BEHAVIOUR OF CROSS - PLY LAMINATED HYBRID COMPOSITE PLATES WITH AN INCLINED CRACK SUBJECTED TO A UNIFORM TEMPERATURE RISE

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ABSTRACT

Thermal buckling analysis of symmetric and antisymmetric cross-ply laminated hybrid composite plates with an inclined crack subjected to a uniform temperature rise are presented in this paper. The first-order shear deformation theory in conjunction with variational energy method is employed in the mathematical formulation. The eight-node Lagrangian finite element technique is used for obtaining the thermal buckling temperatures of hybrid composite laminates. The effects of crack size and lay-up sequences on the thermal buckling temperatures for symmetric and a antisymmetric plates are investigated. The results are shown in graphical form for various boundary conditions. Finally , from this paper, the following main results have been found from which the buckling temperature is affected the larger crack length more than the small crack length, together with other result that the buckling temperature of the plate for every perforation angle is to increase while crack length is increasing

KEYWORDS: hybrid composite plates, thermal buckling, finite element method.

الخلاصة

ان تحليل الانبعاج الحراري للصفائح الرقيقة الهجينة المصنعة من مواد مركبة ذات الطبقات المتماثلة وغير المتماثلة ذات الشق المائل والتي تخضع لإرتفاع منتظم لدرجات الحرارة تمت دراستها في هذا البحث . وقد تم استخدام نظرية تشوه القص ذات الرتبة الاولى المرتبطة بنظرية الطاقة المتغيرة في الصياغة الرياضية لهذا البحث، كذلك تم استخدام تقنية العناصر المحددة بثمانية عقد للحصول على درجات حرارة الانبعاج الحراري للصفائح المركبة الهجينة. كما تم دراسة تأثيرات حجم الشق ومواضع تسلسل درجات حرارة الانبعاج الحراري للصفائح المتماثلة وغير المتماثلة. ان النتائج موضحة على شكل مخطط وبشروط حدية مختلفة.وأخيرا ومن خلال البحث تم الحصول على اهم النتائج ومنها تأثير الانبعاج الحراري على طول الشق الكبير اكثر من الشق الصغير بالاضافة الى ان الأنبعاج الحراري للصفيحة لكل زاوية ناتجة يزداد مع زيادة طول الشق .

INTRODUCTION

Fiber reinforced structures are widely used in so many engineering applications. because of their low weight and high strength. Stability of these structures is important especially at elevated temperatures.

The thermal buckling analyses or orthotropic plates including a crack, were investigated by **Avci , A. and Sahin , O.S.,2005** Thermal buckling analysis of symmetric and antisymmetric cross-ply laminated hybrid composite plates with a hole subjected to a uniform temperature rise for different boundary conditions was studied by using finite elements method by **Avci ,**

A. and Sahin , O.S.,2005. In that paper the effects of hole size, lay-up sequences and boundary conditions on the thermal buckling temperatures were investigated. **Akbulut, H. and Sayman,O.,2001** were studied the buckling behavior of laminated composite plates with central square openings for various boundary conditions and stacking sequences by using finite element method.

The thermal buckling of isotropic and composite plates with a hole by using both closed form solution and finite elements method was investigated by **Chang, J.S.and Shiao,F.J.,1990.** Thermal buckling of antisymmetric cross-ply composite laminates was investigated by **Mathew, T.C. and Rao, G.V.,1992**. Abramovich investigated the thermal buckling behavior of cross-ply symmetric and nonsymmetrical laminated beams employing the first-order deformation theory. **Murphy, K.D. and Ferreira, D.,2000**. Studied theoretical and experimental approaches to obtain the buckling temperature and buckling mode for flat rectangular plates. **Huang, N.N. and Tauchert, T.R., 1992.** Studied the thermal buckling of clamped symmetric angle-ply laminated plates employing a Fourier series approach and the finite element method. **Prabhu, M.R. and Dhanaraj, R., 1994 Thangaratnam, K.R. and Ramaohandran,J.,1989** , also studied the thermal buckling of the laminates subjected to uniform temperature rise or non uniform temperature fields using the finite element approach.

An extensive overview of the general buckling problems of laminated composite plate was made by **Liessa,A.W.,1987.**

In that study, some complicated effects were investigated such as shear deformation, hydrothermal factors and post buckling behavior. The influence of temperature distribution on buckling modes has investigated by **Bednarczyk,H. and Rihter,M.,1985**. The thermally induced buckling of antisymmetic angle-ply laminated plates with Levy-type boundary conditions was investigated by **Chen,L.W. and Liu, W.H., 1993** . Thermal buckling behavior of composite laminated plates with transverse shear deformation was studied by **Sun,L.X. and Hsu,T.R.,1990. Chockalingam, S. ,1992.** investigated the thermal buckling of antisymmetric cross-ply hybrid laminates by using finite element technique based on first-order shear deformation theory. **Chen ,W. and Liu, W.H., 1993**. also studied thermal buckling of laminates subjected to uniform temperature rise or non uniform temperature fields using finite element approach. Local buckling of composite laminar plates was considered and the critical strains of laminated plates with various shaped local delamination and different stacking patterns are obtained by making use of the energy principle. by **Wang, X.and Lu,G.,2003**. Also non-linear thermal buckling for local delamination near the surface of laminated cylindrical shell problem was studied by **Wang, X.and Lu,G.,2003**.

The present paper aims to determine the buckling temperature and buckling mode shapes for hybrid composite laminates with different inclined crack by using the finite element method. The thermal buckling of symmetric and antisymmetric cross-ply laminates with cracks is investigated, based on the first-order deformation theory in conjunction with the variational energy method. The finite clement approach is used for obtaining the thermal buckling temperatures for boron/epoxy-glass/epoxy hybrid laminates. The effects of crack length, crack inclination angle and lay-up sequences on the thermal buckling temperatures have been studied numerically . The buckling behavior of boron/epoxy-glass/epoxy hybrid composite plates was compared with E-glass/epoxy plates.

MATHEMATICAL FORMULATION

The laminated orthotropic construction of the plate is consisted of N layers. Each layer is of thickness t_k, so that h $=\sum_{k=1}^{N} t_k$ is the total thickness of the laminate.

The longitudinal and lateral dimensions of the laminate are *a* and *b* and subjected to uniform temperature difference ΔT between ambient and laminated plate as shown in **Fig. 1**. The linear stress-strain relation for each layer is expressed with *x*, y-axes and has the form**(Whitney,J.M.,1973)** .

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k \begin{Bmatrix}\varepsilon_x & -\alpha_x\Delta T\\ \varepsilon_y & -\alpha_y\Delta T\\ \varepsilon_{xy} & -\alpha_{xy}\Delta T\end{Bmatrix}_k$$

$$\begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix}\bar{Q}_{44} & \bar{Q}_{45}\\ \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \quad (1)$$

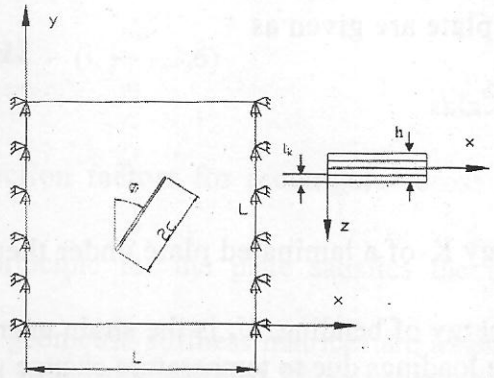

Fig. 1. Geometry of the problem and coordinates

where $\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}$ and τ_{xz} are the stress components, $\bar{Q}_{ij}$ are transformed reduced stiffnesses, which can be expressed in terms of the orientation angle and the engineering constant of the material. ΔT is temperature difference, α_x, and α_y, are the coefficients of thermal expansion in directions of x and .y-axes, respectively. α_{xy} is the apparent coefficient of thermal shear, such as**(Jones, R.M.,1975)**

$$\alpha_x = \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta$$
$$\alpha_y = \alpha_2 \cos^2\theta + \alpha_1 \sin^2\theta \quad (2)$$
$$\alpha_{xy} = 2(\alpha_1 - \alpha_2)\sin\theta . \cos\theta$$

α_1 and α_2 are the thermal expansion coefficients of the lamina along the longitudinal and transverse directions of fibers, respectively.

In this study first-order shear deformation theory is used. The displacements u, v and w can be written as follows:

$$u(x, y, z) = u_0(x, y) + z\,\psi_x(x, y)$$
$$v(x, y, z) = v_0(x, y) + z\,\psi_y(x, y) \quad (3)$$
$$w(x, y, z) = w(x, y)$$

where. u_0, v_0, w are the displacements along to x, y, and z-axes. respectively, at any point of the middle surface, and, ψ_x , ψ_y are the bending rotations of normal to the mid plane about the x and y axes, respectively. The bending strains ε_x , ε_y and transverse shear strains γ_{xy}, γ_{yz} , γ_{xz} at any point of the laminate are **(Jones, R.M.,1975)**

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{vmatrix}\frac{\partial u_o}{\partial x}\\ \frac{\partial v_o}{\partial y}\\ \frac{\partial u_o}{\partial y}+\frac{\partial v_o}{\partial x}\end{vmatrix}+z\begin{vmatrix}\frac{\partial \psi_x}{\partial x}\\ \frac{\partial \psi_y}{\partial y}\\ \frac{\partial \psi_x}{\partial y}+\frac{\partial \psi_y}{\partial x}\end{vmatrix}$$

$$\begin{vmatrix}\gamma_{yz}\\ \gamma_{xz}\end{vmatrix}=\begin{vmatrix}\frac{\partial w}{\partial y}-\psi_y\\ \frac{\partial w}{\partial x}+\psi_x\end{vmatrix} \qquad (4)$$

The resultant forces N_x, N_y and N_{xy}, moments M_x, M_y and M_{xy} and shearing forces Q_x, Q_y per unit length of the plate are given as

$$\begin{bmatrix}N_x & M_x\\ N_y & M_y\\ N_{xy} & M_{xy}\end{bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}(1.z)dz$$

$$\begin{Bmatrix}Q_x\\ Q_y\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}dz \qquad (5)$$

The total potential energy K of a laminated plate under thermal loading is equal to

$$K=U_b+U_s+V \qquad (6)$$

where U_b is the strain energy of bending, U_s is the strain energy of shear and V represents the potential energy of in-plane loadings due to temperature change

$$U_b=1/2\int_{-h/2}^{h/2}\left[\int\int_R\left(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\tau_{xy}\gamma_{xy}\right)dA\right]dz$$

$$U_s=1/2\int_{-h/2}^{h/2}\left[\int\int_R\left(\tau_{xz}\gamma_{xz}+\tau_{yz}\gamma_{yz}\right)dA\right]dz$$

$$V=1/2\int\int_R[\bar{N}_1(\partial w/\partial x)^2+\bar{N}_2(\partial w/\partial y)^2+2\bar{N}_{12}(\partial w/\partial x)(\partial w/\partial y)dA-\int_{\partial R}(\bar{N}_n^b u_n^o+\bar{N}_n^b u_s^o)ds \qquad (7)$$

Here $dA=d_x d_y$, R is the region of a plate excluding the crack. $\bar{N}_n^b$ and $\bar{N}_s^b$ are in-plane loads applied on the boundary ∂R. For the equilibrium, the potential energy K must be stationary. The equilibrium equations of the cross-ply laminated plate subjected to temperature change can be derived from the variational principle through use of stress-strain and swain-displacement relations. One may obtain these equations by using $\delta K=0$.

Finite Element Formulation

In general, a closed form solution is difficult to obtain for buckling problems Therefore numerical methods are usually used for obtaining an approximate solution. In order to study the buckling of the plate, an eight-node Lagrangian finite element analysis is applied in this study. The stiffness matrix of the plate is obtained by using the minimum potential energy principle. Bending stiffness $[K_b]$, shear stiffness $[K_s]$ and geometric stiffness $[K_g]$ matrices can be expressed as

$$[K_b]=\int_A [B_b]^T[D_b][B_b]dA \qquad (8)$$

$$[K_s]=\int_A [B_s]^T[D_s][B_s]dA \qquad (9)$$

and

$$[K_g] = \int_A [B_g]^T [D_g][B_g]\, dA \qquad (10)$$

Where

$$[D_b]=\begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \qquad [D_b] = \begin{bmatrix} K_1^2 A_{44} & 0 \\ 0 & K_2^2 A_{55} \end{bmatrix}$$

and $[D_g]=\begin{bmatrix} \bar{N}_1 & \bar{N}_{12} \\ \bar{N}_{12} & \bar{N}_2 \end{bmatrix}$ (11)

$$(A_{ij}, B_{ij}, D_{ij})= \int_{-h/2}^{h/2} \bar{Q}_{ij}(1,z,z^2)\,dz \quad \text{(i, j= 1,2,6)} \qquad (12)$$

$$(A_{44}, A_{55})= \int_{-h/2}^{h/2} (\bar{Q}_{44}, \bar{Q}_{55})\,dz \qquad (13)$$

A_{44} and A_{55} are the shear correction factors for rectangular cross section are given by $K_1^2 = K_2^2 = 5/6$ (Whitney)

The total potential energy principle for the plate satisfies the assembly of the element equations.

The element stiffness and the geometric stiffness matrices are assembled. The corresponding eigenvalue problem can be solved using any standard Eigen value extraction procedures**(Bathe, K.J.,1982)**

$$\left[[K_\theta] - \lambda_b [K_{\theta g}]\right]\begin{Bmatrix} u_i \\ u_i \\ w_i \end{Bmatrix} = 0 \qquad (14)$$

where

$[K_\theta] = [K_b] + [K_s]$, and $-\lambda_b [K_{\theta g}] = [K_g]$ **(15)**

The product of λ_b and the initial guess value ΔT is the critical buckling temperature T_{cr} . that is

$T_{cr} = \lambda_b\, \Delta T$ **(16)**

NUMERICAL RESULTS AND DISCUSSION

There are many techniques to solve Eigen value problems. In this study the Newton Raphson method is applied to obtain numerical solutions of the problem. For thermal buckling due to a ΔT temperature change in the plate, the uniaxial or biaxial in-plane loads are developed along the rectangular edges, while the crack edges are free.

The E-glass/epoxy, and boron/epoxy are considered as components of hybrid plate and their thermo-elastic properties are given in **Table 1.** Here, E_1 and E_2 are elastic moduli in 1 and 2 directions, respectively. υ_{12} is Poisson's ratio and α_1 and α_2, are thermal expansion coefficients of the materials used in the solution. The effect of α_{12}, is neglected.

Stacking sequence of hybrid composite plates have been taken both symmetric and antisymmetric. Stacking sequences have been represented below. Boron/epoxy and glass/epoxy layers are named B and G, respectively.

Table 1 Material properties

Material	E_1 (Gp$_a$)	E_2 (Gp$_a$)	E_{12} (Gp$_a$)	υ_{12}	α_1(1/°C)	α_2(1/°C)
E-glass / epoxy	15	6	3	0.3	7.0×10^{-6}	2.30×10^{-5}
Boron/ epoxy	101	19	4.8	0.12	4.17×10^{-6}	1.91×10^{-5}

The sequence of 4 Layers symmetric lay up of glass/ epoxy-boron / epoxy is 0°G / 90°B /90°B /0°G The sequence of 4 Layers ant symmetric lay up of glass / epoxy-boron/epoxy is 0°G /90°B /0°G/90°B

Each layer has 0.25 mm thickness and the length of one edge of square plate is 100 mm. 2c/L. ratio. represents the crack size to length of one side of composite plate and ϕ represents the crack inclination angle as shown in **Fig. 1**.

A wide range of boundary conditions can be accommodated, but only one kind of' boundary conditions is chosen as defined below:

Two edges clamped and two edges are free

$At\ x = -\frac{L}{2}\ and\ \frac{L}{2}$ $u = w = \psi_y = \psi_x = 0$

Fig. 2 shows the meshed plate. Four edges of plate have divided into ten parts disregarding the crack size.

Buckling in this paper , ANSYS program version 9 has been used .Behavior of anti symmetrically stacked hybrid composite plate is shown in **Fig**.3. In this figure. T/T_0 ratio is used instead of buckling temperature and 2C/L ratio is used instead of crack length C. Here T_0 represents the buckling temperature of hybrid plate without crack. Thus graph is plotted by using dimensionless axes. It can be seen that when the crack inclination angle ϕ = 90°' and ϕ = 60° T/T_0 ratio decreases while crack length increases. The case of ϕ = 30° and ϕ = 0° T/T_0 ratio increases while crack length increases. If inclination angle decreases, the buckling temperature or T/T_0 ratio decreases. For larger values of crack length, the difference between temperatures for different inclination angles increase. These results show that buckling temperature is effected by both crack length and inclination angle.

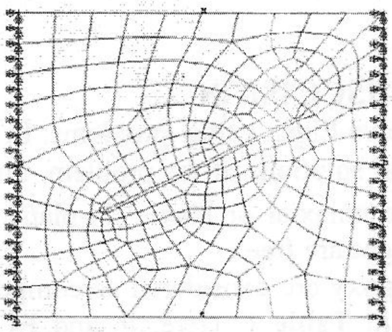

Fig.2.Typical mesh

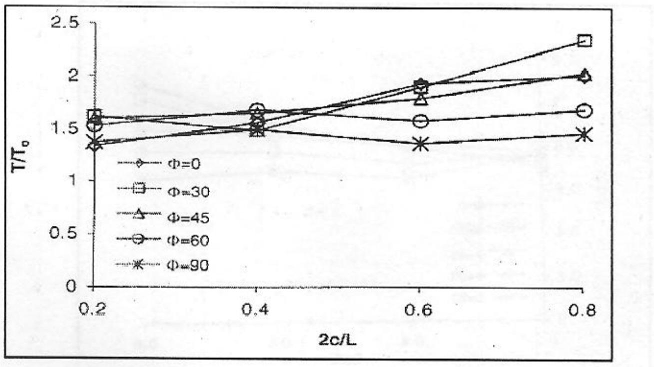

Fig.3.Effect of crack size and position on buckling temperature of antisymmetrically stacked hybrid composites plate.

The buckling behavior of symmetrically stacked hybrid plate is shown in **Fig. 4**. **Fig. 3** shows that, the buckling temperature is depending on crack length and crack inclination angle. But T/To ratio in symmetrically stacked plate is greater than antisymmetrically stacked one for fixed crack length and inclination angle. This result shows that the buckling temperature and consequently the buckling resistance of symmetrically stacked hybrid plate is greater than that of antisymmetrically stacked hybrid plates.

The relationship between crack size and buckling temperature is shown in **Figs. 5** and **6**. Buckling temperature is plotted with respect to crack size for various inclination angels of antisymmetrically stacked E-Glass/epoxy plates in **Fig. 5**. The smallest T/T0 ratios are obtained for crack inclination angle of $\phi = 90°$. On the other hand, it is concluded that, the effect of cracks upon buckling temperature become clear as the crack length increases. Though this behavior is similar for all crack inclination angles, for small inclination angles, this behavior can be seen clearly.

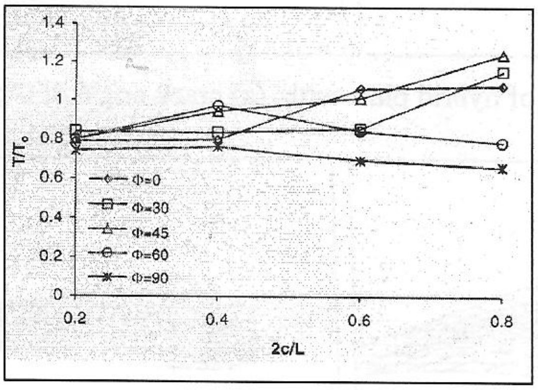

Fig.4. Effect of crack size and position on buckling temperatures of symmetrically stacked hybrid

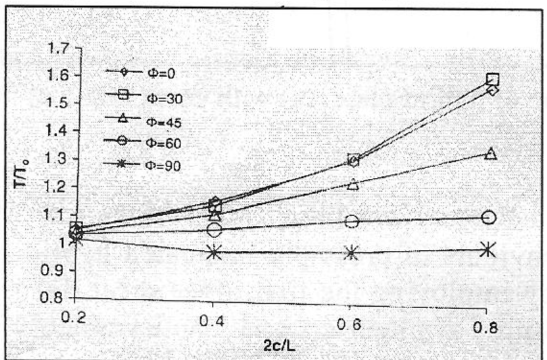

Fig.5.Variation of bucking temperatures of antisymmetrically stacked E-galss/epoxy plate.

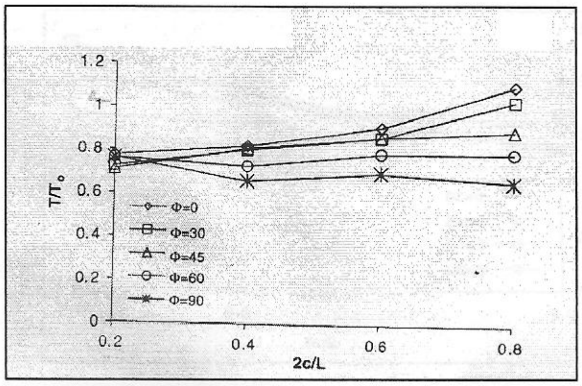

Fig.6.Variation of bucking temperatures of symmetrically stacked E-galss/epoxy plate

The first buckled mode shapes generated glass/epoxy cross-ply laminated plates with inclined crack are shown in **Figs.** 7 and **8**. It is found that critical temperature for crack angle of 0°is 26,284 °C. for crack angle of 60° is 20,851 °C. for crack angle of 90° is 17,148 °C and critical temperature for plate without crack is 24,572 °C, respectively. The mode shapes presented in **Figs. 7 and 8** show considerable skewing for the laminated plates

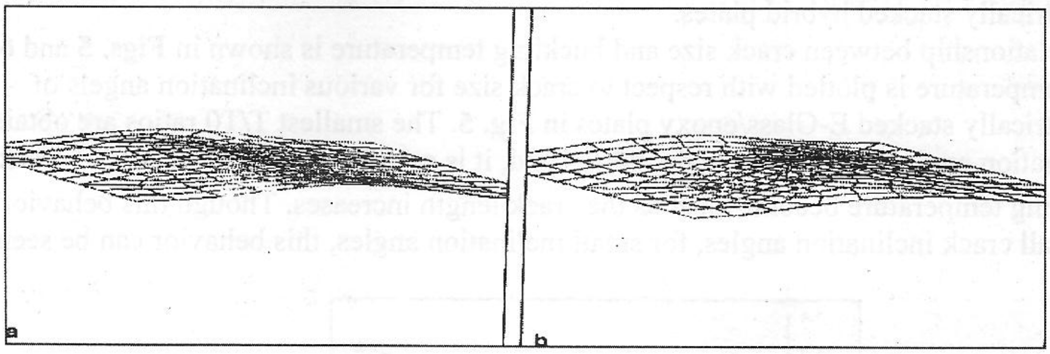

Fig.7.Bukled mode shape of hybrid plate with: (a) crack angle of 0°;(b)crack angle of 60°

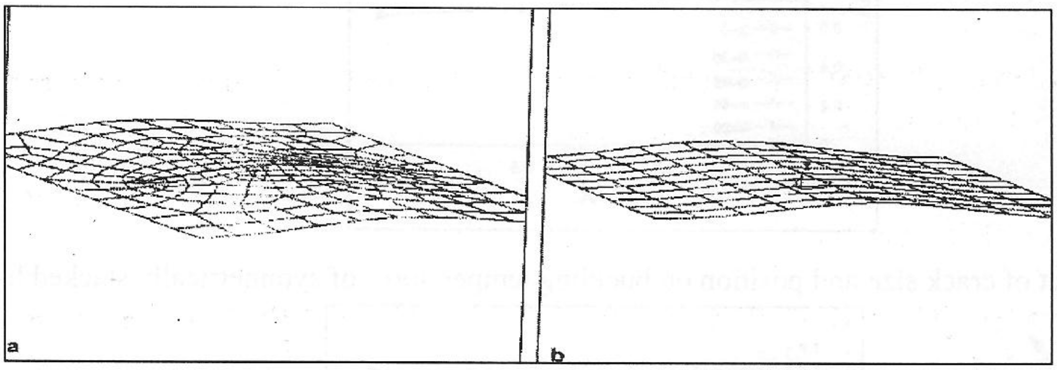

Fig.8.Bukled mode shape of hybrid plate: (a) with crack angle of 90°;(b)without crack

CONCLUSIONS

- Following are the main summarized conclusions drawn from this paper are considered:
- Thermal buckling behaviours of cross-ply laminated hybrid plates with inclined crack have been examined by employing the first-order shear deformation theory and finite lement technique.Both symmetric and antisymmetric lay-up sequence are considered.

- Because of absence of bending-extension coupling, symmetric cross-ply E-glass/epoxy laminates does not yield the highest buckling resistance as expected. Effect of crack upon thermal buckling is minimum while crack inclination angle is 90°.
- As the crack length increases, this effect becomes clear. Effect of cracks upon thermal buckling for hybrid laminated composite plate and E-glass/epoxy laminates are different.
- Effect of crack upon thermal buckling for antisymmetrically stacked hybrid laminates is neative while cracks cause positive effect on symmetrically stacked hybrid plates and effect of cracks upon thermal buckling for antisymmetrically stacked E-glass/epoxy laminates is positive while cracks cause negative effect on symmetrically stacked E-glass/epoxy plates.
- The buckling temperature is affected the larger crack length more than the small crack length as shown in **Figs. 3-6**. For small crack length the high temperature or high thermal stresses can be supported by the imperforated section of the plate. This result can be seen for every perforation angles in **Figs. 3-6**. T/T_0 rates converge the "1" for small perforations at all plates.
- Another result is that buckling temperature of the plate for every perforation angel is to increase while crack length is increasing. This result is expected. Because the higher temperature needs for reaching the same stress level for buckling when the perforation is bigger. If the crack length is great, that means that stressed cross section is fewer so this cross section must be heated much more than when the imperforated section is higher.

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