A COUPLED DYNAMIC FINITE ELEMENT ANALYSIS OF SATURATED SANDS

O.F. al – Damluji
Assistant Professor, Head of Dept. of Civil Engineering, University of Baghdad, Iraq.

Y.J. al – Shakarchi
Professor, Dept. of Civil Engineering, University of Baghdad, Iraq.

Mohammed Yousif Fattah
Lecturer, Dept. of Building and Construction, University of Technology, Iraq.

ABSTRACT
A general mixed finite element formulation (u - w - π) is presented in this paper. This formulation includes the inertia effects and the soil skeleton is considered compressible. The application of this formulation in solving soil dynamic problems of saturated sand is made by governing the boundary conditions concerning the pore fluid. A problem of soil column subjected to an instantly applied surface load is solved. The solid skeleton and pore fluid pressure are each modelled with ten 4-noded isoparametric elements. The results are compared with those obtained by Zienkiewicz et al. (1988). It is concluded that the undamped response of displacement and pore pressure oscillates significantly with the increase of time step length.

الخلاصة
يتضمن البحث تمثيلاً عاماً بطريقة العناصر المحددة للمسائل المزدوجة من نوع (u – w – π) . ويتضمن هذا التمثيل القصور الذاتي ويأخذ بنظر الاعتبار انضغاطية المادة الهيكل الصلب للتربة. وطبق هذا التمثيل في حل مسائل ديناميك التربة للرمال المشبعة بالماء من خلال التحكم بالشروط الحدودية الخاصة بضغط ماء المسام. و تم حل مسألة عمود من التربة الرملية المشبعة معرض إلى حمل ديناميكي يسلط لحظيا على سطح التربة. و مثل كل من هيكل التربة و ضغط ماء المسام بعناصر رباعية العقد. و قد قورنت النتائج مع تلك المستحصلة من قبل (Zienkiewicz et al., 1988)، و تم التوصل إلى نتيجة أن الاستجابة غير المخمدة للأزاحات و ضغط ماء المسام تتذبذب بشكل واضح مع زيادة طول المرحلة الزمنية.

KEY WORD
Plasticity model, saturated sand. Dynamic analysis

SOIL DYNAMIC PROBLEMS
In a saturated soil, with free drainage conditions prevailing, the steady state pore fluid pressures depend only on the hydraulic conditions and they are independent of the soil skeleton response to external loads. Therefore, in that case, a single - phase continuum description of soil behaviour is certainly adequate. Similarly, a single - phase description of soil behaviour is also adequate when no drainage conditions prevail, i.e., there is an interaction between the skeleton strains and the pore fluid flow.
The solutions of dynamic problems require that the soil behaviour be analyzed by incorporating the effects of the transient flow of the pore fluid through the voids, and therefore they require that a two - phase continuum formulation be available for porous media, (Prevost,1987).

As a consequence of the applied cyclic loads, the structure of the cohesionless soil tends to become more compact with a resulting transfer of stress to the pore water and a reduction in stress on the soil grains. As a result, the soil grain structure rebounds to the extent required to keep the volume constant, and this interplay of volume reduction and soil structure rebound determines the magnitude of the increase in pore water pressure in the soil. The basic phenomenon is shown schematically in **Fig. (1),** (Seed, 1979).

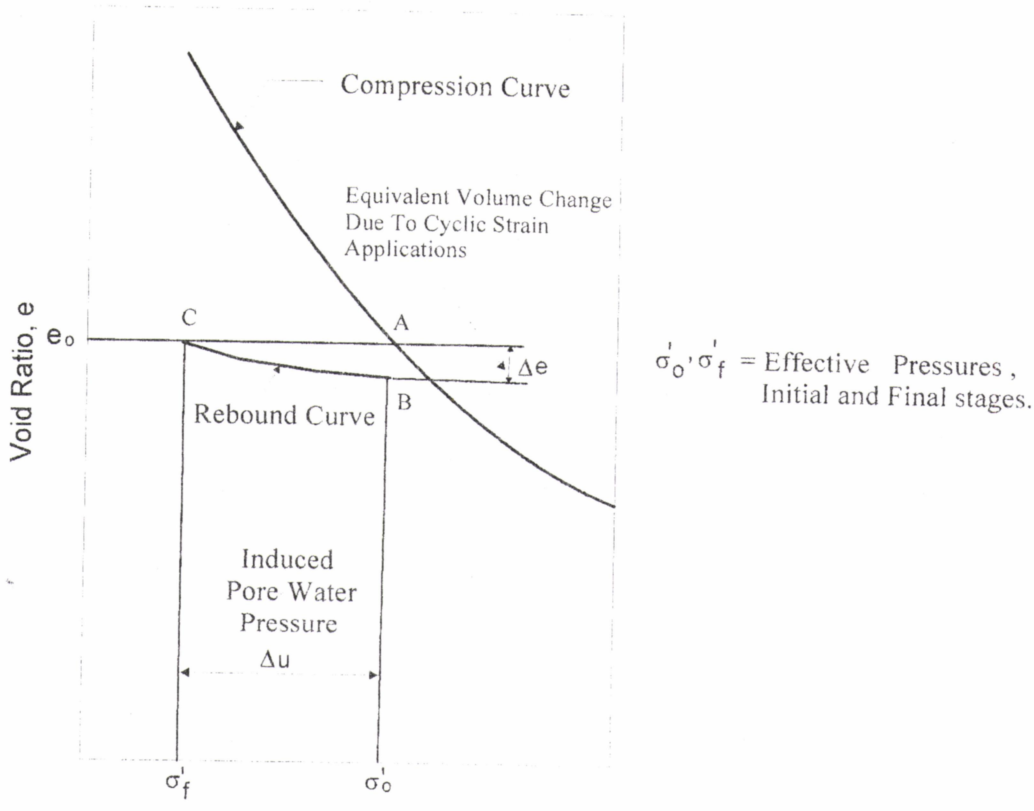

Fig. (1) - Schematic illustration of the mechanism of pore pressure generation during cyclic loading, (after Seed, 1979).

PORE WATER PRESS

The first model for predicting pore water pressures generated by cyclic loading was proposed in 1975 by Martin, Finn and Seed. This model was coupled later to a procedure for dynamic analysis by Martin et al. in 1975, giving the first method for dynamic effective stress analysis. A quantitative relationship between volume reductions occuring during drained cyclic tests and the progressive increase of pore water pressure during undrained cyclic tests had been developed. The use of this relationship enables the build-up of pore water pressure during cyclic loading to be computed theoretically using the basic effective stress parameters of the sand, (Martin et al., 1975). Many other methods for both total and effective stress analyses have been developed.

MODELLING THE DYNAMIC BEHAVIOUR OF SOILS

Soil behaviour under dynamic loading depends on many factors, including, (Daghigh, 1993):

1- The nature of the soil (permeability, relative density, fabric, etc.).
2- The environment of the soil (static stress state and water pressure).
3- The nature of the dynamic loading (strain magnitude, strain rate and number of cycles of loading).

The proper modelling of dynamic behaviour of the soil must take into account the above factors.

Biot (1956) developed the theory of propagation of elastic waves in a porous saturated solid. In this theory, the stress - strain relations for the solid-fluid aggregate were set and the equations that govern the propagation of the waves in a porous medium were derived. This theory predicts one rotational and two dilatational waves existing during vibrations.
Biot (1962a) extended his earlier theory of acoustic propagation in porous media to include anisotropy and visco-elastisity. A "viscodynamic operator" that provides a procedure for the evaluation of the dynamic properties of the fluid in its motion relative to the solid was introduced. A generalized form of Darcy's law from thermodynamic principles was derived in this theory.
Bazant and Krizek (1975) extended Biot's linear elastic theory to the non-linear non-elastic case. An incremental stress - strain relationship that takes into account the nonlinearity and inelasticity of the soil was formulated.
Zienkiewicz and Bettess (1982) extended Biot's formulation to rocks and other porous materials. A physical derivation of the governing equations was presented. Their formulation presented the basic model into which detailed constitutive relationships can be inserted when full analysis is to be carried out.

GOVERNING EQUATIONS FOR DYNAMIC PROBLEMS

The governing equations include the dynamic relations, continuity and the constitutive equations. A complete derivation of the dynamic equations is found in Biot (1956, 1962b).
In a given problem, the loading rate and the permeability of the porous medium play a key role in determining the time scale and the method of solution to be used. When relatively rapid loads are applied and permeability is low, an undrained analysis is possible, i.e., the load rate is greater than the pore fluid diffusion rate.
For situations with relatively slow loading and high permeability, i.e., where load rate is less than the pore fluid diffusion rate, a drained analysis is possible.
The class of problems to be considered here lies between the undrained and drained extremes where dynamic loading is applied and transient pore fluid motion is significant, (Simon et al., 1986).

EFFECTIVE STRESS AND CONSTITUTIVE RELATIONS

Pure statics allows dividing the total stress state into two parts, one of these being the hydrostatic pressure, P, acting externally and internally on the pore fluid (*principle of effective stress*), thus:

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij} P \qquad (1)$$

where σ_{ij} is the total stress, σ'_{ij} is the effective stress and δ_{ij} is Kronecker's delta.
This separation is useful in the description of the stress effects. In soil mechanics, the slight deformations of volumetric nature caused by a pore pressure increase are generally neglected, but in less porous materials, these deformations can be computed as (Zienkiewicz and Bettes, 1982):
where K_s is the average bulk modulus of the solid grains forming the skeleton.

$$d\overline{\varepsilon_{ij}} = \delta_{ij} dP / 3K_s \qquad (2)$$

Having postulated the virtually negligible effect of the pressure P on the total strain, ε_{ij}, it is possible to imply that most of deformations are due to the effective stresses or other extraneous causes such as for instance temperature. Thus, it is possible to write:

$$d\varepsilon_{ij} = d\varepsilon^{\sigma}_{ij} + d\overline{\varepsilon}_{ij} + d\varepsilon^{o}_{ij} \qquad (3)$$

where: $d\varepsilon^{\sigma}_{ij}$ = strains due to stresses.
$d\varepsilon^{o}_{ij}$ = strains due to temperature, creep, etc, (autogeneous strains).
The rate-independent constitutive law relates $d\sigma_{ij}$ to $d\varepsilon^{\sigma}_{ij}$ by, (Zienkiewicz and Bettess, 1982):

$$d\sigma'_{ij} = D_{ijkl} \; d\varepsilon_{kl} \qquad (4)$$

where D_{ijkl} describes the components of the elasticity tensor.

The stress - strain equations for a porous medium are given by Biot (1941) as:

$$\varepsilon_{ij} = \frac{1-\upsilon}{E}\sigma_{ij} - \delta_{ij}\left(\frac{\upsilon}{E}\bar{\sigma} + \frac{P}{3H_1}\right) \quad (5a)$$

$$\theta = \frac{1}{3H_1}\bar{\sigma} + \frac{P}{R_1} \quad (5b)$$

where: ε_{ij} = the components of the strain tensor,
σ_{ij} = the components of the stress tensor,
ν = Poisson's ratio of the soil skeleton,
E = Young's modulus of the soil skeleton,
δ_{ij} = Kronecker's delta,
σ' = effective major principal stress,
= $\sigma_{xx} + \sigma_{yy} + \sigma_{zz}$
P = pore water pressure,

$\frac{1}{H_1}$ = a coefficient which is a measure of the compressibility of the soil for a change in pore water pressure,

θ = a measure of the amount of fluid that has flowed in or out of the soil sample, and

$\frac{1}{R_1}$ = a coefficient which measures the change in water content for a given change in water pressure.

The above equations were derived assuming that the soil is an elastic material. On the other hand, the plastic behaviour of soils can be accounted for using a more general constitutive relation such as:

$$\sigma'_{ij} = D_{ijkl}(\varepsilon_{kl} - \varepsilon^p_{kl}) \quad (6)$$

where ε^p_{ij} is the permanent plastic strain tensor.

The permanent plastic strain is related to the residual pore pressure by (Zienkiewicz et al., 1982):

$$P = -\beta\,\varepsilon^o_v \quad (7)$$

where:

$$\beta = \frac{1}{\frac{1}{K_T} + \frac{n}{K_f}}$$

K_T = tangent bulk modulus of the skeleton,
K_f = bulk modulus of water, and
ε^o_v = volumetric plastic strain = ε^p_{ii}.

DYNAMIC RELATIONS FOR POROUS MEDIA

Biot (1956) developed the theory of propagation of elastic waves in a fluid-saturated porous medium in the absence and presence of dissipation forces, i.e., without or with viscosity, respectively.

The Lagrangian co-ordinates were selected, and Biot started his analysis by considering the kinetic energy, T, of a unit volume of solid-fluid mixture in terms of the average displacement of the solid, u_i and the average displacement of the fluid, U_i:

$$2T = \rho_{11}\dot{u}_i\dot{u}_i + \rho_{12}\dot{u}_i\dot{U}_i + \rho_{13}\dot{U}_i\dot{U}_i \qquad (8)$$

where ρ_{11}, ρ_{12}, ρ_{13} are mass coefficients that take into account the relative fluid flow through the pores which is considered variable and $(\dot{\ })$ represents differentiation with respect to time.
Biot (1962b) reformulated his theory of propagation of stress waves in a porous medium and rewrote the dynamic equations of equilibrium in an alternative form. This form is expressed in terms of total stresses and pore water pressures and uses actual mass densities.
The equations consist of the equation of motion of the bulk soil, of the mixture of fluid and solid and the equation of motion of the liquid relative to the solid, respectively as follows:

$$\sigma_{ij,j} + \rho_t g_i = \rho_t \ddot{u}_i + \rho_w \ddot{w}_i \qquad (9a)$$

$$-P_{,i} + \rho_w g_i = \rho_t \ddot{u}_i + \frac{\beta}{n}\rho_w \ddot{w}_i + \frac{\rho_w g}{k}\dot{w}_i \qquad (9b)$$

where: σ_{ij} = the total stress tensor,
P = pressure acting on the fluid,
u_i = displacement vector of the solid skeleton,
w_i = displacement vector of the fluid with respect to solid,
= $n\,(U_i-u_i)$, n is the effective porosity and nU_i is the volume of the liquid displaced through a unit area perpendicular to the i-direction,
ρ_t = bulk mass density of the liquid-solid mixture,
= $(1-n)\,\rho_s + n\rho_w$
ρ_s = intrinsic mass density of the solid,
ρ_w = intrinsic mass density of the fluid,
g_i = the component of the gravitational constant in the i-direction,
k = Darcy's coefficient of permeability,
β = kinetic energy correction factor, and
$(\ddot{\ })$ = second derivative of ( ) with respect to time.
Kim and Blouin (1984) generalized Biot's approach for the high frequency range by the inclusion of a mass increment factor (r). The values of r range from 1/5 to 1/3 depending on the shape of pores. Thus, the final equations of equilibrium of the fluid will be:

$$-P_{,i} + \rho_w g_i = \rho_t \ddot{u}_i + \frac{\rho_w}{n}\ddot{w}_i(\beta + r) + \frac{\rho_w g}{K}\dot{w}_i \qquad (10)$$

CONTINUITY EQUATION

Zienkiewicz and Bettess (1982) assumed that both the solid grains and water to be incompressible. Mei and Foda (1982) reasoned that the volume changes are due to the existence of air pockets entrapped in the pores. Their equation for a two-phase material in the Eulerian space is given as:

$$n\dot{U}_{i,i} + (1-n)\dot{u}_{i,i} = -\frac{n}{\beta}\dot{P} \qquad (11)$$

where: β = the effective bulk modulus of elasticity.
Zienkiewicz (1981) presented another form of the equation in Lagrangian description as follows:

$$\dot{\varepsilon}_{ii} + \dot{w}_{i,i} + \frac{n}{K_f}\dot{P} = 0 \qquad (12)$$

On the other hand, Prevost et al. (1986) argued that in some soil dynamic applications, the fluid phase could be regarded as incompressible, thus:

$$\dot{\varepsilon}_{ii} + \dot{w}_{i,i} = 0 \qquad (13)$$

Zienkiewicz and Bettes (1982) derived a general form of the continuity equation considering the solid grains and fluids to be compressible, thus:

$$\dot{u}_{i,i} + \dot{w}_{i,i} + \frac{1}{K_b}\dot{P} + \frac{\dot{u}_i}{K_b}P_{,i} + \frac{\dot{w}_i}{K_w}P_{,i} = 0 \qquad (14)$$

where: K_b = the bulk modulus of the soil-water sample.

$$\frac{1}{K_b} = \frac{n}{K_w} + \frac{1-n}{K_s} \qquad (15)$$

K_s = the bulk modulus of the solids.
K_w = the bulk modulus of water.

BOUNDARY CONDITIONS

Two types of boundaries exist; the first, **S**, surrounds the bulk, and the other, $\bar{S}$, surrounds the fluid. At each boundary, the stresses at any time, t, are taken at a portion and the displacement is known at the remaining portion. This can be written mathematically as follows:

$$u_i(x,t) = \hat{u}_i(x,t) \qquad \text{on } S_1 \qquad (16)$$

$$\sigma_{ij}(x,t)n_j(x) = \hat{T}_i(x,t) \qquad \text{on } S_2 \qquad (17)$$

$$w_i(x,t) = \hat{w}_i(x,t) \qquad \text{on } \bar{S}_1 \qquad (18)$$

$$P(x,t)n_i(x) = \hat{P}(x,t) \qquad \text{on } \bar{S}_2 \qquad (19)$$

where n_i is a unit normal vector and the superscript (^) represents a known function.

SOME APPROXIMATIONS TO DYNAMIC EQUATIONS

The use of the full solution of equations (1) to (15) in terms of the u/w variables represents six variables in three dimensions (or four in two dimensions). This formulation is called (u-w) formulation, (Simon et al., 1986).

It is often preferable to reduce the problem by retaining u_i and P as the basic variables. The elimination of the variable w_i is simple if the $\ddot{w}_i$ terms are dropped from the equations on the assumption that the ratio:

$$\frac{\ddot{w}_i}{\ddot{u}_i} \rightarrow 0$$

This formulation is called (u-p) formulation, (Zienkiewicz and Bettess, 1982).

In this work, a more general formulation will be used which is called (u-w-π) model where π is the nodal pore fluid pressure in the finite element discretization. In this model, both the solid grains and fluids are assumed to be compressible. This model also takes into account the fluid inertia effects.

It is rational to think that this model best represents the behaviour of granular materials under dynamic loading and especially blast loading due to the large voids of such materials. This is attributed to the shape, size and arrangement of particles, which allow easy movement of the pore fluid, and this, in turn, increases the fluid inertia.

This formulation can be verified when high frequency loads are applied, (Zienkiewicz and Bettess, 1982).

STAGGERED SOLUTION OF COUPLED PROBLEMS

The staggered solution procedure may be described conceptually as a partitioned solution that can be organized in terms of sequential execution of single-field analyzers. The staggered qualifier arises from the "zigzagging" appearance of the "solution state walkthrough" in the temporal flow diagram representation of the computation process, as illustrated in **Fig. (2),** (Felippa and Park, 1980).

This approach offers two potentially important advantages:

a- program modularity enhancement, and

b- computational efficiency.

The first advantage occurs from the fact that relatively few modifications of the single-field analyzers are required in comparison to the field elimination and simultaneous solution approaches.

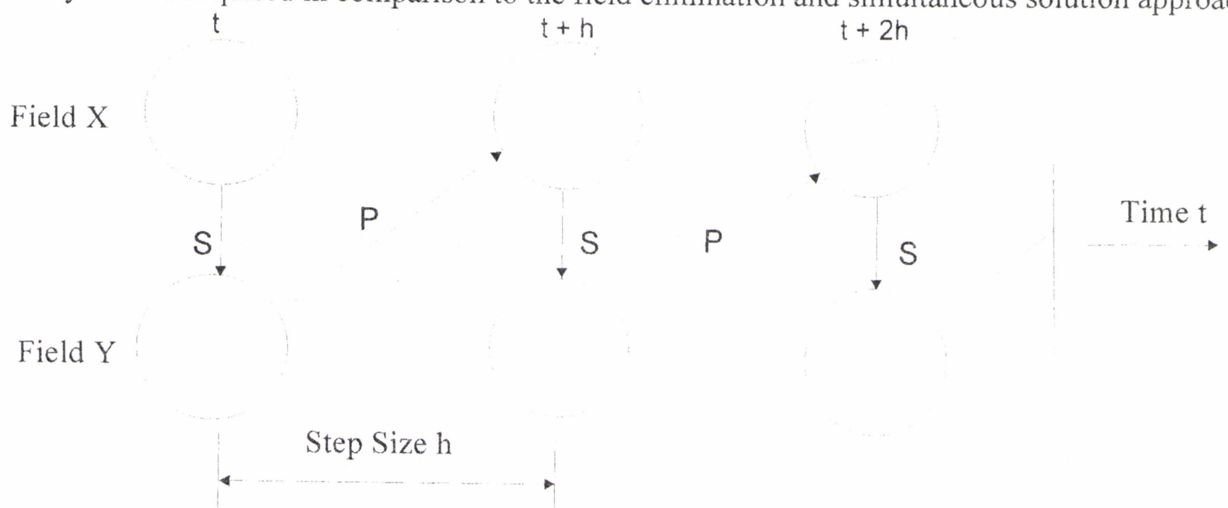

Fig. (2) - Temporal flow diagram illustrating the staggered solution approach for a two - field problem (P = predictor phase, S = substitution phase), (after, Felippa and Park, 1980).

As regards computational efficiency, the staggered solution time step length is roughly the same as that incurred in processing the component fields as separate entities. This is because that the overhead introduced by the flow of information among analyzers becomes comparatively insignificant in large-scale problems. It follows that this approach is attractive and time step size restrictions can be excluded from consideration, (Felippa and Park, 1980).

Zienkiewicz et al. (1980) found that this approach permits decoupling of high frequency and low frequency components of a single system, so that an alternative time marching algorithm can be used in each part.

For the problem under consideration, the discretized form of the equations of motion, which are a system of ordinary differential equations with constant coefficients, can be integrated using the finite difference method to approximate the accelerations and velocities in terms of displacements. Newmark (1959) integration scheme is used herein. This scheme is a forward integration method and requires that the field equations be written at time t+Δt. The accelerations and velocities are given as, (Bathe, 1996):

$$\ddot{u}_{t+\Delta t} = \frac{1}{\alpha}\left[\frac{1}{\Delta t^2}(u_{t+\Delta t} - u_t) - \frac{1}{\Delta t}\dot{u}_t - (\frac{1}{2} - \alpha)\ddot{u}_t\right] \quad (20)$$

and

$$\dot{u}_{t+\Delta t} = (1 - \frac{\delta}{\alpha})\dot{u}_t + (1 - \frac{\delta}{2\alpha})\Delta t\,\ddot{u}_t + \frac{\delta}{\alpha\ \Delta t}(u_{t+\Delta t} - u_t) \quad (21)$$

where α and δ are parameters controlling integration accuracy and stability. The scheme is unconditionally stable if:

$$\delta \geq 0.5 \quad , \alpha' \geq 0.25(\delta + 0.5) \tag{22}$$

On the other hand, the time derivative of the pore pressure is approximated as follows:

$$\dot{P}_{t+\Delta t} = \frac{2}{\Delta t}(P_{t+\Delta t} - P_t) - \dot{P}_t \tag{23}$$

Once the finite difference expressions for the velocity and acceleration are substituted in the discretized equation, the finite element equations become:

$$\underline{\mathbf{K}}^* \underline{\mathbf{U}}_{t+\Delta t} = \underline{\mathbf{F}}^*_{t+\Delta t} \tag{24}$$

where $\underline{K}^*$ is the effective stiffness matrix and $\underline{F}^*$ is the effective force vector.

Certain "nonsymmetries" exist in the equation system, and the primary variables u, w and P have a different structure (and indeed different physical units). Therefore, standard time step algorithms are not easy to apply. It is best to adopt a "staggered" solution process, (Zienkiewicz et al., 1982).

The following steps are followed to obtain a solution of the dynamic problem:

1- Form the mass matrix, $\underline{M}$, the damping matrix, $\underline{C}$ and the stiffness matrix $\underline{K}$.
2- Set time = 0, and select time step, Δt.
3- 3. Form the effective stiffness matrix, $\underline{K}^*$, and the effective force vector, $\underline{F}^*$.
4- 4. Determine the displacement changes, Δu at time t+Δt, using some "extrapolated" value of P, or conversely.
5- 5. Determine $\underline{U}$, $\dot{\underline{U}}$ and $\ddot{\underline{U}}$ at t+Δt.
6- 6. Use the available values of $\underline{U}$ to determine P at time t+Δt.
7- 7. Update the effective force vector, $\underline{F}^*$, and go to step 4 or step 3 if the effective stiffness matrix need to be updated.

THE CAP PLASTICITY MODEL

The cap model is an incremental work-hardening plasticity theory for materials having time and temperature independent properties and undergoing permanent as well as recoverable strain at each loading increment. The loading function for the model is assumed to consist of two parts, (see **Fig.(3):**

1- An ultimate failure envelope which limits the shear stresses in the material, denoted by:

$$f_1 = h(I_1, \sqrt{J_2}) \tag{25}$$

2. A strain hardening surface or "cap" denoted by:

$$f_2 = H(I_1, \sqrt{J_2}, K) \tag{26}$$

where: I_1 = the first invariant of stress tensor.
J_2 = the second invariant of deviatoric stress tensor.
K = a hardening parameter, which is a function of plastic volumetric strain, ε^p_{ii}.

The cap changes as the plastic deformation occurs. As shown in **Fig. (3),** the associated flow rule requires that during cap action, the plastic strain rate vector be directed upward and to the right (if isotropic stress is applied). This implies that the plastic strain rate produces an irreversible decrease in volume in conjunction with the irreversible shear strain, (Sandler and Rubin, 1987).

The equation of the failure envelope was selected to be Chen and Baladi's (1985):

$$f_1 = \sqrt{J_2} - [A - C.\exp(-B.I_1)] \tag{27}$$

$$H = \sqrt{J_2} - \frac{1}{R}\left[\{x(k_1) - L(k_1)\}^2 - \{I_1 - L(k_1)\}^2\right]^{1/2} \tag{28}$$

and the hardening surface:

and the following hardening function is assumed, (Dimaggio and Sandler, 1971):

$$\varepsilon_{KK}^{P} = W\left[1-\exp(-Dx)\right] \tag{29}$$

where A, B, C, D, R and W are constants. W is the maximum plastic volumetric strain.

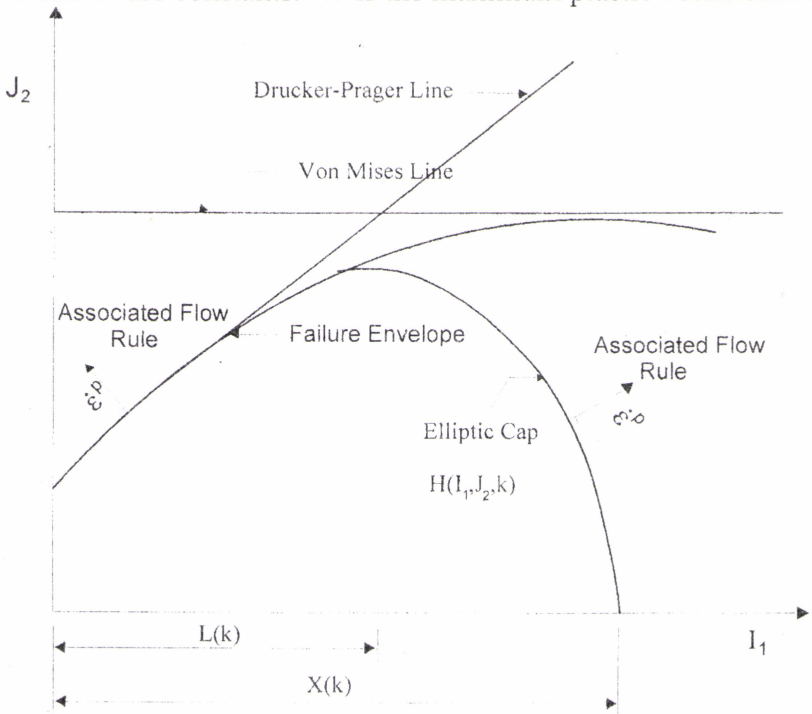

Fig. (3) - Yield surfaces for the selected cap model.

The plastic loading criteria for the function f_2 is given as, (Chen and Baladi, 1985):

$$df = \frac{\partial f}{\partial \tau_{ij}} d\tau_{ij} \begin{cases} > 0 & \textit{Loading} \\ = 0 & \textit{Neutral Loading} \\ < 0 & \textit{Unloading} \end{cases} \tag{30}$$

The plastic strain increment tensor is given by Drucker (1950):

$$d\varepsilon_{ij}^{p} = \begin{cases} d\lambda \dfrac{\partial f}{\partial \tau_{ij}} & \text{if } f = 0 \text{ and } df > 0 \\ 0 & \text{if } f < 0 \text{ or } f = 0 \text{ and } df \le 0 \end{cases} \tag{31}$$

where dλ is a positive factor of proportionality.
The plastic strain can be divided into two components, namely the volumetric, ε^{P}_{kk}, and deviatoric, e^{P}_{ij} , then:

$$d\varepsilon_{ij}^{p} = \frac{1}{3} d\varepsilon_{kk}^{p}\delta_{ij} + de_{ij}^{p}$$

$$d\varepsilon_{ij}^{p} = \frac{1}{3}\left[3d\lambda \frac{\partial f_2}{\partial I_1}\delta_{ij} + d\lambda \frac{\partial f}{\partial S_{ij}}\right]$$

$$d\varepsilon_{ij}^{p} = d\lambda \left[\frac{\partial f}{\partial I_1}\delta_{ij} + \frac{1}{2\sqrt{J_2}}\frac{\partial f}{\partial \sqrt{J_2}} S_{ij}\right] \tag{32}$$

where S_{ij} is the stress tensor.

A full description of the model is found in the text by Chen and Baladi (1985). The cap model parameters are given in **Table (1).**

Table (1) – Cap Model Parameters (from Chen and Baladi, 1985).

Parameter	Value
A	15.0 MPa
B	0.0025 $(MPa)^{-1}$
C	14.9 MPa
D	0.50 $(MPa)^{-1}$
W	0.02

THE COMPUTER PROGRAM

The program (MBLAST) was developed which is an extensive modification of the program (BLAST) developed by Awad (1990) at Colorado State University. The program is modified to take into account different types of loading such as blast, impact and earthquake loading. It is arranged into a modular form in order to minimize the run time.

Since the (u-w-π) formulation adopted in this research results in an unsymmetric effective stiffness matrix, the subroutines ACTCOL and UACTCL (Zienkiewicz, 1977) are used instead of subroutine LDUSKY of the program (BLAST). These subroutines are utilized for symmetric and unsymmetric equations solving, respectively.

DESCRIPTION OF THE PROBLEM - STEP LOADING

Fig (4) shows a soil column subjected to an instantly applied surface load of 1.0 kN/m^2. The boundary conditions are shown in the same figure. The solid skeleton and pore fluid pressure are each modelled with ten 4-noded isoparametric elements. At the vertical boundary of the solid skeleton, only vertical movement is permitted. Pressures at the free surface are taken as zero, (Paul ,1982). The material properties are shown in **Table (2).** The water level is assumed to be at the ground surface.

Table (2) - Soil parameters for the problem, (from Zienkiewicz et al., 1988).

Property	Value
Modulus of elasticity (E)	30 MN/m^2
Poisson's ratio (ν)	0.2
Bulk density (ρ_t)	2000 kg/m^3
Fluid bulk modulus (K_f)	0.10×10^6 N/m^2
Solid bulk modulus (K_s)	0.10×10^{12} N/m^2
Fluid density (ρ_f)	1000 kg/m^3
Porosity (n)	30 %
Permeability (k)	0.001 m/se.
Newmark's Integration Constants:	
α	0.50
δ	0.25

Figs. (5) and (6) show the undamped response (displacement and pore pressure) obtained by the program (MBLAST) for two time steps; Δt = 0.025 and 0.05 sec.

The results of **Figs. (5) and (6)** compare well with those obtained by Zienkiewicz et al. (1988) who used the SSPJ method (single step p-order polynomial, j =1 or 2, the order of the equation, Zienkiewicz et al., 1984) for time integration for the solution of the same problem. The results are accepted in spite of oscillation in displacements and pore pressures.

It is observed that the undamped response of displacement and pore pressure oscillates significantly with the increase of time step length. When large time step lengths are used to obtain the undamped response; Δt = 0.05 sec., the higher frequencies introduce errors and they are the cause of oscillations. Zienkiewicz et al. (1988) reported that improved results are obtained when iterations and numerical damping are introduced or when very small time steps are used to introduce a smooth start.

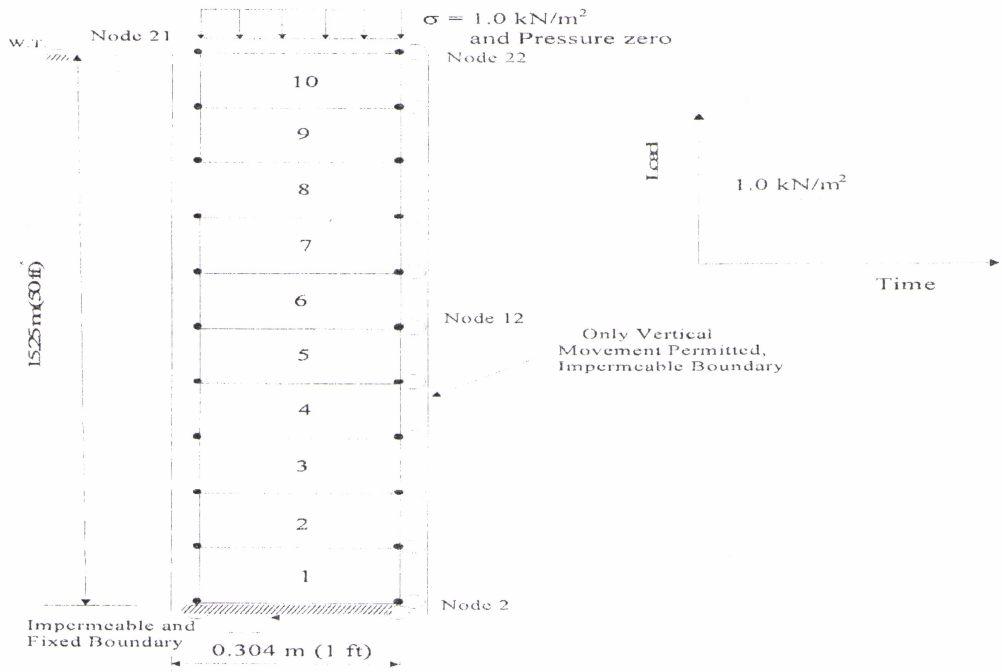

Fig. (4) - Saturated soil column subjected to step loading problem (one- dimensional problem).

CONCLUSIONS

1. A general mixed finite element formulation (u-w-π) is presented in this paper .This formulation included the fluid inertia effects and the soil skeleton is considered compressible. The cap plasticity model is used as a constitutive relation. A computer program is developed by the authors which is capable of analysing coupled dynamic problems including different types of dynamic loads.
2. A problem of soil column subjected to an instantly applied surface load is solved. The solid skeleton and pore fluid pressure are each modelled with ten 4-noded isoparametric elements. The results are compared with those obtained by Zienkiewicz et al. (1988). It is observed that the undamped response of displacement and pore pressure oscillates significantly with the increase of time step length. When large time step lengths are used

to obtain the undamped response; Δt = 0.05 sec., the higher frequencies introduce errors and are the cause of oscillations.

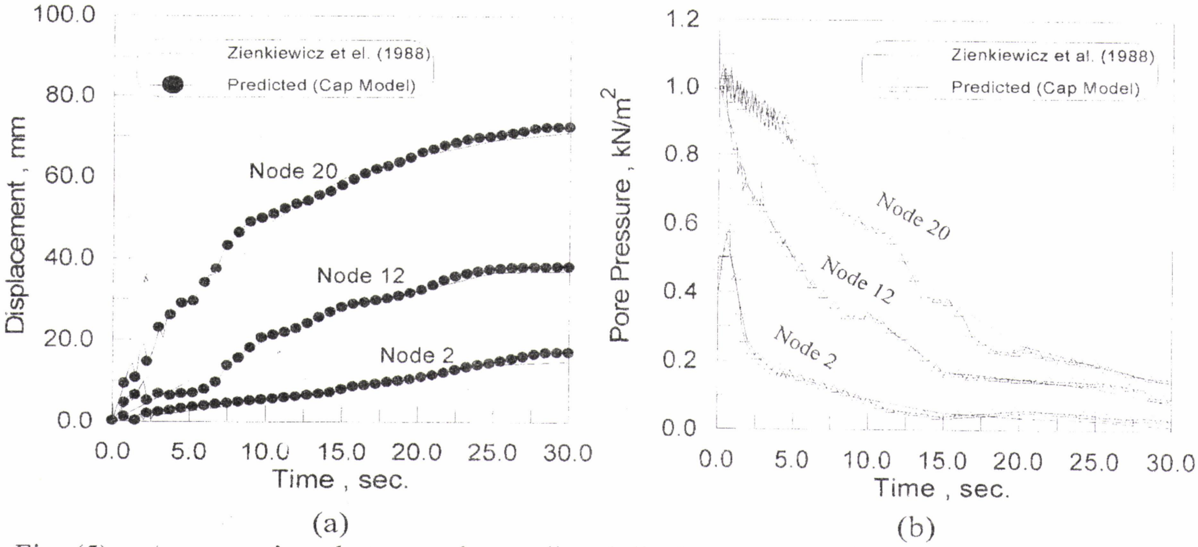

Fig. (5) - A comparison between the predicted displacements and pore pressures at three nodes in the layer with the results of Zienkiewicz et al. (1988) , time step = 0.025 sec.

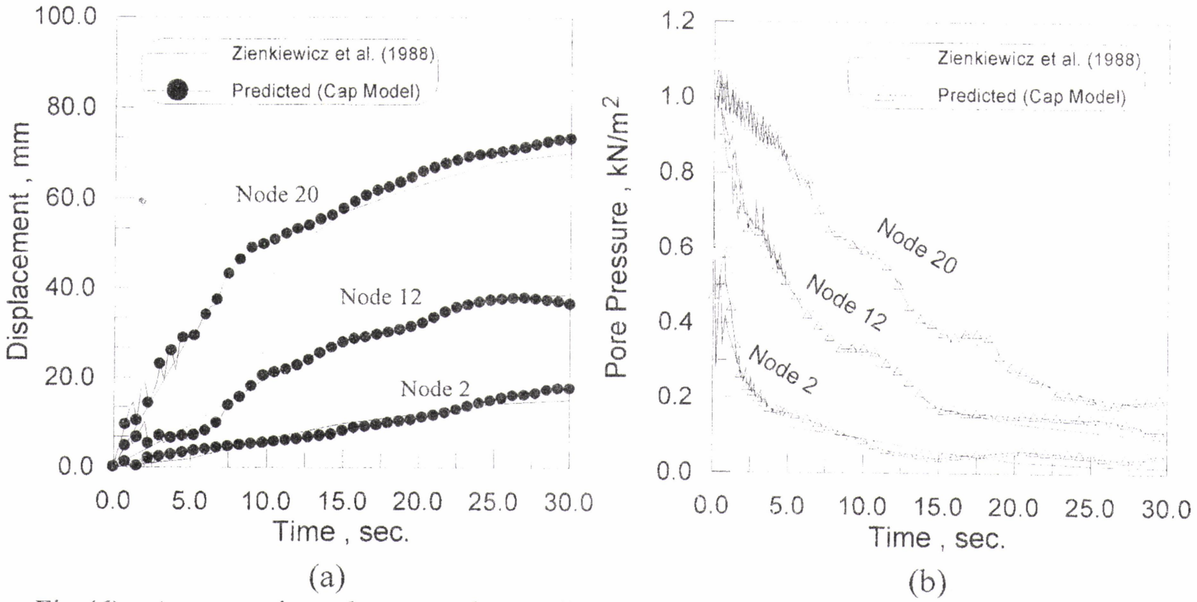

Fig (6) - A comparison between the predicted displacements and pore pessures at three nodes in the layer with the results obtained by Zienkiewicz et al., (1988), time step = 0.05 sec.

REFERENCES

Awad, A. A. A., (1990), A Numerical Model for Blast-Induced Liquefaction Using Displacements-Pore Pressures Formulations, Ph.D. dissertation, Colorado State University.

Bathe, K.J., (1996), Finite Element Procedures, Prentice-Hall.

Bazant, Z. P. and Krizek, R. J., (1975), Saturated Sand as an Inelastic Two-Phase Media, Journal of Engineering Mechanics, ASCE, Vol. 101, EM4, p. p. 317 - 332.

Biot, M. A., (1941), General Theory of Three-Dimensional Consolidation, Journal of Applied Physics, Vol. 12, No. 2, p. p. 155 - 164.

Biot, M. A., (1956), Theory of Propagation of Elastic Waves in a Fluid Saturated Porous Solid), Journal of the Acoustical Society of America, Vol. 28, No. 2, p. p. 168 – 191

Biot, M. A., (1962a), Mechanics of Deformation on Acoustic Propagation in Porous Media, Journal of the Acoustical Society of America, Vol. 34, No. 4, p. p. 1482 - 1498.

Biot, M. A., (1962b), Generalized Theory of Acoustic Propagation in Porous Dissipative Media, Journal of the Acoustical Society of America, Vol. 34, No. 9, p. p. 1254 - 1264.

Chen, W. F. and Baladi, G. Y., (1985), Soil Plasticity-Theory and Implementation, Vol. 38 in Developments in Geotechnical Engineering, Elsevier, Amsterdam.

Daghigh, Y., (1993), Numerical Simulation of Dynamic Behaviour of an Earth Dam During Seismic Loading, Ph.D dissertation, Delft University of Technology.

Dimaggio, F. L. and Sandler, I. S., (1971), Material Model for Granular Soils, Journal of Engineering Mechanics, ASCE, Vol. 97, EM3, p. p. 935 - 950.

Drucker, D. C., (1950), Stress - Strain Relations in the Plastic Range-A Survey of Theory and Experiment, Office of Naval Research, Report No. NR/041/032.

Felippa, C. A. and Park, K. C., (1980), Staggered Transient Analysis Procedures for Coupled Mechanical Systems: Formulation, Journal of Computer Methods in Applied Mechanics and Engineering, Vol. 24, p. p. 61 - 111.

Kim, K. J. and Blouin, S. E., (1984), Response of Saturated Porous Non-linear Materials to Dynamic Loadings, Applied Research Associates Inc., Report Research for Air Force Office of Scientific Research.

Martin, G. R., Finn, W. D. L. and Seed, H. B., (1975), Fundamentals of Liquefaction Under Cyclic Loading, Journal of Geotechnical Engineering Division, ASCE, Vol. 101, GT5, p. p. 423 - 438.

Mei, C. C. and Foda, M. A., (1982), Boundary Layer Theory of Waves in a Poro-Elastic Sea Bed, Chapter 2 in, Soil Mechanics-Transient and Cyclic Loads, edited by G. N. Pande and O. C. Zienkiewicz, p. p. 17 - 35.

Newmark, N. M., (1959), A Method of Computation for Structural Dynamics, Journal of Engineering Mechanics Division, ASCE, Vol. 85, EM3, p. p. 67 - 94.

Paul, D. K., (1982), Effective Dynamic Solutions for Single and Coupled Multiple Field Problems, Ph.D. dissertation, University of Wales.

Prevost, J. H., (1987), Dynamics of Porous Media, Chapter 3 in Geotechnical Modelling and Applications, edited by S. M. Sayed, p. p. 76 - 146, Gulf Publishing Company.

Prevost, J. H., Ferrito, J. M. and Slyh, R. J., (1986), Evaluation and Validation of the Princeton University Effective Stress Model, Naval Civil Engineering Laboratory, Report No. TR 919.

Sandler, I. S. and Rubin, D., (1987), Cap and Critical State Models-Short Course Notes, Second International Conference and Short Course on Constitutive Laws for Engineering Materials, Arizona, p. p. 1 - 29.

Seed, H. B., (1979), Soil Liquefaction and Cyclic Mobility Evaluation for Level Ground During Earthquakes, Journal of Geotechnical Engineering Division, ASCE, Vol. 105, GT2, p. p. 201 - 255.

Simon, B. R., Wu, J. S. S., Zienkiewicz, O. C. and Paul, D. K., (1986), Evaluation of u-w and u-π Finite Element Methods for the Dynamic Response of Saturated Porous Media Using One-Dimensional Models, International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 10, p. p. 461 - 482.

Zienkiewicz, O. C., (1977), The Finite Element Method, McGraw-Hill Book Company.

Zienkiewicz, O. C., (1981), Basic Formulation of Static and Dynamic Behaviour of Soil and Other Porous Media, in Numerical Methods in Geomechanics, Proceedings of the NATO Advanced Study Institute, University of Minho, Braga, Portugal.

Zienkiewicz, O. C., Hinton, E., Leung, K. H. and Taylor, R. L., (1980), Staggered Time Marching Schemes in Dynamic Soil Analysis and a Selective Explicit Extrapolation Algorithm, Proceedings of the Second International Symposium on Innovative Numerical Analysis in Applied Engineering Sciences, Montreal, p. p. 525 - 530.

Zienkiewicz, O. C. and Bettess, P., (1982), Soils and Other Saturated Media Under Transient Dynamic Conditions; General Formulation and the Validity of Various Simplifying Assumptions, Chapter 1 in Soil Mechanics-Transient and Cyclic Loads, edited by G. N. Pande and O. C. Zienkiewicz, p. p. 1 - 16.

Zienkiewicz, O. C., Leung, K. H., Hinton, E. and Chang, C. T., (1982), Liquefaction and Permanent Deformation Under Dynamic Conditions -Numerical Solution and Constitutive Relations, Chapter 5 in Soil Mechanics-Transient and Cyclic Loads, edited by G. N. Pande and O. C. Zienkiewicz, p. p. 71 - 103.

Zienkiewicz, O. C., Wood, W. L., Hine, N. W. and Taylor, R. L., (1984), A Unified Set of Single Step Algorithms-Part 1: General Formulation And Applications, International Journal for Numerical Methods in Engineering, Vol. 20, p. p. 1529 - 1552.

Zienkiewicz, O. C., Paul, D. K. and Chan, A. H. C., (1988), Unconditionally Stable Staggered Solution Procedure for Soil-Pore Fluid Interacrtion Problems, International Journal for Numerical Methods in Engineering, Vol. 26, p. p. 1039 - 1055.