THE EFFECT OF SELF-EQUILIBRATING STRESSES DUE TO MULTI-LINE SPOT WELDED STIFFENERS ON THE NATURAL FREQUENCIES OF PLATE

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ABSTRACT

In this paper an investigation has been made into the effect of residual stresses on the vibration characteristics of a thin rectangular stainless steel plate with multi-line spot welded stiffeners. A new general frequency equation with and without the effect of residual stresses due to multi-line spot welding along the length and width of the plate for different boundary conditions were obtained. The results give that the free ends tries to increase the natural frequencies while the clamped edges try to decrease the natural frequencies; also the central position weld line has the great influence on the natural frequencies.

الخلاصة

تم في هذا البحث دراسة تأثير الاجهادات المتبقية نتيجة اللحام النقطي لعدة خطوط عموديه وافقية على خصائص الاهتزازات بالنسبة الى الصفائح الرقيقة المقواة والمقاومة للصدأ. تم اشتقاق معادلات جديدة للترددات الطبيعية بوجود وعدم وجود تأثير الاجهادات المتبقية لعدد من خطوط اللحام النقطي بالأتجاه الطولي والعرضي للصفائح الرقيقة المقواة لبعض الحالات ذات حدود التثبيت المختلفة. حيث أظهرت النتائج ان النهايات الحرة تعمل على زيادة الترددات الطبيعيةعلى العكس من النهايات المقيدة والتي تعمل على التقليل من الترددات الطبيعية، أضافة" ان قيمة الاجهادات المتبقية وموقع خط اللحام له التأثير الكبير على الترددات الطبيعية.

KEYWORDS

Spot welding, Residual stresses, Vibration, Stiffened plate, Rectangular plate

INTRODUCTION.

The wide use of stiffened structural element in engineering began mainly with the application of steel plates for hulls of ships, steel bridges and aircraft structures. The stiffening usually has a small part of the total weight of the structure, substantially influence their strength and performance under different load conditions. Recently, Guo 2002, Jung 2002, and Nacy, 2002, studied the free vibration analysis of stiffened plates and shells, it was found that the stiffeners shape its distributions have a great effect on the natural frequencies and mode shapes of the plate.

On the other hand, residual stresses are induced at each stage of the life cycle in most engineering component, from original material production to final disposal. Residual stresses are created by welding, forging, casting, rolling, machining, surface treatment, heat treatment etc.

Resistance spot welding is a process used for joining faying surfaces. Major advantages of resistance spot welding are high speed and suitability for automation. Many researches have been published regarding joining strength and residual stresses of spot welds, Berglund 2002, Bae 2003, and Xin 2005. Faiz 2006, studied the effect of spot distributions and residual stresses induced from spot welding, on the vibration characteristics of plates with one array of spot welded stiffeners on the longitudinal centerline of thin rectangular plate. Theoretical method based on the theory of bending of thin plate was used to obtain the governing differential equation; expressions of the exact frequency equation were derived. Finite element modeling was adopted to predict the tendom force produced due spot welding, finding the natural frequencies and mode shapes.

- VIBRATION ANALYSIS OF THE ORTHOTROPIC PLATES.

Due to the existence of stiffeners, the fundamental equation for small deflection theory of bending of thin plates is used to give the details of the theoretical analysis of residual stresses that result from welding and its effect on the natural frequencies and mode shapes. The governing differential equation of deflection for an orthotropic plates, subjected to a force (N_x per unite length) acting on the edges of the orthotropic plate, Timoshenko 1961, can be written as

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = N_x \frac{\partial^2 w}{\partial x^2} \tag{1}$$

Where $H = D_{xy} + 2G_{xy}$ and D_x, D_y, D_{xy}, G_{xy} represents the flexural and torsional rigidities of an orthotropic plate respectively.

The strain energy stored in a plate element, Golf 1976, **Fig. 1**. is the sum of the work done by the bending moments, $M_x dy$ and $M_y dx$ and by the twisting moment $M_{xy} dy$ and $M_{xy} dx$, neglecting the work done by the shearing forces and by any stretching of the middle plane of the plate.

The work done by the bending moments is

½ * moment * angle between the sides of the element after bending.

In the xz plane the angle is: $-\left(\frac{\partial^2 w}{\partial x^2}\right)dx$, and in the yz plane : $-\left(\frac{\partial^2 w}{\partial y^2}\right)dy$

The negative singe occurs because a sagging downwards curvature (positive) has a decreasing slope as x increases. The energy stored due to bending (dU_b) is therefore given by:

$$dU_b = \frac{1}{2}\left(M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} \right) dxdy \tag{2}$$

The relative rotation of the element faces due to twist are $\frac{\partial^2 w}{\partial x \partial y} dx$ and $\frac{\partial^2 w}{\partial x \partial y} dy$

since $M_{xy} dy$ and $M_{yx} dx$ are the twisting moments, and $M_{xy} = M_{yx}$ the same amount of energy is stored by both couples. Then, total energy due to twisting (dU_t) is given by:

$$dU_t = M_{xy} \frac{\partial^2 w}{\partial x \partial y} dxdy \qquad (3)$$

Substituting the expressions for the moments and adding (dU_b) and (dU_t) to produce the total energy stored in an element (dU), we get:

$$dU = \frac{1}{2}\left\{ D_x (\frac{\partial^2 w}{\partial x^2})^2 + 2D_{xy} (\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}) + 4G_{xy} (\frac{\partial^2 w}{\partial x \partial y})^2 + D_y (\frac{\partial^2 w}{\partial y^2})^2 \right\} \qquad (4)$$

This function is composed of the following elements:

½*(x-direction bending moments*rotation); $\frac{1}{2}\left\{ D_x (\frac{\partial^2 w}{\partial x^2})^2 + D_{xy} (\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}) \right\}$

½*(y- direction bending moments*rotation); $\frac{1}{2}\left\{ D_y (\frac{\partial^2 w}{\partial y^2})^2 + D_{xy} (\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}) \right\}$

½*(twisting moments*rotation); $\frac{1}{2}\left\{ G_{xy} (\frac{\partial^2 w}{\partial x \partial y})^2 \right\}$

The strain energy stored in a complete plate is obtained by integration eq. (4) over the surface.

$$U = \frac{1}{2}\iint \left\{ D_x (\frac{\partial^2 w}{\partial x^2})^2 + 2D_{xy} (\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}) + 4G_{xy} (\frac{\partial^2 w}{\partial x \partial y})^2 + D_y (\frac{\partial^2 w}{\partial y^2})^2 \right\} dxdy \qquad (5)$$

The maximum kinetic energy of the element, Len 1989, is:

$$\partial T = \frac{1}{2} \rho h \omega^{\circ} w^2 \partial x \partial y \qquad (6)$$

ω° of the plate my be deduced from the energy balance. Than the angular frequency, Golf 1976:

$$\int dU = \int dT \qquad (7)$$

- RESIDUAL STRESS ANALYSIS.

It is assumed in the following analysis that any cross-section (x and y = constant), the plate is under longitudinal compression distribution uniformly a cross the breadth of the plate, with the equilibrating tension concentrated on the line of spot welding at $(y = r_{yi})$, for each line of welding across the length of the plate. Also, the plate under longitudinal compression distributed uniformly a cross the width of the plate, with the equilibrating tension concentrated on the line of welding at and deflection $(x = r_{xi})$, for each line of spot welding across the width of the plate **Fig. 2**.
The component of maximum strain energy of the element due to the mid-plane forces, Timoshenko 1961, **Fig. 1**, is:

$$\partial U_r = \frac{1}{2} N_x (\frac{\partial w}{\partial x})^2 \partial x \partial y + \frac{1}{2} N_y (\frac{\partial w}{\partial y})^2 \partial x \partial y \qquad (8)$$

The strain energy due to the mid-plane forces may therefore be written, by eq. (8) as

$$U_r = \frac{1}{2} N_x \left\{ \int_{-a}^{a}\int_{-b}^{b} (\frac{\partial w}{\partial x})^2 dxdy - b\int_{-a}^{a} (\frac{\partial w}{\partial x})^2 \Big|_{y=r_{yi}} dx \right\} + \frac{1}{2} N_y \left\{ \int_{-a0}^{a}\int_{-b}^{b} (\frac{\partial w}{\partial y})^2 dxdy - a\int_{-b}^{b} (\frac{\partial w}{\partial y})^2 \Big|_{x=r_{xi}} dy \right\} \qquad (9)$$

In this case N_{xi} and N_{yi} is a negative constant, Golf, 1976. In the case of the mechanical system shown in **Fig. 3**. The total strain energy and kinetic energy are:

$$U = \frac{1}{2}\int_{-a}^{a}\int_{-b}^{b} \left\{ D_x (\frac{\partial^2 w}{\partial x^2})^2 + 2D_{xy} (\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}) + 4G_{xy} (\frac{\partial^2 w}{\partial x \partial y})^2 + D_y (\frac{\partial^2 w}{\partial y^2})^2 \right\} dxdy +$$
$$\sum_{i=1}^{n} \left\{ \frac{1}{2} N_{xi} \left[\int_{-a}^{a}\int_{-b}^{b} (\frac{\partial w}{\partial x})^2 dxdy - b\int_{-a}^{a} (\frac{\partial w}{\partial x})^2 \Big|_{y=r_{yi}} dx \right] \right\} + \sum_{i=1}^{n} \left\{ \frac{1}{2} N_{yi} \left[\int_{-a}^{a}\int_{-b}^{b} (\frac{\partial w}{\partial y})^2 dxdy - a\int_{-b}^{b} (\frac{\partial w}{\partial y})^2 \Big|_{x=r_{xi}} dy \right] \right\} \qquad (10)$$

$$T = \frac{1}{2} \rho h \omega^{\circ} \int_{-a}^{a}\int_{-b}^{b} w^2 dxdy \qquad (11)$$

Then eq. (7) can be written as:

$$\int_{-a}^{a}\int_{-b}^{b} \frac{1}{2} \left\{ D_x (\frac{\partial^2 w}{\partial x^2})^2 + 2D_{xy} (\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2})^2 + 4G_{xy} (\frac{\partial^2 w}{\partial x \partial y})^2 + D_y (\frac{\partial^2 w}{\partial y^2})^2 \right\} dxdy +$$
$$\sum_{i=1}^{n} \left\{ \frac{1}{2} N_{xi} \left[\int_{-a}^{a}\int_{-b}^{b} (\frac{\partial w}{\partial x})^2 dxdy - b\int_{-a}^{a} (\frac{\partial w}{\partial x})^2 \Big|_{y=r_{yi}} dx \right] \right\} +$$
$$\sum_{i=1}^{n} \left\{ \frac{1}{2} N_{yi} \left[\int_{-a}^{a}\int_{-b}^{b} (\frac{\partial w}{\partial y})^2 dxdy - a\int_{-b}^{b} (\frac{\partial w}{\partial y})^2 \Big|_{x=r_{xi}} dy \right] \right\} = \frac{1}{2} \rho h \omega^{\circ} \int_{-a}^{a}\int_{-b}^{b} w^2 dxdy \qquad (12)$$

In general the form of w is not know. However, if assumed form, normally chosen to satisfy the boundary conditions, is substituted in eq. (10). The displacement function $w(x, y, t)$ is approximated by means of the expansion, Kaldas 1981:

$$w(x, y, t) \cong w(x, y)\sin \omega t = \sin \omega t \sum_{ij} C_{ij} X_i(x) Y_j(y) \qquad (13)$$

Let the plate be placed in a coordinate system with the origin at its center and the edge (a) is parallel to X-axis and the edge (b) is parallel to Y-axis.

Table. 1. shows expressions of the displacement functions for different boundary conditions having one free edge. Substituting each expression separately in eq. (12), integrating over the given domains, applying boundary conditions and rearranging terms to obtain the final form of the frequency equation as listed below,

$$C_1 \frac{D_x}{a^4} + C_2 \frac{(D_{xy} + 2G_{xy})}{a^2 b^2} + C_3 \frac{D_y}{b^4} + \sum_{i=1}^{n} \frac{N_{xi}}{a^2} \{A_1 + A_2(\frac{r_{yi}}{b}) + A_3(\frac{r_{yi}}{b})^2 + A_4(\frac{r_{yi}}{b})^3 + A_5(\frac{r_{yi}}{b})^4$$
$$+ A_6(\frac{r_{yi}}{b})^5 + A_7(\frac{r_{yi}}{b})^6 + A_8(\frac{r_{yi}}{b})^7 + A_9(\frac{r_{yi}}{b})^8\} + \sum_{i=1}^{n} \frac{N_{yi}}{b^2} \{B_1 + B_2(\frac{r_{xi}}{a}) + B_3(\frac{r_{xi}}{a})^2 + B_4(\frac{r_{xi}}{b})^3$$
$$+ B_5(\frac{r_{xi}}{a})^4 + B_6(\frac{r_{xi}}{a})^5 + B_7(\frac{r_{xi}}{a})^6 + B_8(\frac{r_{xi}}{a})^7 + B_9(\frac{r_{xi}}{a})^8\} = K\rho h \omega_s^{\circ} \qquad (14)$$

where $C_1, C_2, C_3, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9$ and K are constants depending on the boundary conditions as tabulated in **Table. 2**.

For the stress free condition (no residual stresses $N_{xi} = N_{yi} = 0$), then eq. (14) reduced to:

$$C_1 \frac{D_x}{a^4} + C_2 \frac{(D_{xy} + 2G_{xy})}{a^2 b^2} + C_3 \frac{D_y}{b^4} = K\rho h \omega_s^{\circ} \qquad (15)$$

If $i = 1, r_{xi} = r_{yi} = 0$, this meaning the lines of welding along the length and width of the plate are at the center, and all constant in **Table. 2,** will be zero excepts C_1, C_2, C_3, A_1, B_1, and k.

- RESULTS AND DISCUSSIONS.

An easy procedure was used in this paper to predict the effect of residual stresses due to multi-line of welding along the length and width of a spot welded stiffened plate on the natural frequency. A stainless steel plate of dimensions (120*100*0.6 mm) and a stiffener of (120*20*0.6 mm) were connected by spot welding. The plate and stiffener were assumed of the material with (E=207*10^3 N/mm^2 , G=80*10^3 N/mm^2 and $\upsilon = 0.3$) . In this study three different boundary conditions with one edge is free were discussed (C.C.C.F., C.C.S.F. and S.C.S.F.).

Fig. 4,5 and 6. shows the variation of the fundamental frequencies with welds self-equilibrating stresses on the longitudinal and perpendicular center lines in a thin stiffened rectangular plate with edges C.C.C.F. . As shown in **Fig. 3**, the mid-plane forces N_X exist at any position along the width of the stiffener will decrease the natural frequency by a percent of (21.5% max.) beyond that of free-stress. The frequency change may be understood in terms of energy. For given amplitude of vibration the variation of strain energy in a cycle, largely due to the basic flexural and torsional stiffness of the plate is modified through the work done by the in-plane stresses under mid-plane extension. In the present case the in-plane stresses reduce the total exchange between potential and kinetic energy, and a reduction in frequency therefore follows. On the other, hand the mid-plane forces N_y exist along the length of the plate will increase the natural frequency by a percent of (33.3% max) beyond that of free stress except at the edges in which the natural frequency will decrease beyond that of free stress case. This discussion may be valid also to other cases of boundaries (i.e C.C.S.F. and S.C.S.F.) as shown in **Fig. 7,8,9,10,,11 and 12**, these results gives a good agreement with the analytical results obtained by Dickinson, 1978, without including the residual stress and Golf, 1976, Al-Ammir, 2004, with including the residual stresses.

Fig. 5,8 and 11, shows that the position of spot weld line have a great influence on the natural frequencies when it lays parallel to the clamped edge and it have a little influence a long the free edge parallel to the length of the plate. The changing of residual stress have a significant effect on the magnitude of natural frequency, It is clear that the natural frequency decreased with increasing the magnitude of residual stress spatially near the clamped edge and less pronounced near the free edge as shown in **Fig. 13,15 and 17**. On the other hand **Fig. 6,9 and 12**, indicate that the central position spot weld line increased the natural frequency while the natural frequency decreased when it lays near the edges along the width of the plate.

Fig. 14,16 and 18, gives the effect of magnitude of residual stresses on the natural frequency along the width of the plate, it clear that the natural frequency increased with increasing the magnitude of residual stresses when the spot weld line lays at the middle of plate and decreased with increasing of residual stresses at the two edges of the plate with a little different for case 2 as shown in **Fig. 16**, which have an identical edges along the width of the plate.

CONCLUSIONS

An important conclusion is that the boundary condition and amount of residual stresses have important factors and significant influence on the natural frequency, the free ends tries to increase the natural frequency while the clamped edges try to decrease the natural frequency, this is so clear in **Fig. 5,8 and 11,** therefore in case of C.C.C.F. spot welded plate the free end increased the natural frequency by a percent of 48.39% max., while in case of C.C.S.F. spot welded plate by a percent of 29.05%max., and in case of S.C.S.F. spot welded plate by a percent of 24.28%max.. Also the results give an indication that the central position weld line has the great influence on the natural frequency and this is shown well in **Fig. 6,9 and 12,** so, in case of C.C.C.F. spot welded plate the increase was 48.91% max., while in case of C.C.S.F. spot welded plate by a percent of 35.94%max., and in case of S.C.S.F. spot welded plate by a percent of 26.37%max .

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SYMBOLS:

a, b	Plate side length (mm)
D	Flexural rigidity of an isotropic plate (N.mm)
Dx,Dy	Flexural rigidity of an orthotropic plate in x and y directions
Dxy	Torsional rigidity of an isotropic plate
G	Shear modulus of isotropic material (N/mm^2)
Gxy	Shear modulus of orthotrpic material
H	Enthalpy of material (W.S.C/mm^3)
h	Plate thickness (mm)
Mx,My	Bending moment in x,y direction (N.mm)
Mxy	Twisting moment (N.mm)
Nx, Ny	Edge forces per unit distance (N/mm)
r_{xi}, r_{yi}	Welding position along the x and y-axis (mm)
T	Kinetic energy of the element (W)
t	Time (sec)
U	Strain energy stored in complete plate (W)
Ub	Strain energy stored due to bending (W)
Ut	Strain energy stored due to twisting (W)
Ur	Strain energy stored due to concentrated force (W)
u, v, w	Displacement components in x,y,z directions
x, y, z	Cartesian coordinates
C-C-C-F	Clamped-Clamped-Clamped- Free
C-C-S-F	Clamped-Clamped-Simply- Free
S-C-S-F	Simply-Clamped-Simply-Free
ρ	Mass density (Kg/mm^3)
ω°	Angular frequency (rad/S)

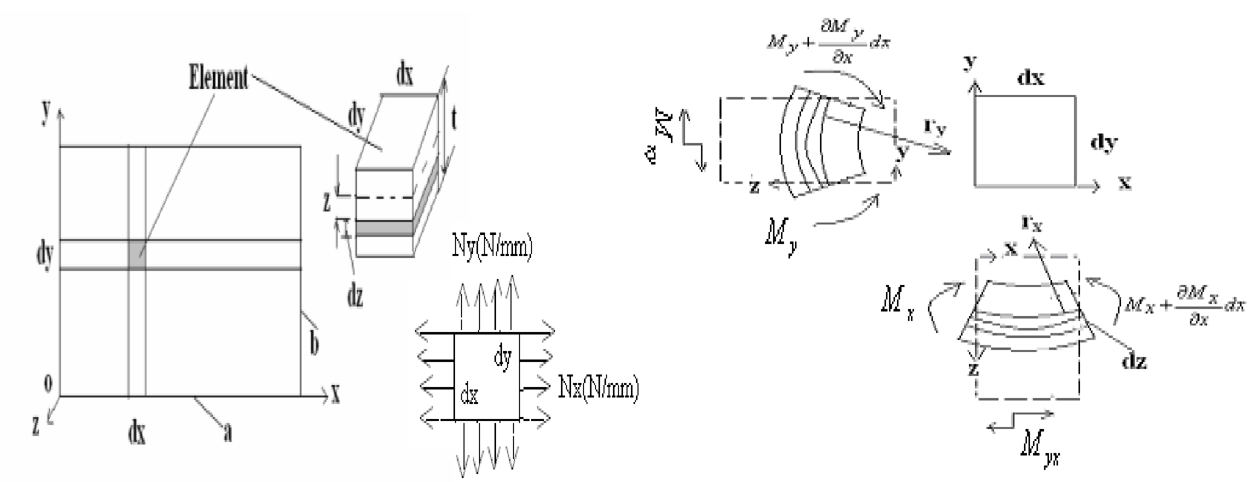

Fig. (1) Thin plate in Bending

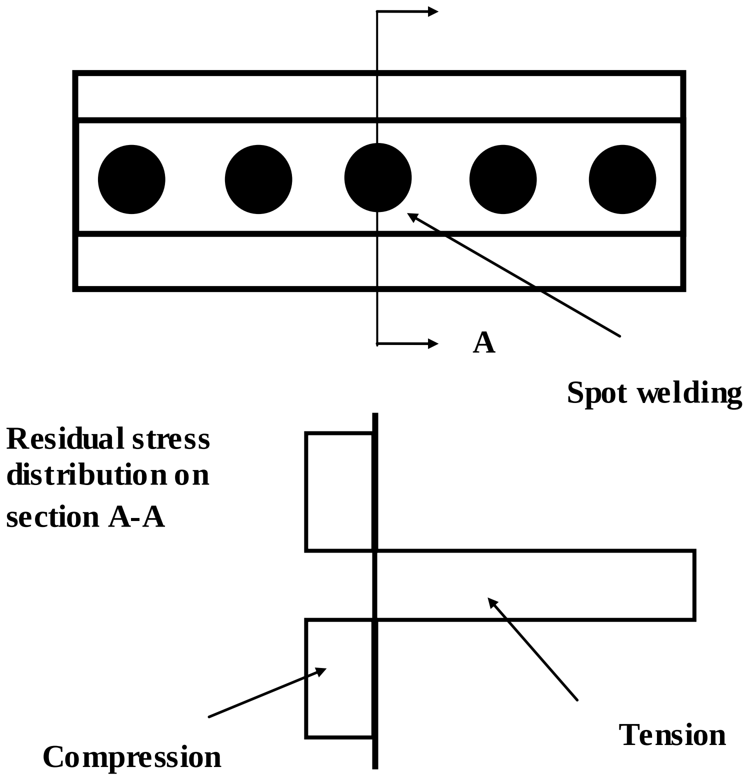

Fig.(2) Plane Loading of a Stiffened Plate.

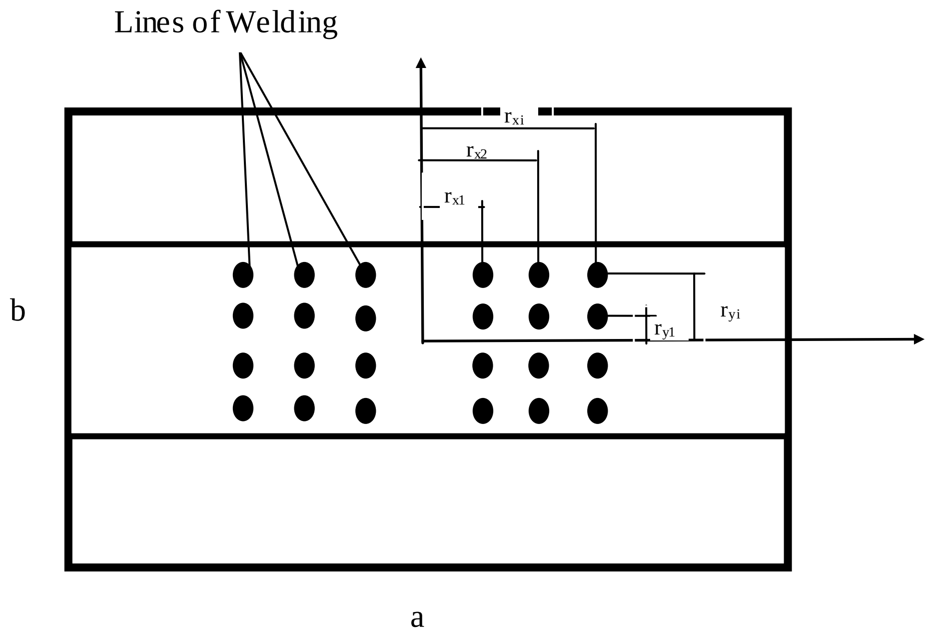

Fig.(3) Plane Loading of a Plate

Table 1 Displacement functions

Boundary Conditions	Displacement Functions
C-C-C-F	$w = C[(\frac{2x}{a})^2 - 1]^2[1 + \frac{56}{17}(\frac{y}{b}) + \frac{24}{17}(\frac{y}{b})^2 - \frac{32}{17}(\frac{y}{b})^3 + \frac{16}{17}(\frac{y}{b})^4]\sin\omega t$
C-C-S-F	$w = C[1 + \frac{x}{a} - 6(\frac{x}{a})^2 - 4(\frac{x}{a})^3 + 8(\frac{x}{a})^4][1 + \frac{56}{17}(\frac{y}{b}) + \frac{24}{17}(\frac{y}{b})^2 - \frac{32}{17}(\frac{y}{b})^3 + \frac{16}{17}(\frac{y}{b})^4]\sin\omega t$
S-C-S-F	$w = C[1 - \frac{6}{5}(\frac{2x}{a})^2 + \frac{1}{5}(\frac{2x}{a})^4][1 + \frac{56}{17}(\frac{y}{b}) + \frac{24}{17}(\frac{y}{b})^2 - \frac{32}{17}(\frac{y}{b})^3 + \frac{16}{17}(\frac{y}{b})^4]\sin\omega t$

Table 2 Values of the constants C_1 , C_2 , ……,K

Constant	C-C-C-F	C-C-S-F	S-C-S-F
C1	1	1	1
C2	69.312/504	64.77/238.73	56.24/97.54
C3	32.358/504	32.358/238.73	32.358/97.54
A1	163428/10253	2941704/194807	4167414/317843
A2	-1028160/10253	-18506880/194807	-26218080/317843
A3	-2134080/10253	-2021760/10253	-54419040/317843
A4	-864000/10253	-15552000/194807	-22032000/317843
A5	1330560/10253	23950080/194807	33929280/317843
A6	-138240/10253	-2488320/194807	-3525120/317843
A7	-967680/10253	-17418240/194807	-24675840/317843
A8	552960/10253	9953280/194807	14100480/317843
A9	-138240/10253	-2488320/194807	-3525120/317843
B1	-1211760/71771	-16899840/1363649	-25317360/2224901
B2	0	-9331200/194807	0
B3	4665600/10253	51321600/194807	69984000/317843
B4	0	9331200/194807	0
B5	-27993600/10253	-205286400/194807	-214617600/317843
B6	0	-298598400/194807	0
B7	74649600/10253	373248000/194807	223948800/317843
B8	0	298598400/194807	0
B9	-74649600/10253	-298598400/194807	-74649600/317843
K	1/504	1/238.73	1/97.54

Case one: C.C.C.F.

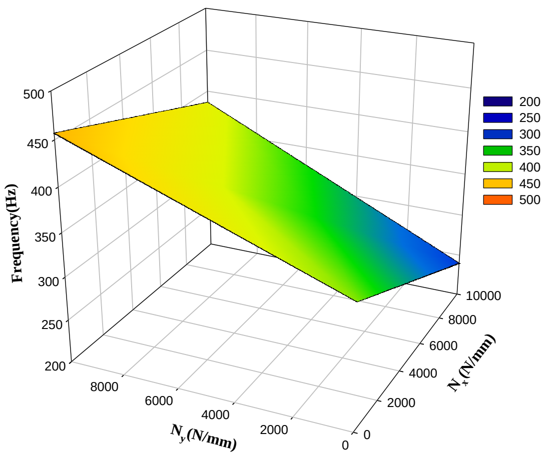

Fig.(4) Fundamental Frequency due to the Self-Equilibrating Stresses at the Longitudinal and Perpendicular Centerlines of a C.C.C.F. Stiffened Plate

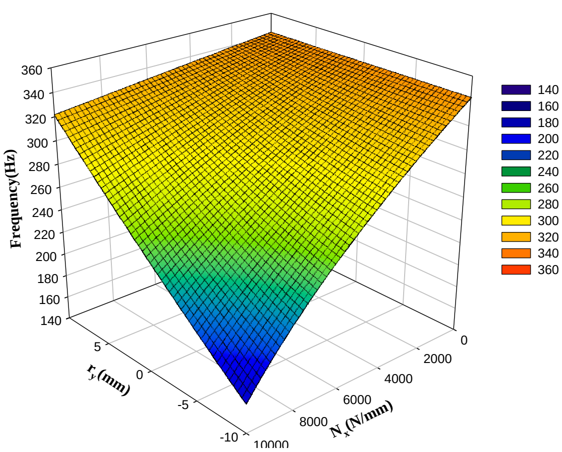

Fig.(5) Fundamental Frequencies of C.C.C.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

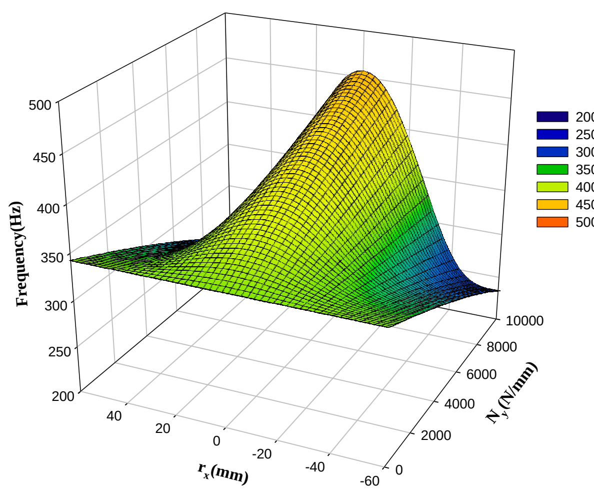

Fig.(6) Fundamental Frequencies of C.C.C.F. Stiffened plate due to the Perpendicular Self-Equilibrating Stresses

Case two: C.C.S.F.

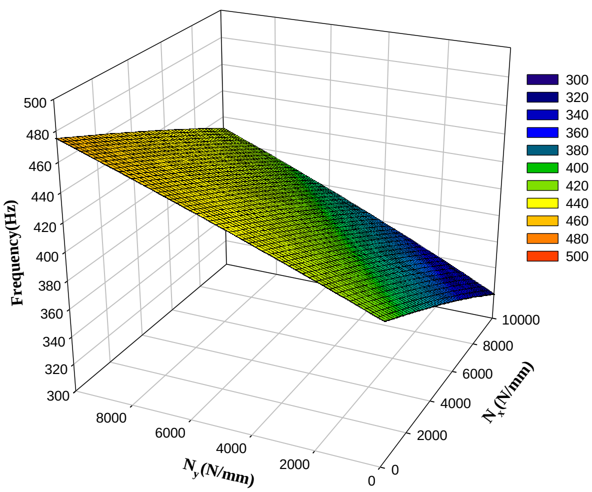

Fig.(7) Fundamental Frequency due to the Self-Equilibrating Stresses at the Longitudinal and Perpendicular Centerlines of a C.C.S.F. Stiffened Plate

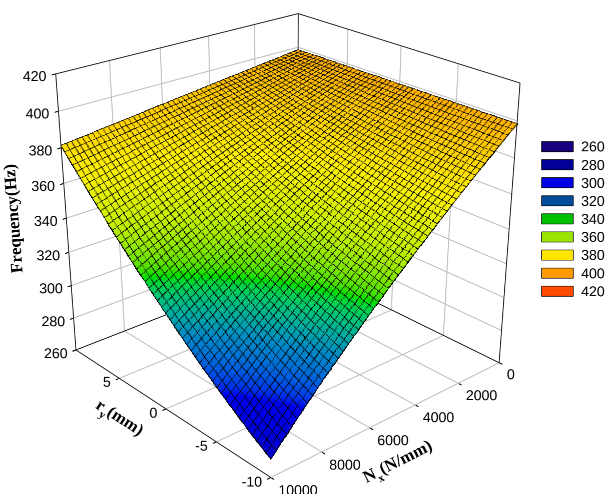

Fig.(8) Fundamental Frequencies of C.C.S.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

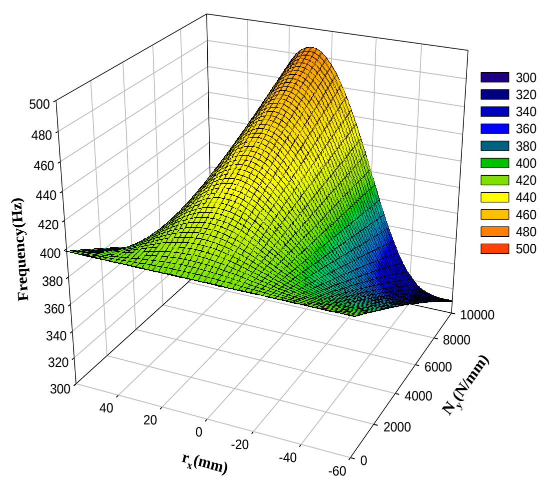

Fig.(9) Fundamental Frequencies of C.C.S.F. Stiffened plate due to the Perpendicular Self-Equilibrating Stresses

Case three: S.C.S.F.

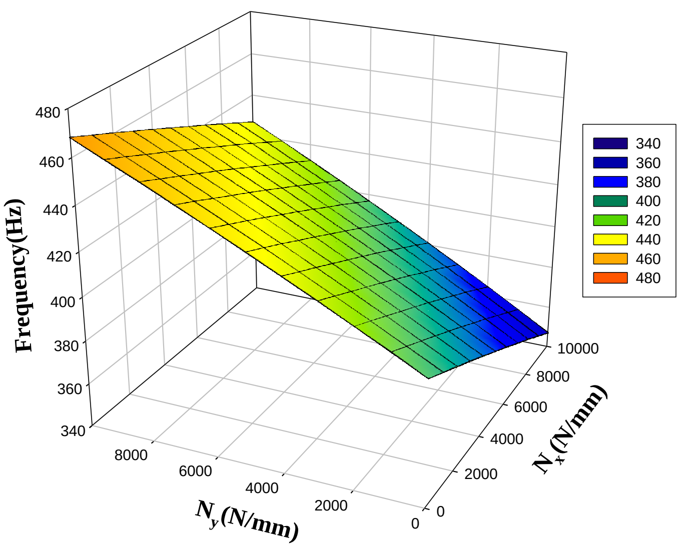

Fig.(10) Fundamental Frequency due to the Self-Equilibrating Stresses at the Longitudinal and Perpendicular Centerlines of a S.C.S.F. Stiffened Plate

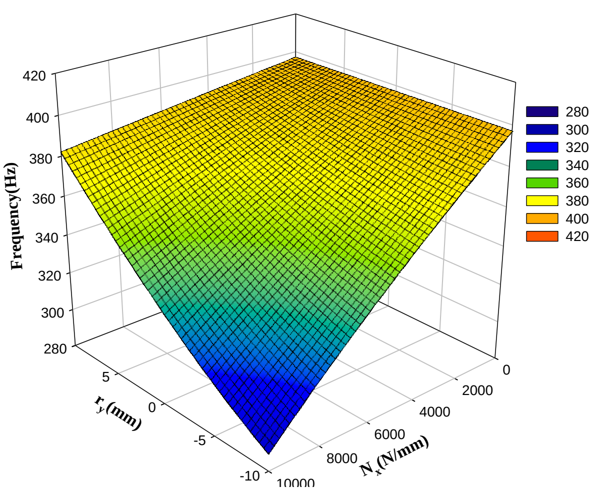

Fig.(11) Fundamental Frequencies of S.C.S.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

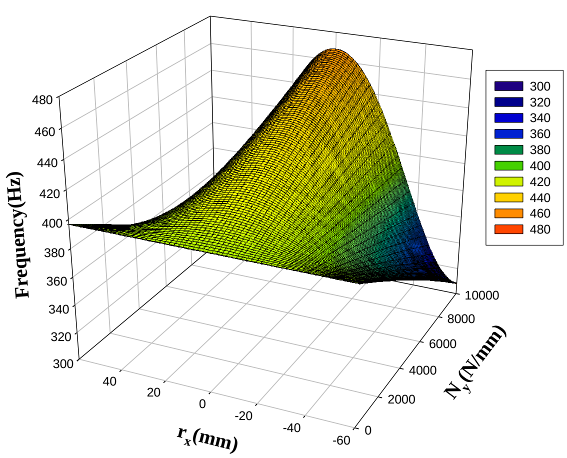

Fig.(12) Fundamental Frequencies of S.C.S.F. Stiffened plate due to the Perpendicular Self-Equilibrating Stresses

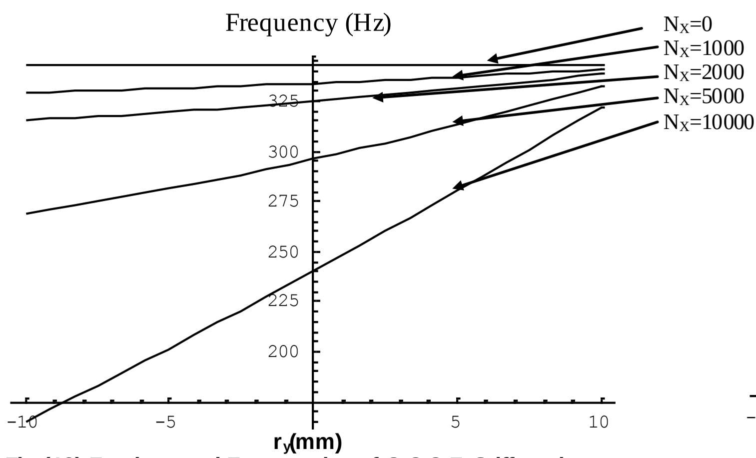

Fig.(13) Fundamental Frequencies of C.C.C.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

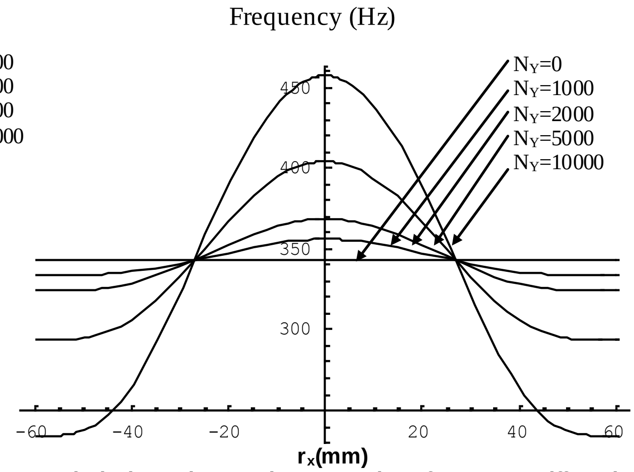

Fig.(14) Fundamental Frequencies of C.C.C.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

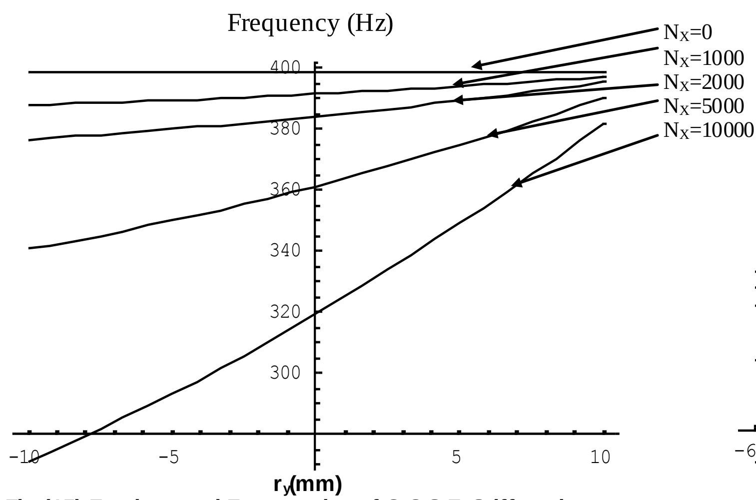

Fig.(15) Fundamental Frequencies of C.C.S.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

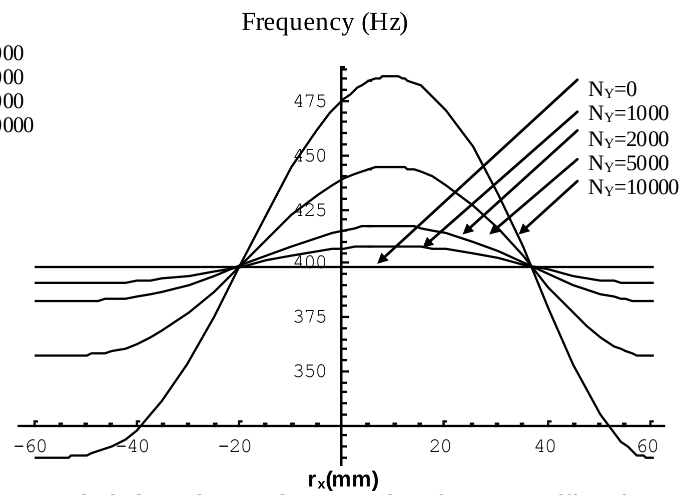

Fig.(16) Fundamental Frequencies of C.C.S.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

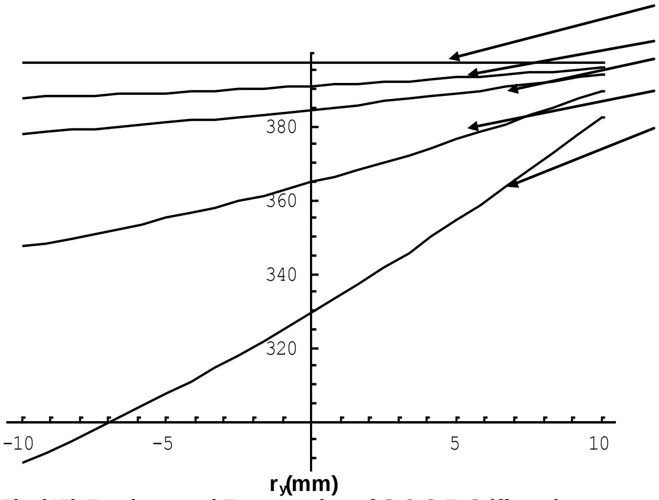

Fig.(17) Fundamental Frequencies of S.C.S.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses

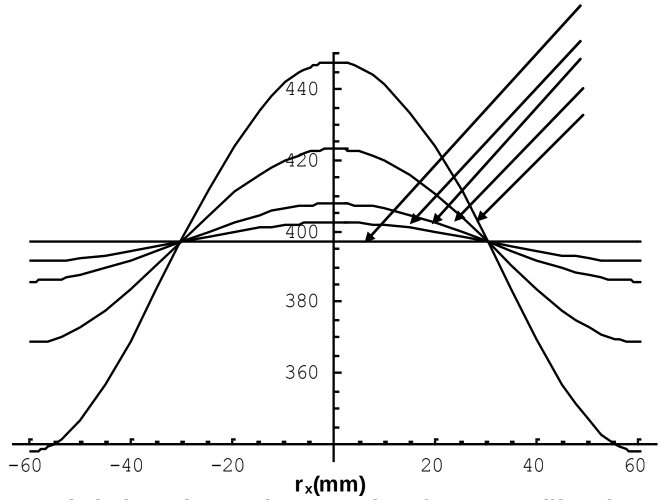

Fig.(18) Fundamental Frequencies of S.C.S.F. Stiffened plate due to the Longitudinal Self-Equilibrating Stresses