , V_{mn} , W_{mn}	Amplitude of (u_o , v_o , w_o) respectively	-
x , y , z	Cartesian coordinate system	m
z	Distance from neutral axis	m
θ	Fiber orientation angle	Degree
α_1, α_2	Coefficient of thermal expansion of composite material	$(1/C^{-1})$ or $(1/K^{-1})$
ρ	Density	(kg/m3)
ϵ_{xx} , ϵ_{yy} , ϵ_{xy}	Strain components	m/m
σ_{xx} , σ_{yy} , σ_{xy}	Stress components	GPa

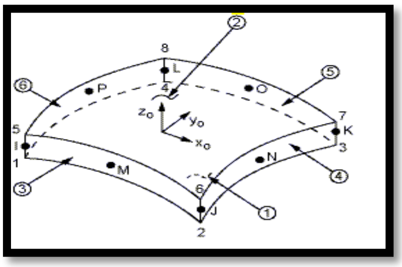

Figure 1. Shell281 geometry [ANSYS 13 Program].

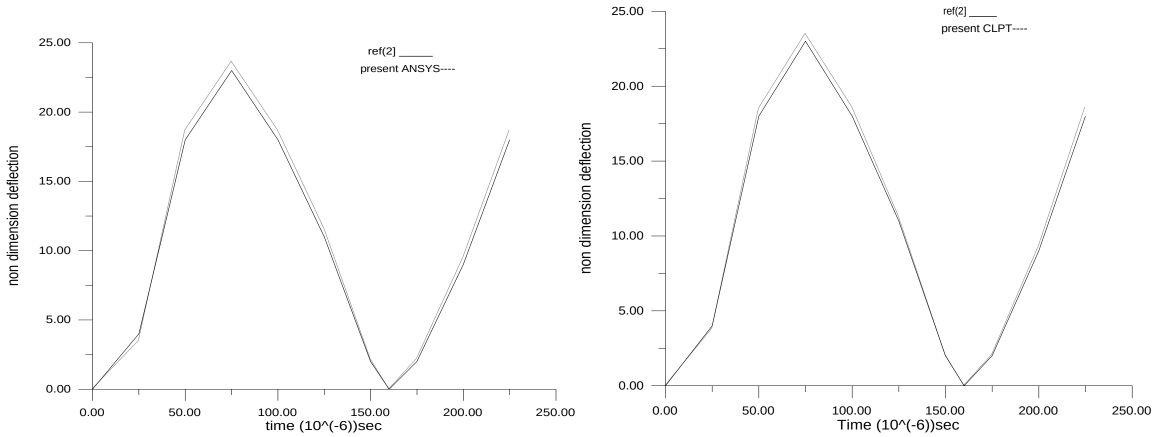

Figure 2. Comparison of the present solution with the numerical solution of [Reddy .J.N, 1982] of two-layer cross-ply (0/90) square plate under suddenly applied sinusoidal loading (a/h=5).

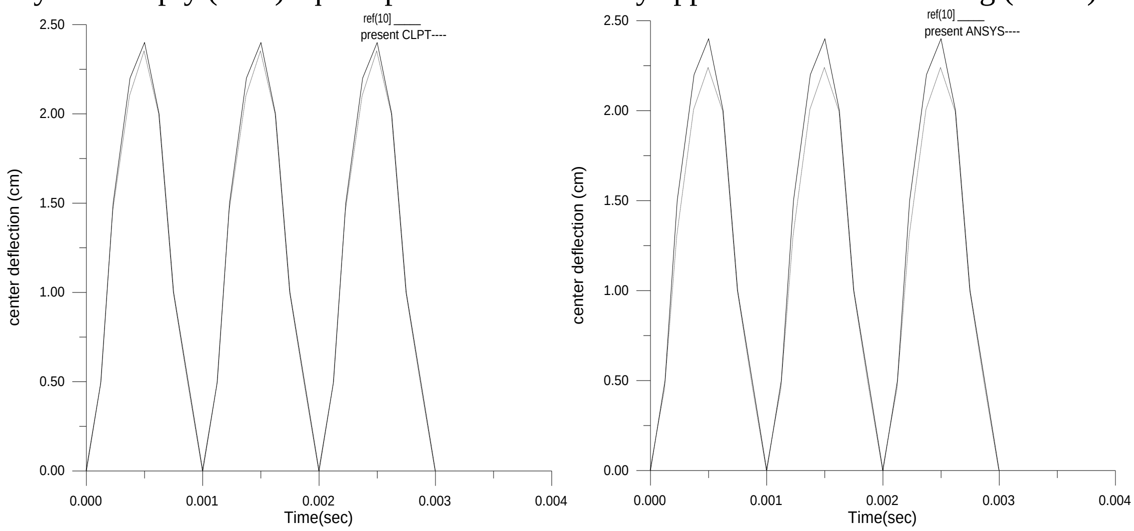

Figure 3. Comparison of present study with **[Hui-Shen Shen et al 2003]** for laminated square plate under thermal loading condition at (Δ T = 200 C^0).

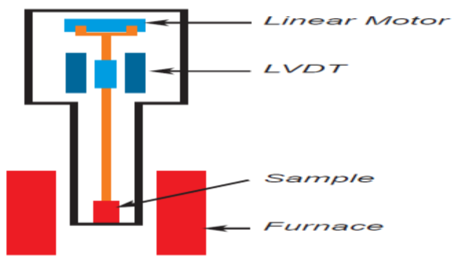

Figure 4. Operating principle of TMA.

Figure 5. TMA PT1000 device**.**

Table1. Experimental value of mechanical and thermal properties of fiber –polyester composite plate for fiber volume fraction= 0.3 changed with temperature.

T C°	E_1 **Mpa**	E_2 **Mpa**	G_{12}=G_{13}= G_{23} **Mpa**	α_1 **E-6/K**	α_2 **E-6/K**
20	24627.0	5588.04	1551.77	14.57	47.81
30	23343.30	4123.11	1618.65	9.03	31.7
40	21775.40	1550.22	2505.75	7.20	29.36
50	15219.80	1515.37	623.423	4.79	25.79
60	6475.41	566.8	114.2336	3.20	21.38
70	2990.82	458.59	113.535	3.18	15.60
80	2555.71	289.27	130.48	3.22	15.59
90	1471.90	210.49	158.83	3.08	15.19
100	903.90	193.84	138.48	2.91	12.58
110	741.31	191.75	131.74	2.75	11.57
120	674.40	187.53	125.23	2.57	10.47
130	644.70	186.51	122.56	2.48	9.34
140	629.01	185.19	117.59	2.45	7.67
150	612.02	182.66	107.81	2.44	5.28
160	597.61	181.88	100.28	2.44	4.19
170	592.41	173.50	95.414	2.44	3.90
180	592.22	164.55	95.04	2.45	3.88
190	591.02	163.91	85.71	2.46	3.66
200	590.57	153.3	83.64	2.47	3

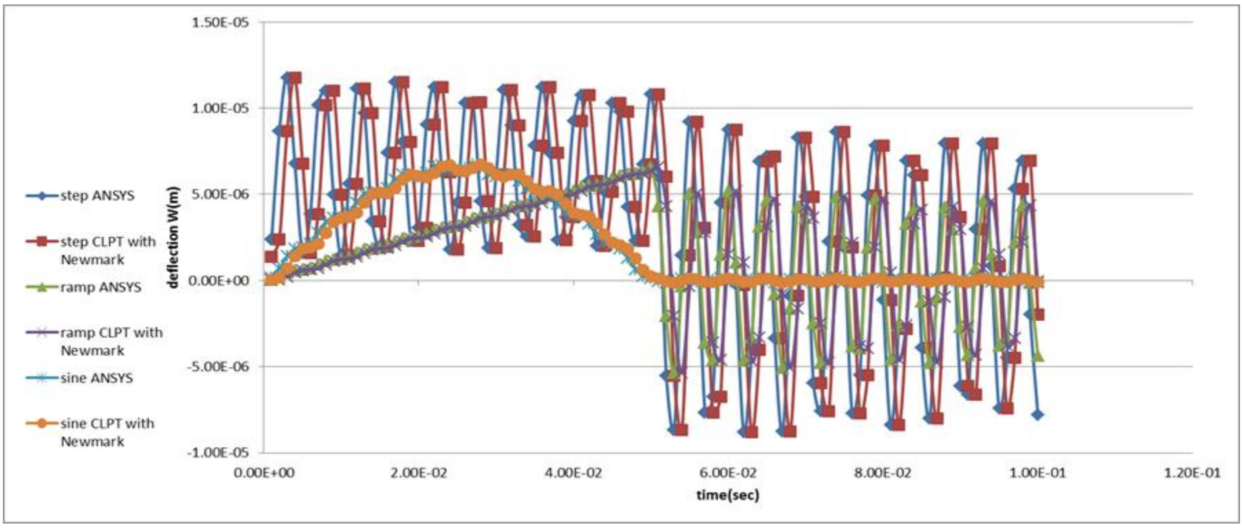

Figure 6. Central deflection of four layers symmetric angle-ply (45/-45/…) laminated plates for variant sinusoidal dynamic load without temperature change.

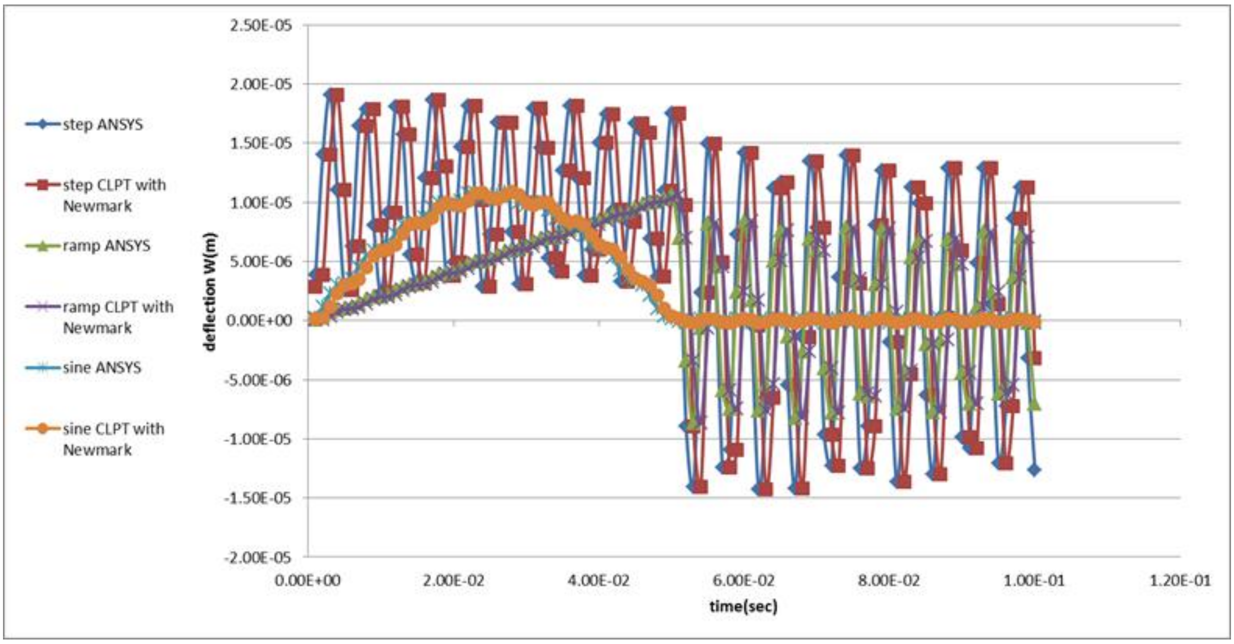

Figure 7. Central deflection of four layers symmetric angle-ply (45/-45/…) laminated plates for variant uniform dynamic load without temperature change.

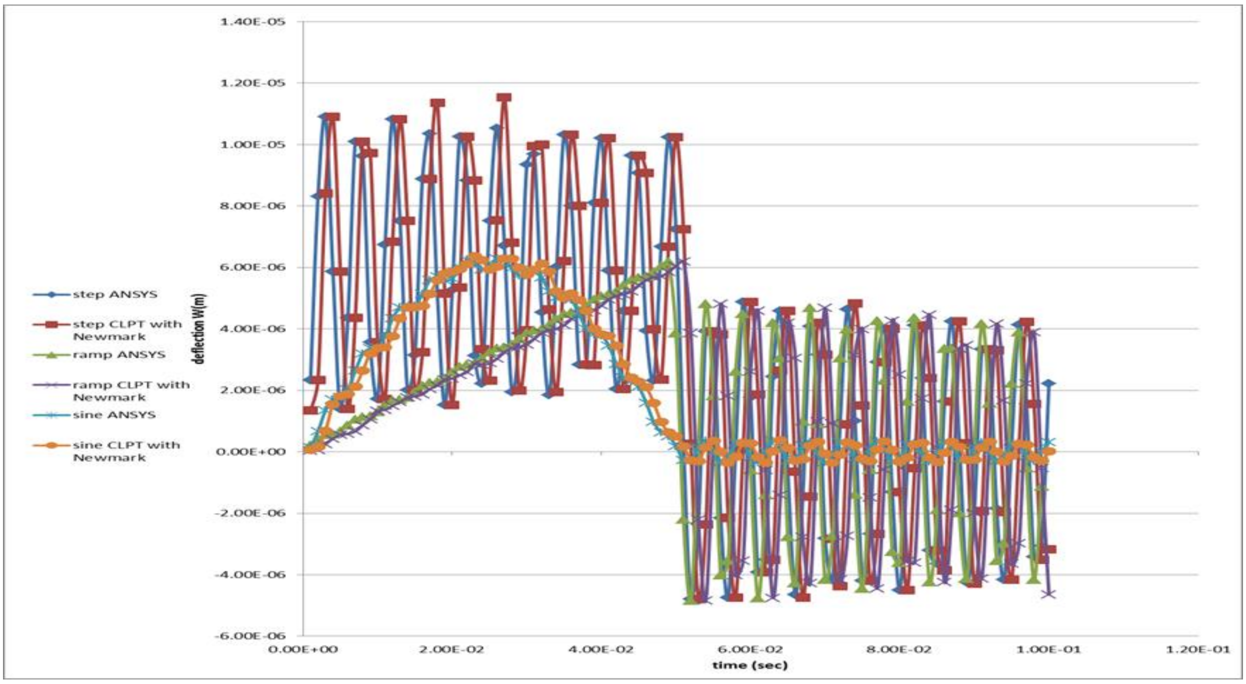

Figure 8. Central deflection of four layers anti-symmetric angle-ply (45/-45/…) laminated plates for variant sinusoidal dynamic load without temperature change.

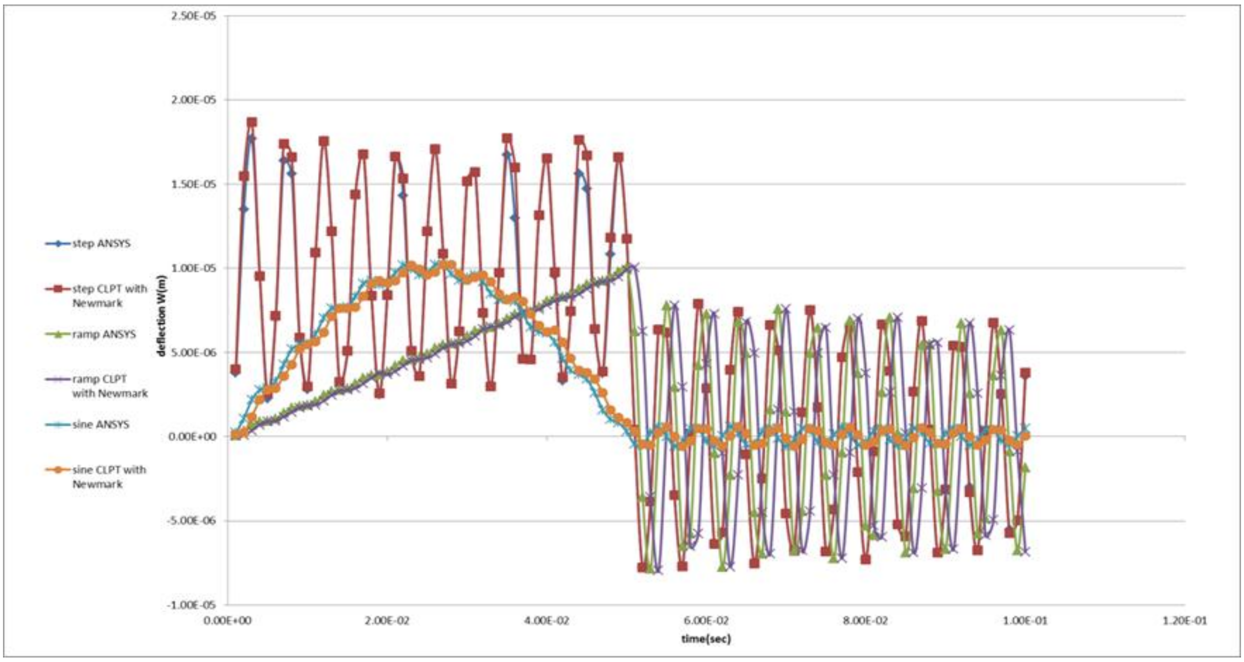

Figure 9. Central deflection of four layers anti-symmetric angle-ply (45/-45/…) laminated plates for variant uniform dynamic load without temperature change.

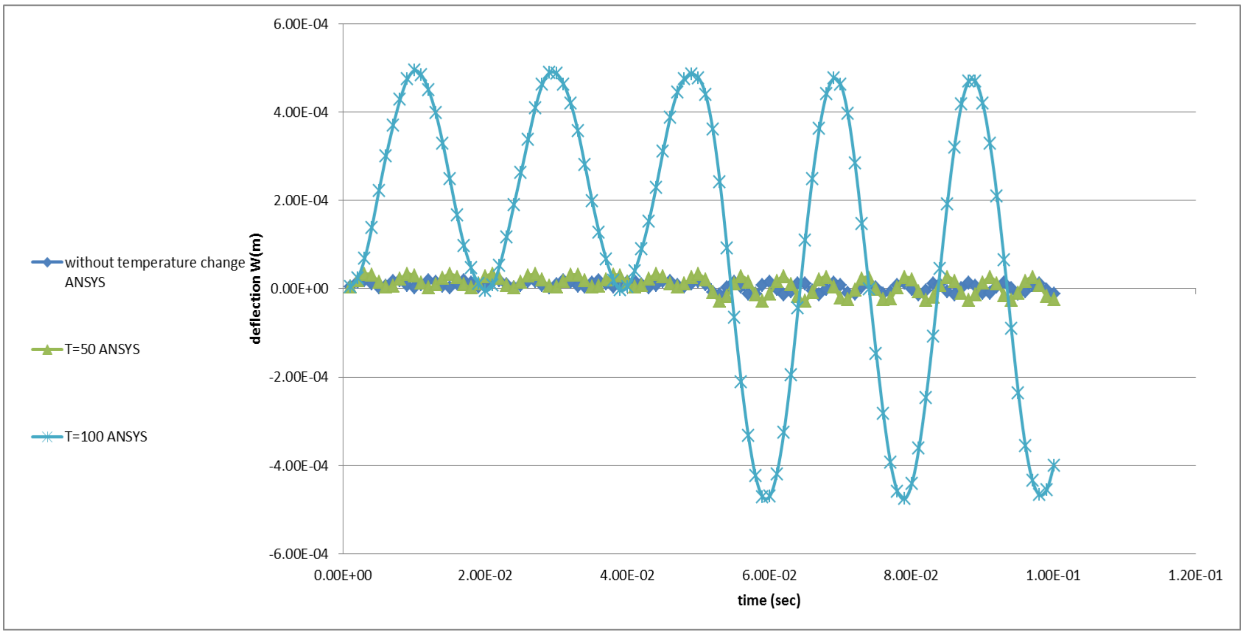

Figure 10. Central deflection of four layers symmetric angle-ply (45/-45/…) laminated plates for step uniform dynamic load with temperature change.

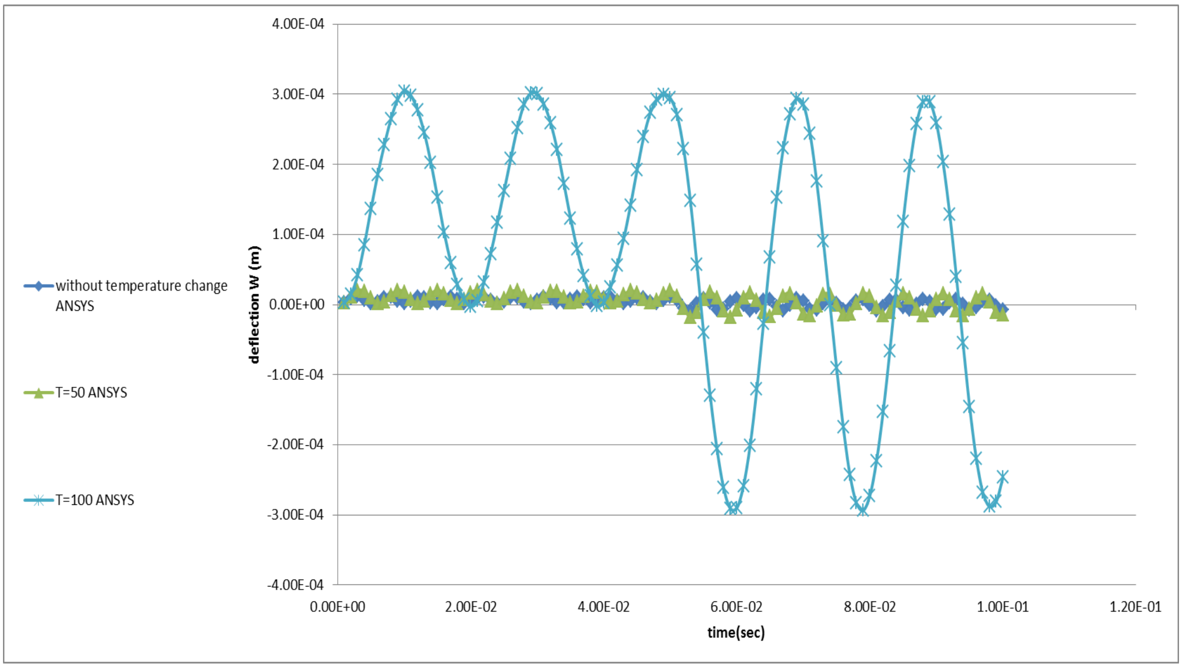

Figure 11. Central deflection of four layers symmetric angle-ply (45/-45/…) laminated plates for step sinusoidal dynamic load with temperature change.

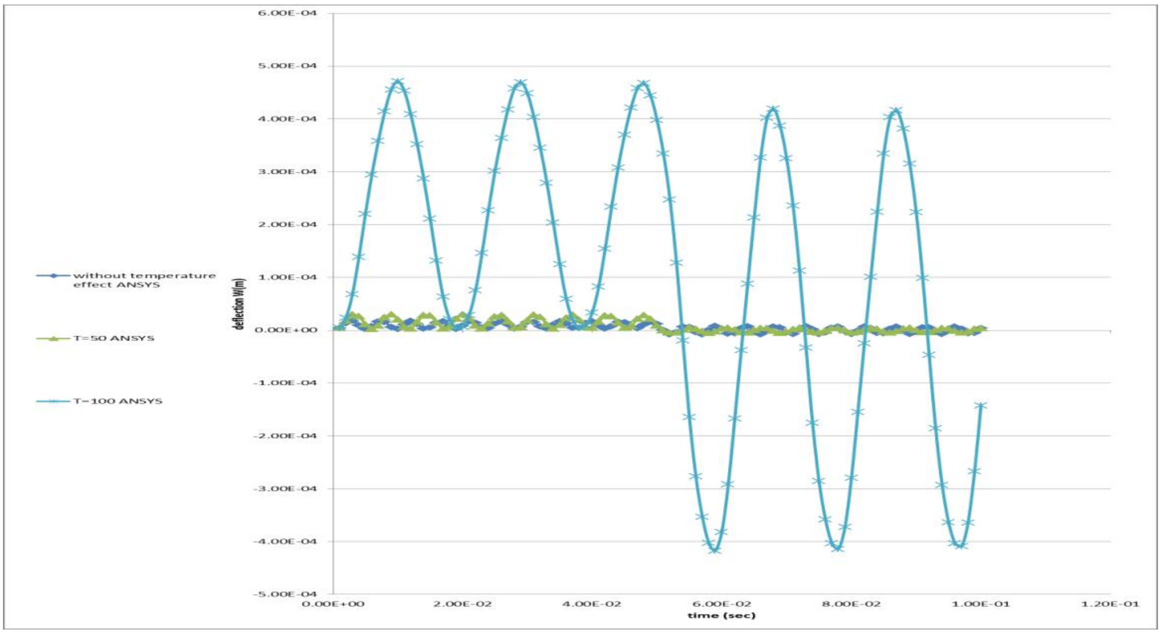

Figure 12. Central deflection of four layers anti- symmetric angle-ply (45/-45/…) laminated plates for step uniform dynamic load with temperature change**.**

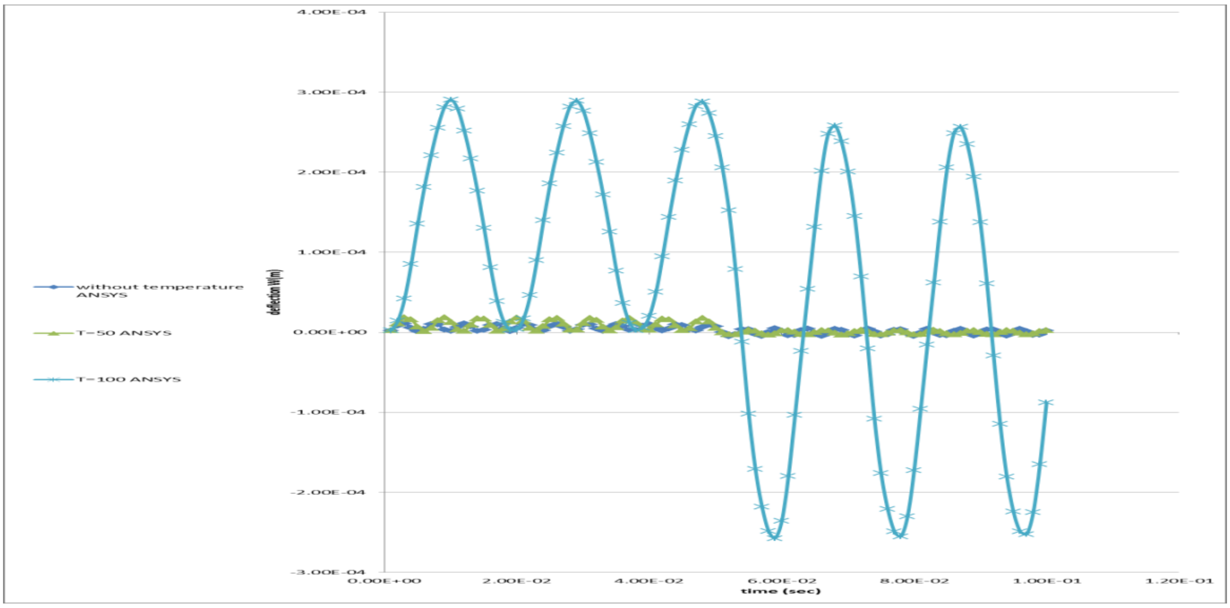

Figure 13. Central deflection of four layers anti- symmetric angle –ply (45/-45/…) laminated plates for step sinusoidal dynamic load with temperature change.

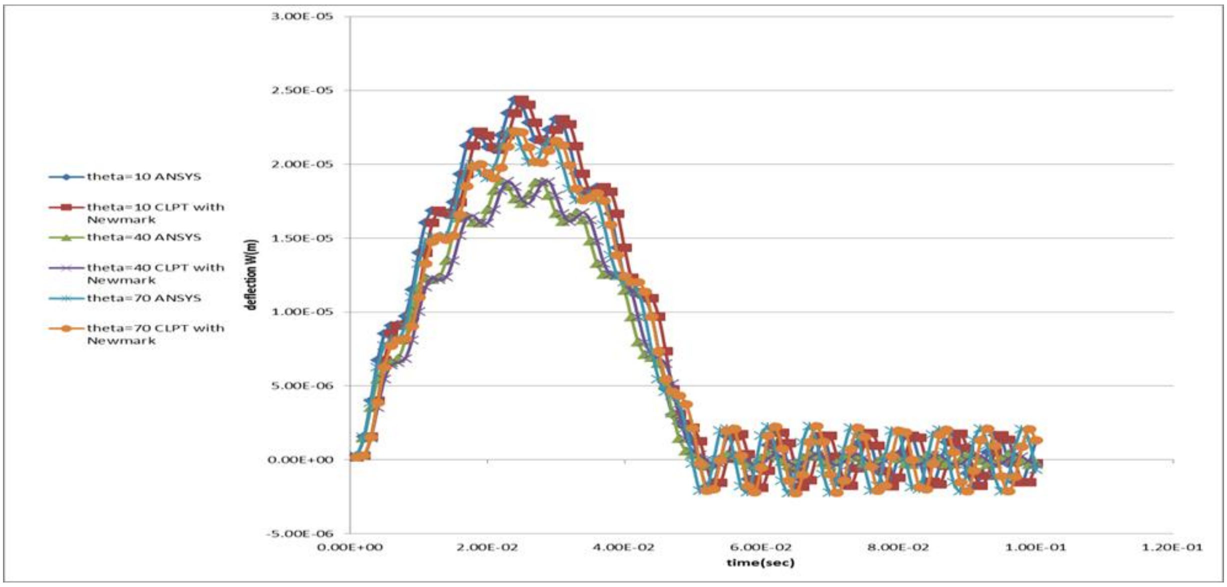

Figure 14. Effect of lamination angle on central deflection of four layers (θ/-θ/-θ/θ) laminated plates for sine uniform dynamic load with temperature equal to 50°C.