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# Analysis Approximate with Using Sumudu Adomian Decomposition Method for Solving SEIVR Epidemic Model

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## ABSTRACT

Whenever a mathematical model is suggested to classing COVID-19. The population must be divided into several groups and our model has five groups namely S(t), E(t), I(t), V(t) and R(t) which represent susceptible, exposed, inflected, vaccinated and recovered individuals respectively. This model is a continuous dynamical system where the derivative is in fractional form. An analysis solution evaluates the positivity of the functions S(t), E(t), I(t), V(t) and R(t) as solutions of the system. The uniformity of the solutions of the system under consideration is also proved. And finding the equilibrium points. To get acceptable results one needs the solutions. Some of the solutions are points called equilibrium points and the other are functions. and studying their stability fractional differential system orders are checked locally and globally. The basic reproduction number is used to prove the stability of all equilibrium points as well as the method of the nature of the eigenvalues of the Jacobian at each equilibrium point. And then studied the local bifurcation to the asymptotically stable and stable equilibrium points. And evaluated approximately. These solutions must satisfy the nature of the problem under consideration for example under certain conditions some of the equilibrium points are stable. Also, the approximate solution must give results close to the real situation. All these demands are shown in this paper. Approximate and Numerical simulation is given through a tables and graphs which shows the efficiency of the method, using the MATLAP to all the figures.

**Keywords:** Bifurcation, Boundedness, Fractional calculus, Stability, Uniformly bounded.

## 1. INTRODUCTION

The World Health Organization declared COVID-19 a pandemic in March 2020. This disease spread very fast in more countries around the world. This leads the governments to make limitations on travelling inside and outside them. The WHO also issued a series of preliminary regulatory determinations for healthcare services against the emerging disease

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and called on all nations to cooperate in its control. The disease then spread to the rest of the world, causing health crises in one region after another. Despite public vaccination in a limited number of countries, programs such as social isolation, physical distancing, and wearing masks are employed as the main control strategies to reduce the exponential growth of COVID-19. Mathematicians together with others submit a large number of models such as SEIR, SEIVR, and SEQAIJRE. A modeling SEQIR for Italy was submitted **(Zahraa and Al_Azzawi, 2022)**. In our country a good number of MSc. and PhD. As **(Naji and Hasan, 2013; Zahraa and Al_Azzawi, 2022)**, the thesis are presented an analytical study of the patterns of the dynamic systems of Covid-19 was carried out using equations of different orders, whether regular or fractional, and their approximate solution was used under specific medical conditions, also, a team of scientists looked at analyzing different-order dynamical systems to understand the evolution and spread of COVID-19, for example **(Naji, 2012)**.

Fractional mathematical models are given on $SEI_1I_2R$ and others **(Zahraa and Al_Azzawi, 2022)**. Upon introducing the extended SEIVR model into a limit-state function and defining the model parameters including transmission, recovery, and mortality rates as random variables, the problem was transformed into a reliability model and analyzed by the Monte Carlo sampling **(Assessment, 2005).** The numerical study of a deterministic mathematical model of an SEIVR, this model incorporates a temporary immune recovery class which involves subsequent dose vaccination for the infants, Hypothetical values were chosen for the parameters to test the validity of the mathematical model, the parameter with the greatest impact on the model was computed using the eigenvalue elasticity and sensitivity analyses and it was found that the parameter of the rate at which the vaccine wanes in the infants has the greatest impact on the mathematical model of **(Kapur, 1985)**. In **(Heffernan et al. 2005)** a suitable numerical simulation method was used to solve a non-linear system that contains multi-variables and multi-parameters with absent real data. A susceptible-vaccinated-exposed-infectious-recovered (SEIVR) epidemic model for an infectious disease that spreads in the host population through horizontal transmission was investigated, it was shown that the model exhibits two equilibria, namely, the disease-free equilibrium and the endemic equilibrium, by constructing a suitable Lyapunov function, it was observed that the global asymptotic stability of the disease-free equilibrium depends on $R_o$ as well as on the treatment rate if $R_0 > 1$, then the endemic equilibrium was globally asymptotically stable with the help of the Li and Muldowney geometric approach applied to four-dimensional systems **(Van den Driessche and Watmough, 2002)**. The references of **(Subahtul and Nusantara, 2014; Ghosh et al., 2022; Ghadeer and Mohammed, 2022)** study the stability of criticality of epidemiological systems for COVID-19. Rather than the local study of the equilibrium points by studying their stability **(Van den Driessche and Watmough, 2002)**.

A number of researchers **(Gao et al., 2018; Ge et al., 2020; Mohammed and Hummady, 2023)** studied the stability of the equilibrium points of a number of medical and environmental models in the world. Several researchers investigated the stability of equilibrium points in various medical and environmental models worldwide, as referenced in **(Liu and Yang, 2012; AL-Azzawi et al., 2017; Nadim et al., 2020)**. Several researchers have explored the stability of equilibrium points in numerous medical and environmental models globally, as cited in references **(Smale and Hirsch, 1974; Pašic et al., 2011; Marsden et al., 2022)**. Several researchers **(Kadhim and Hummady, 2024; Ibraheem and Hummady, 2023; Al-Azaiza and Al-Azzawi, 2023)** have studied the stability of equilibrium points in various medical and environmental models worldwide. Several researchers have examined the stability of equilibrium points in a variety of medical and environmental models globally, as evidenced by references **(Naji and Hasan, 2013; Binuyo et al., 2014; Zahraa and Al_Azzawi, 2022)**. Also in **(Hu et al., 2024)** and since one searches for the global behaviour of the system so we solve it by the Sumudu Adomian decomposition method this method is effective and its computations can be done recursively in simple procedure the basic concepts.

## 2. BASIC CONCEPTS

The following definitions, theorems and examples are essential for this paper and are given by mathematicans in their research and also are necessary in the evaluation.

### 2.1. Existence and Uniqueness

**Theorem 1 (The Fundamental Existence-Uniqueness Theorem) (Pašic et al., 2011):** Let E be an open region in $\mathbb{R}$ containing $x_0$ and f satisfies Lipschitz condition. Then there exists an a > 0 such that the initial value problem is:

$$\dot{X} = f(x), x(0) = x_0 \tag{1}$$

has a unique solution x(t) on the interval [-a, a], a > 0.

**Theorem 2 (Pašic et al., 2011):** If $x_0$ is a stable equilibrium point of (1), then no simple eigenvalue of $Df(x_0)$ has positive real part.

**Corollary 3 (Shakyaand Lamichhane, 2016):** A hyperbolic equilibrium point $x^*$ of the system (1) is either unstable or asymptotically stable.

It should be noted that a saddle point is a hyperbolic equilibrium point if $J(x^*)$ has at least one eigenvalue with a positive real part and at least one eigenvalue with a negative real part.

**Definition 1 (Pašic et al., 2011):** Let F = { $f_i : x \to k, i \in I$ , a family of functions indexed by I where X is an arbitrary set and K in $\mathbb{R}$ or $\mathbb{C}$, then F is uniformly bounded if $\exists$ M > 0, s.t. $|f_i(x)| \le M, \forall\ i \in I\ \&\ x \in X$.

### 2.2 Bifurcation

2.2.1 Local bifurcation **(Kadhimand, 2024):**

Consider the system (1), which has a form that may be stated as when it is determined by a single parameter $\mu$.

$$\frac{dX}{dt} = f(X, \mu), \tag{2}$$

where $\frac{dX}{dt} = \left(\frac{dx_1}{dt}, \frac{dx_2}{dt}, \dots, \frac{dx_n}{dt}\right)^T$, $f = (f_1, f_2, \dots, f_n)^T$ and $\mu \in \mathbb{R}$ is a parameter. $f \in C^1(D \times I)$, where $D$ an open region in $\mathbb{R}^n$, $n \in \mathbb{N}$, and $I \subset \mathbb{R}$ be an interval. Also, $Df(X,\mu) = J$ denoted the Jacobian matrix, and $f_\mu(X,\mu)$ denotes the vector of partial derivatives of the components of $f$ with respect to the parameter $\mu$.

As the vector field $f$ crosses a point $\mu = \mu_0$ in the bifurcation set or the parameter fluctuates through a bifurcation value $\mu = \mu_0$, the qualitative structure of System (2) 's solution set changes.

**Definition 2 (Ibraheem and Hummady, 2023):** Bifurcation is a qualitative (topological) modification in the nature of the System solution that occurs when a system parameter is changed.

**Theorem 4 (Sotomayor theorem) (Ibraheem and Hummady, 2023):** Suppose that the System (2) satisfies that

1. $f(x^*, \mu_0) = 0$, 2. $J = Df(x^*, \mu_0)$ has a simple eigenvalue $\lambda = 0$ with eigenvector $V$. 3. $J^T$ has an eigenvector $\Psi$ corresponding to the eigenvalue $\lambda = 0$.

Assume that $J$ contains $k$ eigenvalues with negative real parts and $(n - k - 1)$ eigenvalues with positive real parts, and that the following requirements are met:

$$\left.\begin{array}{l} \Psi^T f_\mu(x^*, \mu_0) \neq 0 \\ \Psi^T [D^2 f(x^*, \mu_0)(V, V)] \neq 0 \end{array}\right\}. \tag{3}$$

When the parameter $\mu$ crosses over the bifurcation value $\mu_0$, the system (2) experiences a saddle-node bifurcation at the equilibrium point $x^*$. However, if the following conditions are met,

$$\left.\begin{array}{c} \Psi^T f_\mu(x^*, \mu_0) = 0 \,, \Psi^T \left[D f_\mu(x^*, \mu_0) V\right] \neq 0 \\ \Psi^T [D^2 f(x^*, \mu_0)(V, V)] \neq 0 \end{array}\right\}. \tag{4}$$

Then the system (2) experiences a Transcortical bifurcation at the equilibrium point $x^*$ when the parameter $\mu$ crosses over the bifurcation value $\mu_0$. Finally, if the third requirement in (2) is not met and the following conditions are met instead,

$$\left.\begin{array}{c} \Psi^T f_\mu(x^*, \mu_0) = 0 \,, \Psi^T \left[D f_\mu(x^*, \mu_0) V\right] \neq 0 \\ \Psi^T [D^2 f(x^*, \mu_0)(V, V)] = 0 \\ \Psi^T [D^3 f(x^*, \mu_0)(V, V, V)] \neq 0 \end{array}\right\}. \tag{5}$$

Then the system (2) experiences a pitchfork bifurcation at the equilibrium point $x^*$ when the parameter $\mu$ crosses over the bifurcation value $\mu_0$.

**Definition 3 (Heffernan et al., 2005):** An epidemic is an unusually large short-term outbreak of a disease, such as Cholera and Aids... etc. The spread of diseases depends on many factors such as:

- The mode of transmission.
- The type of diseases.
- Susceptibility.
- Infections period.
- Resistance. and any other factors.

**Definition 4 (Van den Driessche and Watmough, 2002):** A mathematical model that describes the contagious diseases which spread in the population is known as the Epidemiological model.

**Example 1:** SI model **(Shakya and Lamichhane, 2016)** The SI model assumes that, for a given population, there are no recovered and everyone is either susceptible to the disease or else infected
with the disease. Thus from equation (2.1) we obtain S + I = N.
Accordingly, the general form for a mathematical model that describes the dynamics of SI epidemic model has the following structure:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI \qquad (6)$$

Where β is the infection rate.

### 2.3 Basic Reproduction Number

The basic reproduction number, commonly denoted by is a measure of the possibility for disease diffusion in a population. In mathematical epidemiology, is a sill for stability of a disease-free equilibrium and is linked to the peak and final size of an epidemic. There are different methods that can be used for the derivation of, but the two main lines of analytical method for derivation of are namely by next-generation method **(Van den and Watmough, 2002; Heffernan et al., 2005)**. Survival method **(Hu et al., 2024)**. A brief description of the next-generation method used is shown below **(James, 2021)**: Suppose that the following system (with nonnegative initial conditions) represents a disease transmission model with compartments of **(Heffernan et al., 2005)**. Preliminary and Basic Concepts which for (m$< n$) the first compartments correspond to the states with infection:

$$\frac{dx_i}{dt} = F_i(x) \qquad (7)$$

where, x(t)∈ $\bar{R}_+^n$(nonnegative $R^n$vector) and $x_i$(t) the number of individuals in its compartment. Assume that $X_S$ be the set of all disease-free states of the system (7) is defined by $X_S$ = {x∈ $\bar{R}_+^n$|$x_i$=0 for 1≤ $i \leq m$}

Then system (7) can be rearranged as

$$\frac{dx_i}{dt} = \mathcal{F}_i(x) - \mathcal{V}_i(t) \qquad (8)$$

**Definition 5 (Allman and Rhodes, 2004):** The basic reproduction number, denoted as $R_0$, is an important threshold quantity which determines whether COVID-19 will continuously spread in the population or disappear. It is defined as the average number of secondary cases produced by one infected individual introduced into a population of susceptible individuals **(James, 2021)**.

**Example 2 (Allman and Rhodes, 2004):** Suppose the following system of nonlinear ordinary differential equations, which constituted the SEIRV model used in the outstudy

$$\frac{dS(t)}{dt} = \Delta - \beta S(t)I(t) - \alpha S(t) - \mu S(t),$$
$$\frac{dE(t)}{dt} = \beta S(t)I(t) - \gamma E(t) + \sigma\beta I(t)V(t) - \mu E(t),$$
$$\frac{dI(t)}{dt} = \gamma E(t) - \delta I(t) - \mu I(t),$$

$$\frac{dR(t)}{dt} = \delta I(t) - \mu R(t),$$
$$\frac{dV(t)}{dt} = \alpha S(t) - \sigma\beta I(t)V(t) - \mu V(t).$$

For the set containing all infected individuals (E(t) and I(t)), we defined

$$X(t) = \begin{bmatrix} E(t) \\ I(t) \end{bmatrix}$$

There is $\frac{dX}{dt} = \begin{bmatrix} \frac{dE}{dt} \\ \frac{dI}{dt} \end{bmatrix} = \begin{bmatrix} -(\gamma+\mu)E(t) + \beta(S(t)+\sigma V(t))I(t) \\ \gamma E(t) - (\delta+\mu)I(t) \end{bmatrix}$

$$= \begin{bmatrix} \beta(S(t)+\sigma V(t))I(t) \\ 0 \end{bmatrix} - \begin{bmatrix} (\gamma+\mu)E(t) \\ -\gamma E(t) + (\delta+\mu)I(t) \end{bmatrix} = F(X) - V(X)$$
$$F'(X) = \left[\frac{\partial F(X)}{\partial X^T}\right] = \begin{bmatrix} 0 & \beta(S(t)+\sigma V(t))I(t) \\ 0 & 0 \end{bmatrix}, V'(X) = \left[\frac{\partial V(X)}{\partial X^T}\right] = \begin{bmatrix} (\gamma+\mu) & 0 \\ -\gamma & (\delta+\mu) \end{bmatrix}$$

The basic reproduction number $R_0$ was defined as the spectral radius of the next-generation matrix $F'(X)V'^{-1}(X)$ as the following, and more detailed calculation was illustrated in Supplementary Note S3.1 **(James, 2021)**

$R_0 = \frac{\gamma\beta(S(t)+\sigma V(t))}{(\gamma+\mu)(\delta+\mu)}$ The reproduction number $R_0$ is used to measure the transmission potential of a disease. Intuitively, we can expect that if $R_0<1$ then the number of new cases of COVID-19 will decrease, and the number of new cases will increase if $R_0>1$.

**Example 3 (Frölich and Vazquez-Alvarez, 2009):** Consider the SI vaccination model
$$\dot{S} = (1-p)\pi - \mu S - (\beta I + \beta_V I_V)S$$
$$\dot{S}_V = P\pi - \mu S_V - (1-r)(\beta I + \beta_V I_V)S_V$$
$$\dot{I} = (\beta I + \beta_V I_V)S - (\mu+\alpha)I$$
$$\dot{I}_V = (1-r)(\beta I + \beta_V I_V)S_V - (\mu+\alpha_V)I_V$$
$$V = \begin{bmatrix} \mu+\alpha & 0 \\ 0 & \mu+\alpha_V \end{bmatrix}$$

## 3. THE MAIN RESULTS

### 3.1 The Modified Mathematical Model

The model consists of five different orders of fractional differential equations where $\hat{S}$, $\hat{E}$, $\hat{I}$, $\hat{V}$, and $\hat{R}$ represent susceptible, exposed, infected, vaccinated and recovered individuals respectively, besides the second iteration of the logistic map.
The logistic map is: $Q_\mu(\hat{S})=\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)$. Whose curve is shown in **Fig. 1**
So that $Q_\mu$: $[0,\tilde{k}] \to [0,\tilde{k}]$, the parameter is must belong to the interval [0,4] from the figure (4.1) we notice that the population increases in $[0,\frac{\tilde{k}}{2}]$ and decreasing in $[\frac{\tilde{k}}{2},\tilde{k}]$ but it is known that the growth rate increases and decreases alternatively in $[0,\tilde{k}]$. The $n-th$ iteration of the logistic map $Q_\mu{}^{[n]}$ satisfies these demands and we consider $Q_\mu{}^{[2]}(\hat{S})$

where $Q_\mu^{[2]}(\hat{S}) = Q_\mu(Q_\mu(\hat{S}))$. This function has two maximum points and are minimum points in $[0, \tilde{k}]$. Therefore the new fractional-order system is:

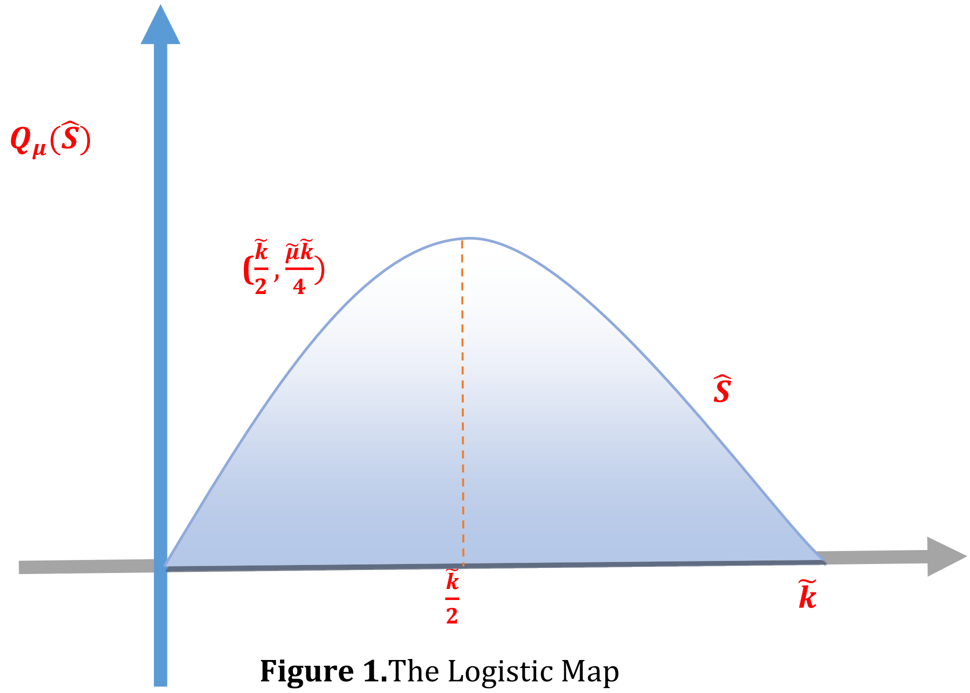


**Figure 1.**The Logistic Map

$$\frac{d^{\alpha_1}\hat{S}}{dt^{\alpha_1}} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right) - \frac{\tilde{\beta}\hat{S}\hat{I}}{\tilde{\Psi}(\hat{I})} + \tilde{v}_4\hat{V} \equiv \hat{g}_1$$

$$\frac{d^{\alpha_2}\hat{E}}{dt^{\alpha_2}} = \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} - (\tilde{v}_1 + \tilde{\mu}_1)\hat{E} \equiv \hat{g}_2$$

$$\frac{d^{\alpha_3}\hat{I}}{dt^{\alpha_3}} = \tilde{v}_1\hat{E} - (\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3)\hat{I} \equiv \hat{g}_3 \qquad (9)$$

$$\frac{d^{\alpha_4}\hat{V}}{dt^{\alpha_4}} = \tilde{\mu}\tilde{\xi} + \tilde{v}_3\hat{I} - (\tilde{\mu}_1 + \tilde{v}_4)\hat{V} \equiv \hat{g}_4$$

$$\frac{d^{\alpha_5}\hat{R}}{dt^{\alpha_5}} = \tilde{v}_2\hat{I} - \tilde{\mu}_1\hat{R} \equiv \hat{g}_5$$

The initial conditions of the above system are: $\hat{S}(0)$ = So > 0, $\hat{E}(0)$ = Eo ≥ 0, $\hat{I}(0)$ = Io ≥ 0, $\hat{V}(0)$ =Vo ≥ 0, $\hat{R}(0)$ = Ro ≥ 0 and $0 < \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_4 < 1$. Here in system (9), the growth rate of the population is denoted by $\tilde{\mu}$. The parameter $\tilde{\beta}$ is the disease contact rate and $\tilde{k}$ represents the cairn capacity of the fraction of individuals to be vaccinated. The natural death rate is denoted by $\tilde{\mu}_1$ and the disease related death rate is shown by $\tilde{v}_0$. The exposed individuals are infected at the rate of $\tilde{v}_1$ and $\tilde{v}_2$ shows the rate of recovery from infection. The inf.ected individuals are vaccinated/treated at the rate of $\tilde{v}_3$. The vaccinated individuals loose their immunity at the rate of $\tilde{v}_4$. The amounts of vaccinated is $\tilde{\xi}$. by assumed the same transmission rate in the form of $\tilde{\beta}\hat{S}\hat{I}$ / ψ ($\hat{I}$), where ψ represents a positive function such that ψ (0) = 1 and ψ ($\hat{I}$) ≥ 0. This generalizes the mass action incidence (i.e. ψ ($\hat{I}$) = 1), and the incidence rate

$\tilde{\beta}\hat{S}\hat{I}/1+\tilde{k}\hat{I}$ . For small Î, the function Î/ ψ (Î) is increasing while decreasing for large Î, that is ψ (Î)=1+cÎ$^2$, cÎ$^2$< 1. This describes the "psychological" effect: for a very large number of infective individuals, the infection force may decrease as the number of infective individuals increases, because in the presence of large number of infective individuals, the population may tend to reduce the number of contacts per unit time.

## 3.2 Positivity and Boundedness

It is known that the solutions S(t), E(t), I(t), V(t) and R(t) are functions are these quantities are positive and bounded therefore we must show that those functions are positive and bounded which reflect that the model is well defined.

3.2.1. Positivity

All the solutions of system (9) are positive.

since

Let $T(t) = \frac{c\,t^{\alpha}}{\Gamma(\alpha+1)} + T_0$

Where c & $T_0$ are constants, then

$\frac{d^{\alpha}\hat{S}}{dt^{\alpha}} = \frac{d^{\alpha}\hat{S}(T)}{dt^{\alpha}} = \dot{\hat{S}}(T).\frac{d^{\alpha}T}{dt^{\alpha}} = \frac{c\dot{S}(T)}{\Gamma(\alpha+1)}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1)} = c\dot{\hat{S}}(T)$, [ By division and multiplication by the term $\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1)}$ to with suppose $\frac{1}{\Gamma(\alpha+1)} = c$]

Here, the system equations were converted from the fractional derivative to the ordinary differential derivative. At first must show the positivity of $\hat{S}(t)$ and similarly for the others.

By taking $0 < \alpha_1 \leq 1$, put $\alpha_1 = 1$ in the 1$^{st}$ equation from the system (9):

So that

$$\dot{\hat{S}} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right) - \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} + \tilde{v}_4\hat{V}$$

Thus

$$\frac{d^{\alpha_1}\hat{S}}{dt^{\alpha_1}} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right) - \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} + \tilde{v}_4\hat{V}$$

And since $\frac{d^{\alpha}\hat{S}}{dt^{\alpha}} = c\dot{\hat{S}}(T)$, and $\frac{d^{\alpha_1}\hat{S}}{dt^{\alpha_1}} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right) - \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} + \tilde{v}_4\hat{V}, where\ \alpha_1 = \alpha = 1$, then : $c\dot{\hat{S}} = \frac{d\hat{S}}{dt} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right) - \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} + \tilde{v}_4\hat{V}$, thus :

$c\dot{\hat{S}} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right)-\frac{\tilde{\beta}\hat{S}\hat{I}}{\tilde{\Psi}(\hat{I})}+\tilde{v}_4\hat{V} \Rightarrow \frac{d\hat{S}}{\hat{S}} = \frac{1}{c}[\tilde{\mu}(\tilde{\mu}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right)-\frac{\tilde{\beta}\hat{S}\hat{I}}{\tilde{\Psi}(\hat{I})}+$ $\tilde{v}_4\frac{\hat{V}}{\hat{S}}]dt$ . [ After multiplying both sides by the term : $\frac{dt}{\hat{S}}$ ]

Integrating both sides to get:

$\text{Ln}\,\hat{S}(t) = \text{Ln}[\frac{1}{c}[\tilde{\mu}(\tilde{\mu}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right)-\frac{\tilde{\beta}\hat{S}\hat{I}}{\tilde{\Psi}(\hat{I})}+\tilde{v}_4\frac{\hat{V}}{\hat{S}}] + c_1$

Now by taking the exponential function to both sides we get :

$\hat{S}(t) = \hat{S}_0\, e^{\int\frac{1}{c}[\tilde{\mu}(\tilde{\mu}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right)-\frac{\tilde{\beta}\hat{S}\hat{I}}{\tilde{\Psi}(\hat{I})}+\tilde{v}_4\frac{\hat{V}}{\hat{S}}]dt} > 0$ , $[e^{c_1} = \hat{S}_0, \textit{both } c_1 \,\&\, \hat{S}_0 \textit{ are constants}]$

And similarly for the others equations on the system (9).

3.2.2. Uniformly Boundedness

All the solutions of the system (9) are uniformly bounded.

Any solution of system (9) with nonnegative initial let $\left(\hat{S}(t),\hat{E}(t),\hat{I}(t),\hat{V}(t),\hat{R}(t)\right)$ condition $\left(\hat{S}(0),\hat{E}(0),\hat{I}(0),\hat{V}(0),\hat{R}(0)\right) \in \mathbb{R}^5$. Let $\hat{H}(t) = \hat{S}(t)+\hat{E}(t)+\hat{I}(t)+\hat{V}(t)+\hat{R}(t)$

$\frac{1}{c}[\dot{\hat{S}}(t)+\dot{\hat{E}}(t)+\dot{\hat{I}}(t)+\dot{\hat{H}}(t) = \frac{1}{c_1}\dot{\hat{S}}(t)+\frac{1}{c_2}\dot{\hat{E}}(t)+\frac{1}{c_3}\dot{\hat{I}}(t)+\frac{1}{c_4}\dot{\hat{V}}(t)+\frac{1}{c_5}\dot{\hat{R}}(t) \Rightarrow \dot{\hat{H}}(t) \le \dot{\hat{V}}(t)+\dot{\hat{R}}(t)]$

$= \frac{1}{c}[\frac{\tilde{\mu}^3\hat{S}^4+(\tilde{\mu}^3\tilde{k}^2-\tilde{\mu}^2\tilde{k})\hat{S}^2+\tilde{\mu}^2\tilde{k}^2\hat{S}}{\tilde{k}^3}-\frac{\tilde{\beta}\hat{S}\hat{I}}{\psi(\hat{I})}+\tilde{v}_4\hat{V}+\frac{\tilde{\beta}\hat{S}\hat{I}}{\psi(\hat{I})}-(\tilde{v}_1+\tilde{\mu}_1)\hat{E}+\tilde{v}_1\hat{E}-(\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3)\hat{I}+\tilde{\mu}\tilde{\xi}+\tilde{v}\hat{I}-(\tilde{\mu}_1+\tilde{v}_4)\hat{V}+\tilde{v}_2\hat{I}-\tilde{\mu}_1\hat{R}]$, Where $c$ is max $\{c_1, c_2, c_3, c_4, c_5\}$

$= \frac{1}{c}[\frac{\tilde{\mu}^3\hat{S}^4+(\tilde{\mu}^3\tilde{k}^2-\tilde{\mu}^2\tilde{k})\hat{S}^2+\tilde{\mu}^2\tilde{k}^2\hat{S}}{\tilde{k}^3}+(\tilde{v}_4-\tilde{\mu}_1-\tilde{v}_4)\hat{V}+(\tilde{v}_1-\tilde{v}_1-\tilde{\mu}_1)\hat{E}+(\tilde{v}_3+\tilde{v}_2-\tilde{\mu}_1-\tilde{v}_0-\tilde{v}_2-\tilde{v}_3)\hat{I}-\tilde{\mu}_1\hat{R}+\tilde{\mu}\tilde{\xi}] \le \frac{1}{c}[\tilde{\mu}\hat{S}+\tilde{v}_1\hat{E}+\tilde{v}_3\hat{I}+\tilde{v}_4\hat{V}-\tilde{\mu}_1\hat{R}+\tilde{\mu}\tilde{\xi}]$

Now $\dot{\hat{H}}(t) \le \frac{1}{c}[-\tilde{\mu}(\hat{S}+\hat{E}+\hat{I}+\hat{V}+\hat{R})+2\tilde{\mu}\hat{S}-\frac{\tilde{\mu}\hat{S}^2}{\tilde{k}}+\tilde{\mu}\tilde{\xi}+(\tilde{v}_2-\tilde{v}_1)\hat{E}-\tilde{v}_0\hat{I}]$, Since $\hat{S}(t) \le \tilde{k}$ so $\hat{E} < \tilde{k}$

Therefore $\dot{\hat{H}}(t) \le \frac{1}{c}[-\tilde{\mu}H+\tilde{\mu}\tilde{\xi}+2\tilde{\mu}\tilde{k}+(\tilde{v}_2-\tilde{v}_1)\tilde{k}] \Rightarrow \dot{\hat{H}}(t)+\frac{\tilde{\mu}}{c}H \le \tilde{\mu}\tilde{\xi}+(2\tilde{\mu}\tilde{k}+(\tilde{v}_2-\tilde{v}_1))\tilde{k}$

Let $L = \tilde{\mu}\tilde{\xi}+(2\tilde{\mu}\tilde{k}+(\tilde{v}_2-\tilde{v}_1))\tilde{k}$. Then $\dot{\hat{H}}(t)+\frac{\tilde{\mu}}{c}H \le L$ 1st order differential inequality, $H e^{\frac{\tilde{\mu}}{c}t} \le \int Le^{\frac{\tilde{\mu}}{c}t}dt + c_1 = \frac{Lc}{\tilde{\mu}}e^{\frac{\tilde{\mu}}{c}t}+c_1$

Therefore $H(t) \leq \frac{L\tilde{\mu}}{c} + c_1 e^{-\frac{\tilde{\mu}}{c}t} \Rightarrow \lim_{t\to\infty} H(t) \leq \frac{L\tilde{\mu}}{c} \Rightarrow \therefore H(t)$ *is* uniformly bounded.

### 3.3 Equilibrium Points

the equilibrium points of system (9) are determined by equalities the R.H.S. to zero. Consider the first four equation by equating there four conditions to zero to get the equilibrium points:

From the 2nd & 3rd equations of system (9):

$(\tilde{\beta}\hat{S} - \frac{(\tilde{\mu}_1+\tilde{\nu}_1)(\Psi(\hat{I})((\tilde{\mu}_1+\tilde{\nu}_0+\tilde{\nu}_2+\tilde{\nu}_3)}{\tilde{\nu}_1})\hat{I} = 0$, And rom this eq. get two cases :-

**<u>Case 1</u>:-** $\hat{I}=0$, then $\hat{E}=\hat{R}=0$ & $\hat{V}=\frac{\Delta\tilde{\xi}}{\tilde{\mu}_1+\tilde{\nu}_4}$ and $\tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right)+\frac{\tilde{\nu}_4\tilde{\mu}\tilde{\xi}}{\tilde{\mu}_1+\tilde{\nu}_4}=0$

$$\frac{\tilde{\mu}^3\hat{S}^4 + (\tilde{\mu}^3\tilde{k}^2 - \tilde{\mu}^2\tilde{k})\hat{S}^2 + \tilde{\mu}^2\tilde{k}^2\hat{S}}{\tilde{k}^3} + \frac{\tilde{\nu}_4\tilde{\mu}\tilde{\xi}}{\tilde{\mu}_1+\tilde{\nu}_4} = 0$$

$$\hat{S}^4 + (\tilde{k}^2 - \frac{1}{\tilde{\mu}}\tilde{k})\hat{S}^2 + \frac{1}{\tilde{\mu}}\tilde{k}^2\hat{S} + \frac{\tilde{k}\tilde{\nu}_4\tilde{\mu}\tilde{\xi}}{\tilde{\mu}^3(\tilde{\mu}_1+\tilde{\nu}_4)} = 0$$

Where

(1) $(\tilde{k}^2 - \frac{1}{\tilde{\mu}}\tilde{k}) = \tilde{a}_1$

(2) $\frac{1}{\tilde{\mu}}\tilde{k}^2 = \tilde{a}_2$ (3) $\frac{\tilde{k}\tilde{\nu}_4\tilde{\mu}\tilde{\xi}}{\tilde{\mu}^3(\tilde{\mu}_1+\tilde{\nu}_4)} = \tilde{a}_3$

By Ferrari method **(Kadhim and Hummady, 2024. )** the equilibrium points are:

$$\hat{S}^4 + \tilde{a}_1\hat{S}^2 + \tilde{a}_2\hat{S} + \tilde{a}_3 = 0$$

$$\hat{S}_1 = \frac{1}{2}\left(\sqrt{\frac{1}{3}(u+\bar{u}) + \frac{2\tilde{a}_2}{3}} + \sqrt{\frac{1}{3}(ju+\bar{j}\bar{u}) + \frac{2\tilde{a}_2}{3}} + \sqrt{\frac{1}{3}\left(j^2u+\overline{j^2}\bar{u}\right) + \frac{2\tilde{a}_2}{3}}\right)$$

$$\hat{S}_2 = \frac{1}{2}\left(-\sqrt{\frac{1}{3}(u+\bar{u}) + \frac{2\tilde{a}_2}{3}} - \sqrt{\frac{1}{3}(ju+\bar{j}\bar{u}) + \frac{2\tilde{a}_2}{3}} - \sqrt{\frac{1}{3}\left(j^2u+\overline{j^2}\bar{u}\right) + \frac{2\tilde{a}_2}{3}}\right)$$

$$\hat{S}_3 = \frac{1}{2}\left(-\sqrt{\frac{1}{3}(u+\bar{u}) + \frac{2\tilde{a}_2}{3}} + \sqrt{\frac{1}{3}(ju+\bar{j}\bar{u}) + \frac{2\tilde{a}_2}{3}} + \sqrt{\frac{1}{3}\left(j^2u+\overline{j^2}\bar{u}\right) + \frac{2\tilde{a}_2}{3}}\right)$$

$$\hat{S}_4 = \frac{1}{2}\left(-\sqrt{\frac{1}{3}(u+\bar{u}) + \frac{2\tilde{a}_2}{3}} - \sqrt{\frac{1}{3}(ju+\bar{j}\bar{u}) + \frac{2\tilde{a}_2}{3}} + \sqrt{\frac{1}{3}\left(j^2u+\overline{j^2}\bar{u}\right) + \frac{2\tilde{a}_2}{3}}\right)$$

And if $\hat{S}_i < 0$, then it's neglect, so $\hat{S}_i$, $i = 2, 3, 4$ are neglect.

The 1st equilibrium point is $(\hat{S}, \hat{E}, \hat{I}, \hat{V}, \hat{R}) = (\hat{S}_1, 0, 0, \frac{\Delta\tilde{\xi}}{\tilde{\mu}_1+\tilde{\nu}_4}, 0)$

**<u>Case 2</u>:-**

$$\tilde{\beta}\hat{S}-\frac{(\tilde{\mu}_1+\tilde{v}_1)(\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3)}{\tilde{v}_1}\Psi(\hat{I})=0$$

$\therefore \hat{S}=\frac{\tilde{k}_1}{\tilde{\beta}}(1+\hat{I}^2)$ , where $\tilde{k}_1=\frac{(\tilde{\mu}_1+\tilde{v}_1)(\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3)}{\tilde{v}_1}$ (10)

Substituting (10) in the 1st equation of the system (9) to get a forth degree equation in I:

$$\hat{I}^4+\frac{2\tilde{a}_2-1}{\tilde{a}_2}\hat{I}^2-\frac{\tilde{a}_3-\tilde{k}_1}{\tilde{a}_1\tilde{a}_2}\hat{I}-\frac{\tilde{a}_4+\tilde{a}_1-\tilde{a}_1\tilde{a}_2}{\tilde{a}_1\tilde{a}_2}=0$$

Where

(1) $\frac{\tilde{\mu}\tilde{k}_1}{\tilde{\beta}}=\tilde{a}_1$, (2) $\frac{\tilde{k}_1}{\tilde{\beta}\tilde{k}}=\tilde{a}_2$, (3) $\frac{\tilde{v}_4\tilde{v}_3}{\tilde{\mu}_1+\tilde{v}_4}=\tilde{a}_3$ , (4) $\frac{\tilde{v}_4\tilde{\mu}\tilde{\xi}}{\tilde{\mu}_1+\tilde{v}_4}=\tilde{a}_4$

By Ferrari method **(Mohammed and Hummady, 2023)** the equilibrium points are in **Table 1**

**Table 1.** The critical points

| $\hat{I}_i$ | $\hat{S}_i=\frac{\tilde{k}_1}{\tilde{\beta}}(1+\hat{I}_1^2)$ | $\hat{E}_i=\frac{\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3}{\tilde{v}_1}*\hat{I}_i$ | $\hat{V}_I=\frac{\tilde{\mu}\tilde{\xi}+\tilde{v}_I\hat{I}_i}{\tilde{\mu}_1+\tilde{v}_4}$ | $\hat{R}_i=\frac{\tilde{v}_2\hat{I}_i}{\tilde{\mu}}$ |
|---|---|---|---|---|
| $\hat{I}_0=\hat{e}_1$ | $\hat{S}_0=\frac{\tilde{k}_1}{\tilde{\beta}}(1+\hat{e}_1^{\,2})$ $\equiv\hat{w}_1$ | $\hat{E}_0=\frac{\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3}{\tilde{v}_1}*\hat{e}_1$ $\equiv\hat{t}_1$ | $\hat{V}_0=\frac{\tilde{\mu}\tilde{\xi}+\tilde{v}_I\hat{e}_1}{\tilde{\mu}_1+\tilde{v}_4}$ $\equiv\hat{f}_1$ | $\hat{R}_0=\frac{\tilde{v}_2\hat{e}_1}{\tilde{\mu}}$ $\equiv\hat{h}_1$ |
| $=\hat{e}_2\hat{I}_1$ | $=\frac{\tilde{k}_1}{\tilde{\beta}}(1+\hat{e}_2^{\,2})\hat{S}_1$ $\equiv\hat{w}_2$ | $=\frac{\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3}{\tilde{v}_1}*\hat{e}_2\hat{E}_1$ $\equiv\hat{t}_2$ | $\hat{V}_1=\frac{\tilde{\mu}\tilde{\xi}+\tilde{v}_I\hat{e}_2}{\tilde{\mu}_1+\tilde{v}_4}$ $\equiv\hat{f}_2$ | $=\frac{\tilde{v}_2\hat{e}_2}{\tilde{\mu}}=\hat{R}_1$ $\equiv\hat{h}_2$ |
| $=\hat{e}_3\hat{I}_2$ | $=\frac{\tilde{k}_1}{\tilde{\beta}}(1+\hat{e}_3^{\,2})\hat{S}_2$ $\equiv\hat{w}_3$ | $=\frac{\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3}{\tilde{v}_1}=\hat{e}_3\hat{E}_2$ $\equiv\hat{t}_3$ | $\hat{V}_2=\frac{\tilde{\mu}\tilde{\xi}+\tilde{v}_I\hat{e}_3}{\tilde{\mu}_1+\tilde{v}_4}$ $\equiv\hat{f}_3$ | $=\frac{\tilde{v}_2\hat{e}_3}{\tilde{\mu}}=\hat{R}_2$ $\equiv\hat{h}_3$ |
| $=\hat{e}_4\hat{I}_3$ | $=\frac{\tilde{k}_1}{\tilde{\beta}}(1+\hat{e}_4^{\,2})\hat{S}_3$ $\equiv\hat{w}_4$ | $=\frac{\tilde{\mu}_1+\tilde{v}_0+\tilde{v}_2+\tilde{v}_3}{\tilde{v}_1}*\hat{e}_4\hat{E}_3$ $\equiv\hat{t}_4$ | $\hat{V}_3=\frac{\tilde{\mu}\tilde{\xi}+\tilde{v}_I\hat{e}_4}{\tilde{\mu}_1+\tilde{v}_4}$ $\equiv\hat{f}_4$ | $=\frac{\tilde{v}_2\hat{e}_4}{\tilde{\mu}}=\hat{R}_3$ $\equiv\hat{h}_4$ |

Therefore this system has five equilibrium points namely : $A_1=(\hat{S}_1,0,0,\frac{\tilde{\mu}\tilde{\xi}}{\tilde{\mu}_1+\tilde{v}_4},0)$,

$A_2=(\hat{S}_0,\hat{E}_0,\hat{I}_0,\hat{V}_0,\hat{R}_0)=(\hat{w}_1,\hat{t}_1,\hat{e}_1,\hat{f}_1,\hat{h}_1)$, $A_3=(\hat{S}_5,\hat{E}_1,\hat{I}_1,\hat{V}_1,\hat{R}_1)=(\hat{w}_2,\hat{t}_2,\hat{e}_2,\hat{f}_2,\hat{h}_2)$, $A_4=(\hat{S}_6,\hat{E}_2,\hat{I}_2,\hat{V}_2,\hat{R}_2)=(\hat{w}_3,\hat{t}_3,\hat{e}_3,\hat{f}_3,\hat{h}_3)$,

$A_5=(\hat{S}_7,\hat{E}_3,\hat{I}_3,\hat{V}_3,\hat{R}_3)=(\hat{w}_4,\hat{t}_4,\hat{e}_4,\hat{f}_4,\hat{h}_4)$

Now, since

$\tilde{k}_1=2*10^6$ , $\tilde{v}_0=0.001$ , $\tilde{v}_1=0.009$ , $\tilde{v}_2=0.07$ , $\tilde{v}_3=0.04$, $\tilde{v}_4=0.2$ , $\tilde{\beta}=0.002$, let $\tilde{\xi}=1$ , $\tilde{\mu}=0.025$, $\tilde{\mu}_1=0.005$. Then the five equilibrium points are :

1. $A_1 = (400.04,0,0,0.122,0)$,

2. $A_2 = (\ 911731\ ,10.6\ ,0.7\ ,0.2\ ,5.6\ )$,

3. $A_3 = (4591085.7,38.51\ ,2.6,0.24,20.4)$,

4. $A_4 = (\ 183570\ ,7.6\ ,0.5\ ,0.144\ ,4\ )$,

5. $A_5 = (673090,21.14,1.4,0.2,11.2)$.

### 3.4 Stability of the Equilibrium Point

Checking the stability by the eigen value method is difficult so the stability will be checked by the basic reproduction number $R_0$.

One can evaluate $R_0$ by applying the following steps:

1. Take the equations of $\frac{d^{\alpha_2}E}{dt^{\alpha_2}}$ & $\frac{d^{\alpha_3}I}{dt^{\alpha_3}}$ of system (9).

2. Let F = $\begin{bmatrix} terms\ containg\ multiplication \\ terms \end{bmatrix} = \begin{bmatrix} \frac{\beta SI}{1+I^2} \\ 0 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}_{2x1}$,

W=$-\begin{bmatrix} \text{single} \\ terms \end{bmatrix} = -\begin{bmatrix} (\tilde{v}_1 + \tilde{\mu}_1)\hat{E} \\ -\tilde{v}_1\hat{E} + (\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3)\hat{I} \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}_{2x1}$,

3. Compute $\dot{F}$ & $\dot{W} \Rightarrow \dot{F} = \begin{bmatrix} \frac{df_1}{dE} & \frac{df_1}{dI} \\ \frac{df_2}{dE} & \frac{df_2}{dI} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\beta SI(1-I^2)}{(1+I^2)^2} \\ 0 & 0 \end{bmatrix}$

& $\dot{W} = \begin{bmatrix} \frac{dg_1}{dE} & \frac{dg_1}{dI} \\ \frac{dg_2}{dE} & \frac{dg_2}{dI} \end{bmatrix} = \begin{bmatrix} (\tilde{v}_1 + \tilde{\mu}_1) & 0 \\ -\tilde{v}_1 & (\ \tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3) \end{bmatrix}$,

4. Evaluate $\dot{F}$ & $\dot{W}$ at the equilibrium points:-

(1) $\dot{F}\ |_{A_1} = \begin{bmatrix} 0 & 0.8001 \\ 0 & 0 \end{bmatrix}$,

(2) $\dot{F}\ |_{A_2} = \begin{bmatrix} 0 & 39.640435 \\ 0 & 0 \end{bmatrix}$,

(3) $\dot{F}\ |_{A_3} = \begin{bmatrix} 0 & 150.9233 \\ 0 & 0 \end{bmatrix}$,

(4) $\dot{F}\ |_{A_4} = \begin{bmatrix} 0 & 229.5 \\ 0 & 0 \end{bmatrix}$,

(5) $\dot{F}\ |_{A_5} = \begin{bmatrix} 0 & -30.53 \\ 0 & 0 \end{bmatrix}$,

5. Let $W^*|_{A_i} = \begin{bmatrix} (\tilde{v}_1 + \tilde{\mu}_1) & 0 \\ -\tilde{v}_1 & (\ \tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3) \end{bmatrix}$, $i$ = 1, 2, 3, 4, 5.

Find $W^{*-1}$, since $W^{*-1}$= $\begin{bmatrix} 1 & 0 \\ 0.8 & 1.2 \end{bmatrix}$,

6. Find $F^{*}.W^{*-1}$:

(1) $F^{*} = \dot{F}\,|_{A_1} = \begin{bmatrix} 0 & 0.8001 \\ 0 & 0 \end{bmatrix} \Rightarrow F^{*}.W^{*-1} = \begin{bmatrix} 0 & 0.8001 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1.2 \end{bmatrix} = \begin{bmatrix} 0.6401 & 0.1 \\ 0 & 0 \end{bmatrix}$, and since $R_o$ is the highest non negative value of the main diagonal elements ⇒ $R_o = 0.6401 < 1$. Thus, $A_1$ is Asymptotically stable equilibrium point.

(2) $F^{*} = \dot{F}\,|_{A_2} = \begin{bmatrix} 0 & 39.640435 \\ 0 & 0 \end{bmatrix} \Rightarrow F^{*}.W^{*-1} = \begin{bmatrix} 0 & 39.640435 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1.2 \end{bmatrix} = \begin{bmatrix} 31.71235 & 47.6 \\ 0 & 0 \end{bmatrix}$, and since $R_o$ is the highest non negative value of the main diagonal elements ⇒ $R_o = 31.71235 > 1$. Thus, $A_2$ is unstable equilibrium point.

(3) $F^{*} = \dot{F}\,|_{A_3} = \begin{bmatrix} 0 & 150.9233 \\ 0 & 0 \end{bmatrix}$

$F^{*}.W^{*-1} = \begin{bmatrix} 0 & 150.9233 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1.2 \end{bmatrix} = \begin{bmatrix} 120.74 & 181.12 \\ 0 & 0.2 \end{bmatrix}$, and since $R_o$ is the highest non negative value of the main diagonal elements ⇒ $R_o = 120.74 > 1$. Thus, $A_3$ is unstable equilibrium point.

(4) $F^{*} = \dot{F}\,|_{A_4} = \begin{bmatrix} 0 & 229.5 \\ 0 & 0 \end{bmatrix} \Rightarrow F^{*}.W^{*-1} = \begin{bmatrix} 0 & 229.5 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1.2 \end{bmatrix} = \begin{bmatrix} 183.6 & 275.52 \\ 0 & 0 \end{bmatrix}$, and since $R_o$ is the highest non negative value of the main diagonal elements ⇒ $R_o = 183.6 > 1$. Thus, $A_4$ is unstable equilibrium point.

(5) $F^{*} = \dot{F}\,|_{A_5} = \begin{bmatrix} 0 & -30.53 \\ 0 & 0 \end{bmatrix}$,

$F^{*}.W^{*-1} = \begin{bmatrix} 0 & -30.53 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1.2 \end{bmatrix} = \begin{bmatrix} -2.824 & -36.64 \\ 0 & 0 \end{bmatrix}$, and since $R_o$ is the highest non negative value of the main diagonal elements ⇒ $R_o = 0 < 1$. Thus, $A_5$ is stable equilibrium point.■

### 3.5 The Local Bifurcation

Local bifurcation analysis near $\tilde{A}_5$: because it happens at $A_5$ only.

$$\frac{d^{\alpha_1}S}{dt^{\alpha_1}} = \tilde{\mu}(\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)\left(1-\frac{\tilde{\mu}\hat{S}\left(1-\frac{\hat{S}}{\tilde{k}}\right)}{\tilde{k}}\right) - \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} + \tilde{v}_4\hat{V} \equiv f_1$$

$$\frac{d^{\alpha_2}E}{dt^{\alpha_2}} = \frac{\tilde{\beta}\hat{S}\hat{I}}{\Psi(\hat{I})} - (\tilde{v}_1 + \tilde{\mu}_1)\hat{E} \equiv f_2 \tag{11}$$

$$\frac{d^{\alpha_3}I}{dt^{\alpha_3}} = \tilde{v}_1\hat{E} - (\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3)\hat{I} \equiv f_3$$

$$\frac{d^{\alpha_4}V}{dt^{\alpha_4}} = \tilde{\mu}\tilde{\xi} + \tilde{v}_3\hat{I} - (\tilde{\mu}_1 + \tilde{v}_4)\hat{V} \equiv f_4$$

$$\frac{d^{\alpha_5}R}{dt^{\alpha_5}} = \tilde{v}_2\hat{I} - \tilde{\mu}_1\hat{R} \equiv f_5,$$

Now to study the Bifurcation at the equilibrium point $\tilde{A}_5$ by the following theorem :

The equilibrium point $\tilde{A}_5$ of (9) is saddle node bifurcation when the following criteria are fulfilled:

$\tilde{v}_3 = \frac{402.24\tilde{\beta}\tilde{\mu}_1 + 402.24\tilde{\beta}\tilde{v}_4}{\tilde{v}_4}$

(12)

$$€_1 = \frac{(\tilde{v}_1 + \tilde{\mu}_1)(\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3) + 402.24\tilde{\beta}\tilde{v}_1}{3.1\tilde{\beta}}, €_2 = \frac{(\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3)}{\tilde{v}_1},$$

$$€_3 = \frac{-402.24\tilde{\beta}}{\tilde{v}_4}, €_4 = \frac{\tilde{v}_2}{\tilde{\mu}_1} \text{ , and } €_5 = \frac{(\tilde{\mu}_1 + \tilde{v}_4)}{\tilde{v}_4}.$$

with the parameter $\tilde{v}_3$ passes through the value $\widehat{\tilde{v}_3} = \tilde{v}_3$, has a saddle − node bifurcation, but neither a trans critical nor a pitchfork bifurcation at $\tilde{A}_5$.

According to the Jacobian matrix ($\tilde{A}_5$ ) given by (9),

$$\hat{J} = \begin{vmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \tilde{a}_{14} & \tilde{a}_{15} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \tilde{a}_{24} & \tilde{a}_{25} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} & \tilde{a}_{34} & \tilde{a}_{35} \\ \tilde{a}_{41} & \tilde{a}_{42} & \tilde{a}_{43} & \tilde{a}_{44} & \tilde{a}_{45} \\ \tilde{a}_{51} & \tilde{a}_{52} & \tilde{a}_{53} & \tilde{a}_{54} & \tilde{a}_{55} \end{vmatrix} = \begin{vmatrix} \frac{d\hat{f}_1}{d\hat{S}} & \frac{d\hat{f}_1}{d\hat{E}} & \frac{d\hat{f}_1}{d\hat{I}} & \frac{d\hat{f}_1}{d\hat{V}} & \frac{d\hat{f}_1}{d\hat{\mathcal{R}}} \\ \frac{d\hat{f}_2}{d\hat{S}} & \frac{d\hat{f}_2}{d\hat{E}} & \frac{d\hat{f}_2}{d\hat{I}} & \frac{d\hat{f}_2}{d\hat{V}} & \frac{d\hat{f}_2}{d\hat{\mathcal{R}}} \\ \frac{d\hat{f}_3}{d\hat{S}} & \frac{d\hat{f}_3}{d\hat{E}} & \frac{d\hat{f}_3}{d\hat{I}} & \frac{d\hat{f}_3}{d\hat{V}} & \frac{d\hat{f}_3}{d\hat{\mathcal{R}}} \\ \frac{d\hat{f}_4}{d\hat{S}} & \frac{d\hat{f}_4}{d\hat{E}} & \frac{d\hat{f}_4}{d\hat{I}} & \frac{d\hat{f}_4}{d\hat{V}} & \frac{d\hat{f}_4}{d\hat{\mathcal{R}}} \\ \frac{d\hat{f}_5}{d\hat{S}} & \frac{d\hat{f}_5}{d\hat{E}} & \frac{d\hat{f}_5}{d\hat{I}} & \frac{d\hat{f}_5}{d\hat{V}} & \frac{d\hat{f}_5}{d\hat{\mathcal{R}}} \end{vmatrix}$$

the system (9) at equilibrium point

$\tilde{A}_5 = \left(\frac{\tilde{k}_1}{\tilde{\beta}}(1 + \hat{e}_4^2), \frac{\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3}{\tilde{v}_1}\tilde{e}_4, \hat{e}_4, \frac{\tilde{\mu}\tilde{\xi} + \tilde{v}_3\tilde{v}_4\tilde{e}_4}{\tilde{\mu}_1 + \tilde{v}_4}, \frac{\tilde{v}_2\hat{e}_4}{\tilde{\mu}_1}\right)$, has no positive eigenvalue, say: $R_o = 0 < 1$, hence the Jacobian matrix $J(\tilde{A}_5)$ becomes: Let $\widehat{H}_{[5]}$ be any non zero vector such that $\widehat{H}_{[5]} = (\widehat{H}_1, \widehat{H}_2, \widehat{H}_3, \widehat{H}_4, \widehat{H}_5)$

Where $\widehat{H}_{[5]}^T = (\widehat{H}_1, \widehat{H}_2, \widehat{H}_3, \widehat{H}_4, \widehat{H}_5)^T$

$$D\left(\dot{\widehat{H}}, \dot{\widehat{H}}_{[5]}\right) =$$

$$\begin{bmatrix} \frac{4\tilde{\mu}\hat{S}^3 + \tilde{\mu}^2\tilde{k}^2 - (2\tilde{\mu}^3\tilde{k}^2\hat{S} + 2\tilde{\mu}^2\tilde{k})\hat{S}}{\tilde{k}^3} - \frac{\tilde{\beta}\hat{I}}{1+\hat{I}} & 0 & -\frac{\tilde{\beta}\hat{S}\,(1+\hat{I}^2)}{(1+\hat{I}^2)^2} & \tilde{v}_4 & 0 \\ \frac{\tilde{\beta}\hat{I}}{1+\hat{I}^2} & -(\tilde{v}_1 + \tilde{\mu}_1) & \frac{\tilde{\beta}\hat{S}\,(1+\hat{I}^2)}{(1+\hat{I}^2)^2} & 0 & 0 \\ 0 & \tilde{v}_1 & -(\tilde{\mu}_1 + \tilde{v}_0 + \tilde{v}_2 + \tilde{v}_3) & 0 & 0 \\ 0 & 0 & \tilde{v}_3 & -(\tilde{\mu}_1 + \tilde{v}_4) & 0 \\ 0 & 0 & \tilde{v}_2 & 0 & -\tilde{\mu}_1 \end{bmatrix}$$

$$D^2\left(\dot{\hat{T}},\dot{\hat{T}}_{[5]}\right)=\begin{bmatrix}\frac{12\tilde{\mu}^2\hat{S}^2-2\tilde{\mu}^3\tilde{k}^2-2\tilde{\mu}^2\tilde{k}}{\tilde{k}^3}-\frac{\tilde{\beta}(1-\hat{I}^2)}{(1+\hat{I}^2)^3} & 0 & \frac{-\tilde{\beta}(1-\hat{I}^2)}{(1-\hat{I}^2)^2}+\frac{2\tilde{\beta}\hat{S}\hat{I}}{(1-\hat{I}^2)^2} & 0 & 0\\ \frac{\tilde{\beta}(1+\hat{I}^2)}{(1+\hat{I}^2)^2} & 0 & \frac{\tilde{\beta}(1-\hat{I}^2)-2\tilde{\beta}\hat{I}}{(1-\hat{I}^2)^2}+\frac{2\tilde{\beta}\hat{S}\hat{I}}{(1-\hat{I}^2)^3} & 0 & 0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix}$$

$$D^3\left(\dot{\hat{Z}},\dot{\hat{Z}}_{[5]}\right)=$$

$$\begin{bmatrix}\frac{24\tilde{\mu}\hat{S}}{\tilde{k}^3}-\frac{2\tilde{\beta}\hat{I}(1-\hat{I}^2)}{(1+\hat{I}^2)^3} & 0 & \frac{2\tilde{\beta}\hat{I}(1+\hat{I}^2)^2+4\tilde{\beta}\hat{I}(1+\hat{I}^4)}{(1+\hat{I}^2)^4} & 0 & 0\\ 0 & 0 & \frac{2\tilde{\beta}^2\hat{I}(1+\hat{I}^2)^2-4\tilde{\beta}\hat{I}(1+\hat{I}^2)^2-2\tilde{\beta}\hat{I}}{(1+\hat{I}^2)^4}+\frac{2\tilde{\beta}\hat{S}(1+\hat{I}^2)^3-12\tilde{\beta}\hat{S}^2\hat{I}^3(1+\hat{I}^2)^2}{(1+\hat{I}^2)^6} & 0 & 0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\end{bmatrix}$$

let $A_5=\left(\hat{S},\hat{E},\hat{I},\hat{V},\hat{R}\right)\cong A_5=(673090,21.14,1.4,0.2,11.2)$, and under the stability condition $\tilde{k}<\frac{2\tilde{\mu}\hat{S}}{\tilde{\mu}^3\hat{S}+2\tilde{\mu}^2}$ . *Thus* $\tilde{v}_3=\frac{402.24\tilde{\beta}\tilde{\mu}_1+402.24\tilde{\beta}\tilde{v}_4}{\tilde{v}_4}$

$\hat{H}[5]=\left[\hat{H}_1,\hat{H}_2,\hat{H}_3,\hat{H}_4,\hat{H}_5\right]=\left[€_1\hat{H}_3,€_2\hat{H}_3,\hat{H}_3,€_3\hat{H}_3,€_4\right]$, where $\hat{H}_3$is non zero number.

Now find $\left[\hat{J}_{[5]}\right]^{T}$ and multiplying with vector $\hat{\phi}[5]$.

$0,\hat{\phi}_4,0]$, such that $\hat{\phi}_4$ any non zero real number.$\hat{\phi}_5^{[5]}=\left[\hat{\phi}_1,\hat{\phi}_2,\hat{\phi}_3,\hat{\phi}_4,\hat{\phi}_5\right]=\left[€_5\hat{\phi}_4,0,\right.$

Consider $\tilde{\mu}$ is the dependent parameter for System $f_i$, and to derivative the system (9) with respect to parameter $\tilde{k}$, such that $\frac{df_i}{d\tilde{k}}.\Rightarrow\frac{d\hat{f}_i}{d\tilde{k}}=f_{\tilde{k}}\left(A_i,\tilde{k}\right)=\left(\frac{d\hat{f}_1}{d\tilde{k}},\frac{d\hat{f}_2}{d\tilde{k}},\frac{d\hat{f}_3}{d\tilde{k}},\frac{d\hat{f}_4}{d\tilde{k}},\frac{d\hat{f}_5}{d\tilde{k}}\right)^{T}=$

$\left(\frac{\tilde{k}^3((2\tilde{k}\tilde{\mu}^3-\tilde{\mu}^2)\hat{S}^2+2\tilde{k}\tilde{\mu}^2\hat{S}-3\tilde{k}^2(\tilde{\mu}^3\hat{S}^4+(\tilde{\mu}^3\tilde{k}^2-\tilde{\mu}^2\tilde{k})\hat{S}^2+\tilde{\mu}^2\tilde{k}^2\hat{S})}{\tilde{k}^6},0,0,0,0\right)^{T}$ . So , $f_{\tilde{k}}\left(A_5,\tilde{k}\right)=$

. $\left(\frac{\tilde{k}^3((2\tilde{k}\tilde{\mu}^3-\tilde{\mu}^2)807.13+56.82\tilde{k}\tilde{\mu}^2-3\tilde{k}^2(651.5\tilde{\mu}^3+807.13(28.41\tilde{\mu}^3\tilde{k}^2-\tilde{\mu}^2\tilde{k})+\tilde{\mu}^2\tilde{k}^2)}{\tilde{k}^6},0,0,0,0\right)^{T}\neq 0$

$\left[\hat{\phi}_5^{[5]}\right]^{T}.f_{\tilde{k}}\left(A_5,\tilde{k}\right)=\frac{\tilde{k}^3((2\tilde{k}\tilde{\mu}^3-\tilde{\mu}^2)807.13+56.82\tilde{k}\tilde{\mu}^2-3\tilde{k}^2(651.5\tilde{\mu}^3+807.13(28.41\tilde{\mu}^3\tilde{k}^2-\tilde{\mu}^2\tilde{k})+\tilde{\mu}^2\tilde{k}^2)}{\tilde{k}^6}€_5\hat{\phi}_4,\hat{\phi}_4\neq 0$.Therefore, according to [Sotomayor's theorem for Local Bifurcation [18]]. By substituting $\hat{H}[5]$ in Eq.12: $\left[\hat{\phi}_5^{[5]}\right]^{T}.f_{\tilde{k}}\left(A_5,\tilde{k}\right)=\frac{\tilde{\mu}_1+\hat{v}_4}{\hat{v}_4}\hat{\phi}_4\left[\left(\frac{4\tilde{\mu}\hat{S}^3+\tilde{\mu}^2\tilde{k}^2-(2\tilde{\mu}^3\tilde{k}^2\hat{S}+2\tilde{\mu}^2\tilde{k})\hat{S}}{\tilde{k}^3}-\frac{\tilde{\beta}\hat{I}}{1+\hat{I}}\right)\tilde{a}_1-\frac{\tilde{\beta}\hat{S}(1-\hat{I}^2)}{(1+\hat{I}^2)^2}\tilde{a}_3+\tilde{v}_3\tilde{a}_4\right]\hat{H}_3+\hat{\phi}_4[\tilde{v}_3\tilde{a}_3-(\tilde{\mu}_1+\hat{v}_4)\tilde{a}_4]\hat{H}_3\neq 0$. Thus, according to the Sotomayor's theorems, System (11) has saddle node bifurcation at $A_5$ with parameter $\tilde{k}$.■

## 4. APPROXIMATE SOLUTIONS

In this section an approximate solution at the system (9) is evaluated by Sumudu Adomain decomposition method.

Take Sumudu transformation to both sides of (9) to get :

$S[D_*^{\alpha_1}s] = S[\mu(\mu S\left(1-\frac{S}{k}\right)\left(1-\frac{\mu S\left(1-\frac{S}{k}\right)}{k}\right)-\beta sI/\Psi(I) \ + v_4\ V]$

$S[D_*^{\alpha_2}E] = S[\ \beta SI/\Psi(I)\ - v_1\ E - \mu_1 E]$

$S[D_*^{\alpha_3}I] = S[v_1 E - (\mu_1 + v_0 + v_2 + v_3\ )I]$ (13)

$S[D_*^{\alpha_4}V] = S[\mu\xi + v_3\ I - \mu_1 V - v_4\ V]$

$S[D_*^{\alpha_5}R] = S[v_2\ I - \mu_1 R]$

then

$u^{-\alpha_1}[S(s) - S(0)] = S\left[\mu(\mu S\left(1-\frac{S}{k}\right)\left(1-\frac{\mu S\left(1-\frac{S}{k}\right)}{k}\right)-\frac{\beta sI}{\Psi(I)}+v_{\ 4}V\right]$

$u^{-\alpha_2}[S(E) - E(0)] = S[\ \beta SI/\Psi(I)\ - v_1\ E - \mu_1 E]$

$u^{-\alpha_3}[S(I) - I(0)] = S[v_1 E - (\mu_1 + v_0 + v_2 + v_3\ )I]$

$u^{-\alpha_4}[S\ (V) - V(0)] = S[\mu\xi + v_3\ I - \mu_1 V - v_4\ V]$

$u^{-\alpha_5}[S(R) - R(0)] = S[v_2\ I - \mu_1 R]$

Now substituting the initial conditions:

$S(0)=S_0$, $E(0)=E_0$, $I(0)=I_0$, $V(0)=V_0$ and $R(0)=R_0$;

and taking the invers transformation :

$S(t) = S_0 + S^{-1}\left[u^{\alpha_1} S\left\{\mu(\mu S\left(1-\frac{S}{k}\right)\left(1-\frac{\mu S\left(1-\frac{S}{k}\right)}{k}\right)-\frac{\beta SI}{\Psi(I)}+v_{\ 4}V\right\}\right]$; (14)

$E(t) = E_0 + S^{-1}[u^{\alpha_2} S\{\beta SI/\Psi(I)\ - v_1\ E - \mu_1 E\}]$;

$I(t) = I_0 + S^{-1}[u^{\alpha_3} S\{v_1 E - (\mu_1 + v_0 + v_2 + v_3\ )I\}]$;

$V(t) = V_0 + S^{-1}[u^{\alpha_4} S\{\mu\xi + v_3\ I - \mu_1 V - v_4\ V\}]$;

$R(t) = R_0 + S^{-1}[u^{\alpha_5} S\{v_2\ I - \mu_1 R\}]$;

This system is non linear so Sumudu transformation is not applicable but if we use Adomian decomposition method then at system (9) becomes linear and to do so we apply ADM as follows suppose :

$S=\sum_{i=0}^{\infty} S_i; E = \sum_{i-0}^{\infty} E_i; I = \sum_{i=0}^{\infty} I_i; V = \sum_{i=0}^{\infty} V_i; R = \sum^{\infty}{}_{i=0} R_i;$

Suppose $SI=\sum_{i=0}^{\infty} P_i$ ; *where* , $P_i = \sum_{j=0}^{\infty} I_j\, S_{i-j}$;

Therefore (14) becomes :

$$\sum_{i=1}^{\infty} S_i = S_0 + S^{-1}[u^{\alpha_1} S\{\mu(\mu \sum_{i=0}^{\infty} S_i \left(1 - \frac{\sum_{i=0}^{\infty} S_i}{k}\right)\left(1 - \frac{\mu \sum_{i=0}^{\infty} S_i \left(1 - \frac{\sum_{i=0}^{\infty} S_i}{k}\right)}{k}\right) - \frac{\beta \sum_{i=0}^{\infty} P_i}{\Psi(\sum_{i=0}^{\infty} I_i)} + \text{v}_4 \sum_{i=0}^{\infty} V_i\}];$$

$$\sum_{i=1}^{\infty} E_i = E_0 + S^{-1}\,[u^{\alpha_2} S\{\frac{\beta \sum_{i=0}^{\infty} P_i}{\Psi(\sum_{i=0}^{\infty} I_i)} - \text{v}_1 \sum_{i=0}^{\infty} E_i - \mu_1 \sum_{i=0}^{\infty} E_i\}];$$

$$\sum_{i=1}^{\infty} I_i = I_0 + S^{-1}[u^{\alpha_3} S\{\text{v}_1 \sum_{i=0}^{\infty} E_i - (\mu_1 \sum_{i=0}^{\infty} I_i + \text{v}_0 \sum_{i=0}^{\infty} I_i + \text{v}_2 \sum_{i=0}^{\infty} I_i + \text{v}_3 \sum_{i=0}^{\infty} I_i)\}];$$

$$\sum_{i=1}^{\infty} V_i = V_0 + S^{-1}[u^{\alpha_4} S\{\mu\xi + \text{v}_3 \sum_{i=0}^{\infty} I_i - \mu_1 \sum_{i=0}^{\infty} V_i - \text{v}_4 \sum_{i=0}^{\infty} V_i\}];$$

$$\sum_{i=1}^{\infty} R_i = R_0 + S^{-1}[u^{\alpha_5} S\{\text{v}_2 \sum_{i=0}^{\infty} I_i - \mu_1 \sum_{i=o}^{\infty} R_i\}];$$

Therefore

$$S_{n+1} = S^{-1}\,[u^{\alpha_1} S\{\mu(\mu S_n \left(1 - \frac{S_n}{k}\right)\left(1 - \frac{\mu S_n\left(1 - \frac{S_n}{k}\right)}{k}\right) - \frac{\beta P_n}{\Psi(I_n)} + \text{v}_4 V_n\}];$$

$$E_{n+1} = S^{-1}\left[u^{\alpha_2} S\left\{\frac{\beta P_n}{\Psi(I_n)} - \text{v}_1 E_n - \mu_1 E_n\right\}\right];$$

$$I_{n+1} = S^{-1}[u^{\alpha_3} S\{\text{v}_1 E_n - (\mu_1 I_n + \text{v}_0 I_n + \text{v}_2 I_n + \text{v}_3 I_n)\}];$$

$$V_{n+1} = S^{-1}[u^{\alpha_4} S\{\mu\xi + \text{v}_3 I_n - \mu_1 V_n - \text{v}_4 V_4\}];$$

$$R_{n+1} = S^{-1}[u^{\alpha_5} S\{\text{v}_2 I_n - \mu_1 R_n\}];$$

(1) If n = 0, 1.

If n=0, then the 1st iteration $S_1, E_1, I_1, V_1$ *and* $R_1$ are given by:-

$$S_1 = x_1 \frac{t^{\alpha_1}}{\Gamma(\alpha_1 + 1)}; where$$

$$x_1 = \mu(\mu S_0 \left(1 - \frac{S_0}{k}\right)\left(1 - \frac{\mu S_0 \left(1 - \frac{S_0}{k}\right)}{k}\right) - \frac{\beta S_0}{\Psi(I_0)} + \text{v}_4 V_0$$

$$E_1 = x_2 \frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)}; where\ x_2 = \frac{\beta P_0}{\Psi(I_0)} - \text{v}_1 E_0 - \mu_1 E_0;$$

$$I_1 = x_3 \frac{t^{\alpha_3}}{\Gamma(\alpha_3 + 1)}; where\ x_3 = \text{v}_1 E_0 - (\mu_1 + \text{v}_0 + \text{v}_2 + \text{v}_3) I_0;$$

$V_1 = x_4 \frac{t^{\alpha_4}}{\Gamma(\alpha_4+1)}$; *where* $x_4 = \mu\xi + v_3 I_0 - \mu_1 V_0 - v_4 V_0$;

$R_1 = x_5 \frac{t^{\alpha_5}}{\Gamma(\alpha_5+1)}$; *where* $x_5 = v_2 I_0 - \mu_1 R_0$;

(2) If n=1, then the 2nd iteration $S_2, E_2, I_2, V_2$ *and* $R_2$ are given by:-

$S_2 = S^{-1}\left[u^{\alpha_1} S\left\{\mu(\mu S_1\left(1-\frac{S_1}{k}\right)\left(1-\frac{\mu S_1\left(1-\frac{S_1}{k}\right)}{k}\right) - \frac{\beta P_1}{\Psi(I_1)} + v_4 V_1\right\}\right]$; and since $P_i = \sum_{j=0}^{i} I_j S_{i-j}$;

$\Rightarrow P_1 = S_0 I_1 + S_1 I_0$;

$S_2 = \mu^2 x_1 \frac{t^{2\alpha_1}}{\Gamma(2\alpha_1+1)} - \left(\frac{\mu^3}{k} + \frac{\mu^2}{k}\right) {x_1}^2 \frac{t^{3\alpha_1}}{\Gamma(3\alpha_1+1)} + 2\frac{\mu^3}{k^2} {x_1}^3 \frac{t^{4\alpha_1}}{\Gamma(4\alpha_1+1)} - \frac{\mu^3}{k^3} {x_1}^4 \frac{t^{5\alpha_1}}{\Gamma(5\alpha_1+1)} - \frac{\beta S_0 x_3 t^{\alpha_1+\alpha_3}}{\Psi(I_1)\Gamma(\alpha_1+\alpha_3+1)} + v_4 x_4 \frac{t^{\alpha_1+\alpha_4}}{\Gamma(\alpha_1+\alpha_4+1)}$

$E_2 = S^{-1}\left[u^{\alpha_2} S\left\{\frac{\beta S_1 P_1}{\Psi(I_1)} - v_1 E_1 - \mu_1 E_1\right\}\right]$;

$= \frac{\beta S_0 x_3}{\Psi(I_1)} \frac{t^{\alpha_1+\alpha_3}}{\Gamma(\alpha_1+\alpha_3+1)} + \frac{\beta I_0 x_1}{\Psi(I_1)} \frac{t^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1+\alpha_2+1)} - (v_1+\mu_1) x_2 \frac{t^{2\alpha_2}}{\Gamma(2\alpha_2+1)}$

$I_2 = S^{-1}[u^{\alpha_3} S\{v_1 E_1 - (\mu_1 + v_0 + v_2 + v_3)\}]$

$= v_1 x_2 \frac{t^{\alpha_2+\alpha_3}}{\Gamma(\alpha_2+\alpha_3+1)} - x_3(\mu_1 + v_0 + v_2 + v_3) \frac{t^{2\alpha_3}}{\Gamma(2\alpha_3+1)}$

$V_2 = S^{-1}[u^{\alpha_4} S\{\mu\xi + v_3 I_1 - \mu_1 V_1 - v_4 V_1\}]$

$= v_3 x_3 \frac{t^{\alpha_3+\alpha_4}}{\Gamma(\alpha_3+\alpha_4+1)} - (\mu_1 + v_4) x_4 \frac{t^{2\alpha_4}}{\Gamma(2\alpha_4+1)}$

$R_2 = S^{-1}[u^{\alpha_5} S\{v_2 I_1 - \mu_1 R_1\}]$

$= v_2 x_3 \frac{t^{\alpha_3+\alpha_5}}{\Gamma(\alpha_3+\alpha_5+1)} - \mu_1 x_5 \frac{t^{2\alpha_5}}{\Gamma(2\alpha_5+1)}$

Also the exposed population is given by :

1- $S(t) = S_0 + S_1 + S_2$

$S(t) = S_0 + x_1 \frac{t^{\alpha_1}}{\Gamma(\alpha_1+1)} + \mu^2 x_1 \frac{t^{2\alpha_1}}{\Gamma(2\alpha_1+1)} - \left(\frac{\mu^3}{k} + \frac{\mu^2}{k}\right) {x_1}^2 \frac{t^{3\alpha_1}}{\Gamma(3\alpha_1+1)} + 2\frac{\mu^3}{k^2} {x_1}^3 \frac{t^{4\alpha_1}}{\Gamma(4\alpha_1+1)} - \frac{\mu^3}{k^3} {x_1}^4 \frac{t^{5\alpha_1}}{\Gamma(5\alpha_1+1)} - \frac{\beta S_0 x_3 t^{\alpha_1+\alpha_3}}{\Psi(I_1)\Gamma(\alpha_1+\alpha_3+1)} + v_4 x_4 \frac{t^{\alpha_1+\alpha_4}}{\Gamma(\alpha_1+\alpha_4+1)}$

2- $E(t) = E_0 + E_1 + E_2$

$E(t) = E_0 + x_2 \frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)} + \frac{\beta S_0 x_3}{\Psi(I_1)} \frac{t^{\alpha_1+\alpha_3}}{\Gamma(\alpha_1+\alpha_3+1)} + \frac{\beta I_0 x_1}{\Psi(I_1)} \frac{t^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1+\alpha_2+1)} - (v_1+\mu_1) x_2 \frac{t^{2\alpha_2}}{\Gamma(2\alpha_2+1)}$

3- $I(t) = I_0 + I_1 + I_2$

$I(t) = I_0 + x_3 \frac{t^{\alpha_3}}{\Gamma(\alpha_3+1)} + v_1 x_2 \frac{t^{\alpha_2+\alpha_3}}{\Gamma(\alpha_2+\alpha_3+1)} - x_3(\mu_1 + v_0 + v_2 + v_3) \frac{t^{2\alpha_3}}{\Gamma(2\alpha_3+1)}$

4- $V(t) = V_0 + V_1 + V_2$

$V(t) = V_0 + x_4 \frac{t^{\alpha_4}}{\Gamma(\alpha_4+1)} + v_3 x_3 \frac{t^{\alpha_3+\alpha_4}}{\Gamma(\alpha_3+\alpha_4+1)} - (\mu_1 + v_4) x_4 \frac{t^{2\alpha_4}}{\Gamma(2\alpha_4+1)}$

5- R(t)= $R_0 + R_1 + R_2$

$$R(t)= R_0 + x_5 \frac{t^{\alpha_5}}{\Gamma(\alpha_5+1)} + v_2 x_3 \frac{t^{\alpha_3+\alpha_5}}{\Gamma(\alpha_3+\alpha_5+1)} - \mu_1 x_5 \frac{t^{2\alpha_5}}{\Gamma(2\alpha_5+1)} \quad (15)$$

## 5. NUMARICAL SIMULATION

**Case (1):** Now if we take $S_0 = 1, E_0 = 1;\ I_0 = 1; V_0 = 1;\ R_0 = 1;\ \mu = 0.025, \mathrm{k} = 8 * 10^6, \nu_0 = 0.001, \nu_1 = 0.009, \nu_2 = 0.07, \nu_3 = 0.04, \nu_4 = 0.2, \tilde{\beta} = 0.002$, let $\tilde{\xi} = 0, \mu = 0.025, \mu_1 = 0.005. R_0 = 5;\ \Psi(I_0) = \Psi(I_1) = \Psi(I_2) = 1;\ \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 = 1$, thus (1), (2), (3), (4) and (5) in (15) we get :

S(t) = $1+0.2t +0.33002t^2 - 0.00000000001t^3 + 3.2552083t^4 - 0.000000000001t^5$

E(t) = $1 + 0.2t +0.033t^2$

I(t) = $1 - 0.071t + 0.0031t^2$

V(t) = $1 + 9.9t + 0.34t^2$

R(t) = $1 + 0.002\,t - 0.0001\,t^2$

**Table 2.** The interval points for SEIVR, when $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 = 1$

| t | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 1 |
|---|---|---|---|---|---|---|
| **S(t)** | 1 | 1.023 | 2.22 | 2.99 | 3.2 | 4.8 |
| **E(t)** | 1 | 1.02033 | 1.04132 | 1.061 | 1.081 | 1.233 |
| **I(t)** | 1 | 0.99321 | 0.985924 | 0.97897 | 0.97209 | 0.9321 |
| **V(t)** | 1 | 1.9934 | 3.016 | 4.0006 | 5.0144 | 11,24 |
| **R(t)** | 1 | 1.00019 | 1.00039 | 1.00059 | 1.00079 | 1.0009 |

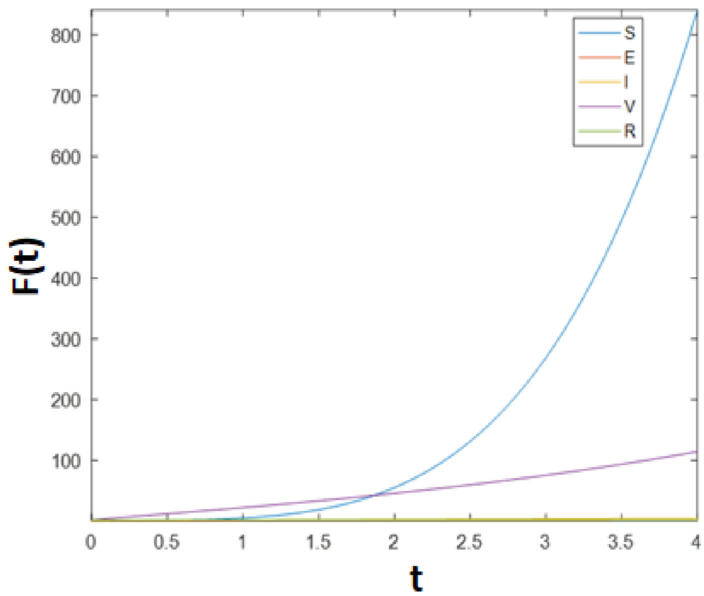


**Figure 2.** Graph of S(t), under the conditions : $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ E(t), I(t),V(t) and R(t) = 1, t $\in [0, 4]$.

From **Fig. 2** and **Table 2** we notice :

(1) The susceptible population S(t) increases rapidly at the beginning because they do not know the danger then it stabilities at acceptable number.
(2) The exposed E(t) also increases because either they don't know the infected or the infected people don't declare their disease after that according to the limitation ordered by medical organization the number of E(t) decreases.

Similar interpretation for I(t), V(t) and R(t).

**Case (2):** If $\alpha_1 = 0.7, \alpha_2 = 0.6, \alpha_3 = 0.8, \alpha_4 = 0.9, \alpha_5 = 0.91$, with $S_0 = E_0 = I_0 = V_0 = R_0 = 1$, thus (1), (2), (3), (4) and (5) in (15) we get :
$S(t) = 1+0.12t^{0.7} - 0.001t^{1.4} - 0.0000000024t^{1.4} + 0.043t^{2.8} + 1.3t^{1.6} - 1.2t$

$E(t) = 1 - 0.63t^{0.6} - 0.12t^{1.2}$

$I(t) = 1 - 0.05t^{0.8} - 0.0001t^{1.2} + 0.0004t^{1.6}$

$V(t) = 1 + 10.6325t^{0.9} - 0.001t^{1.7} - 0.9t^{1.8}$

$R(t) = 1 + 0.0015t^{0.91} - 0.0013t - 0.000002t^{1.82}$

**Table 3.** The interval points for SEIVR, when $\alpha_1 = 0.7, \alpha_2 = 0.6, \alpha_3 = 0.8, \alpha_4 = 0.9, \alpha_5 = 0.91$

| t | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 1 |
|---|---|---|---|---|---|---|
| S(t) | 1 | 0.94 | 0.89 | 0.887 | 0.886 | 1.26 |
| E(t) | 1 | 0.8 | 0.7 | 0.6 | 0.5 | 0.25 |
| I(t) | 1 | 0.992 | 0.986 | 0.980 | 0.976 | 0.950 |
| V(t) | 1 | 2.352 | 3.547 | 4.7007 | 5.8337 | 12.5312 |
| R(t) | 1 | 1.00005 | 1.00008 | 1.00011 | 1.00013 | 1.00019 |

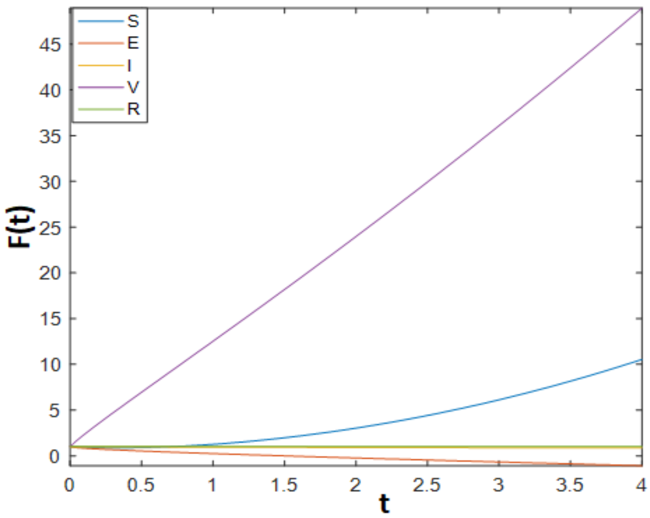


**Figure 3.** Graph of S(t), E(t), I(t),V(t) and R(t) under the conditions: $\alpha_1 = 0.7, \alpha_2 = 0.6, \alpha_3 = 0.8, \alpha_4 = 0.9, \alpha_5 = 0.91$, $t \in [0, 4]$.

From **Fig. 3** and the **Table 3** we notice :

(1) The susceptible population S(t), V(t) and R(t) are increases rapidly at the beginning because they do not know the danger then it stabilities at acceptable number.
(2) The exposed E(t) and infective I(t) are decreases rapidly but not equal to zero because either they don't know the infected or the infected people don't declare their disease after that according to the limitation ordered by medical organization the number of E(t) decreases.

**Case (3):** If $\alpha_1 = 0.8, \alpha_2 = 0.61, \alpha_3 = 0.7, \alpha_4 = 0.6, \alpha_5 = 0.8$, with $S_0 = E_0 = I_0 = V_0 = R_0 = 1$, thus (1), (2), (3), (4) and (5) in (15) we get
S(t) = $1+0.111t^{0.8} + 1.7t^{1.4} + 0.001t^{1.6} - 0.0000000002t^{2.1} + 0.611t^{3.21} - 1.042t^4$

E(t) = $1-0.634t^{0.61} - 0.005t^{1.5}$

I(t) = $1 - 0.01t^{0.7} - 0.0001t^{1.31} + 0.001t^{1.4}$

V(t) = $1 + 12.63t^{0.6} - 0.002t^{1.3} - 0.0021t^{1.2}$

R(t) = $1 + 0.00111t^{0.8} - 0.0014t^{1.5} - 0.000002t^{1.6}$

**Table 4.** The interval points for SEIVR, when $\alpha_1 = 0.8, \alpha_2 = 0.61, \alpha_3 = 0.7, \alpha_4 = 0.6, \alpha_5 = 0.8$.

| **t** | **0** | **0.1** | **0.2** | **0.3** | **0.4** | **1** |
|---|---|---|---|---|---|---|
| **S(t)** | 1 | 1.085 | 1.211 | 1.36 | 1.53 | 2.38 |
| **E(t)** | 1 | 0.8 | 0.7 | 0.6 | 0.5 | 0.36 |
| **I(t)** | 1 | 0.998 | 0.996 | 0.994 | 0.992 | 0.990 |
| **V(t)** | 1 | 4.17227 | 5.80807 | 7.1321 | 8.2872 | 13.6259 |
| **R(t)** | 1 | 1.00013 | 1.00018 | 1.00019 | 1.00017 | 0.9997 |

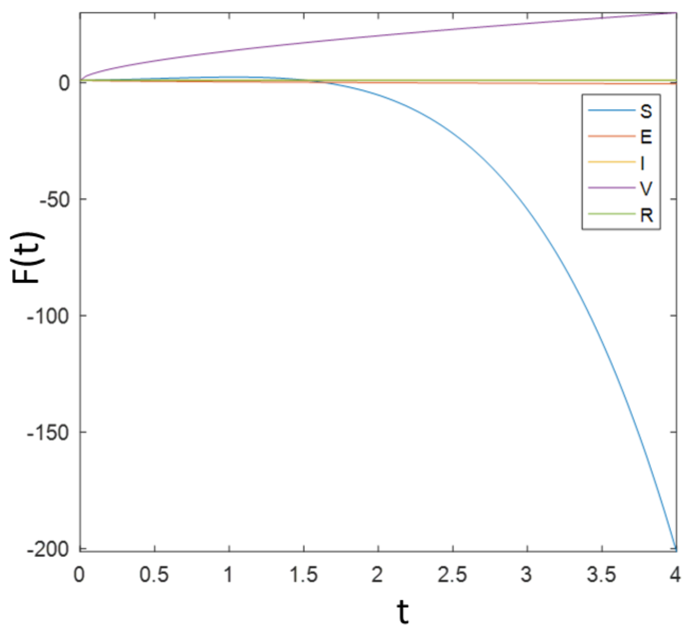


**Figure 4.** Graph of S(t), E(t), I(t),V(t) and R(t) under the conditions: $\alpha_1 = 0.8, \alpha_2 = 0.61, \alpha_3 = 0.7, \alpha_4 = 0.6, \alpha_5 = 0.8$, $t \in [0, 4]$.

From the **Fig. 4** and **Table 4**, we notice :

(1) The susceptible population S(t) decreases rapidly at the beginning because they do not know the danger then it stabilities at acceptable number.
(2) The exposed E(t) increases because either they don't know the infected or the infected people don't declare their disease after that according to the limitation ordered by medical organization the number of E(t) decreases.

Similar interpretation for I(t), V(t) and R(t) they are also increases.

## 6. CONCLUTIONS

The dynamical behaviors of model (9) are discussed in this paper. Model (9) contains five fractional differential equations with different orders concerning COVID-19 is given and the five parameters are S(t), E(t), I(t), V(t) and R(t) which represent susceptible, exposed, inflected, vaccinated and recovered individuals respectively. An analysis solution is evaluated positivity of the functions S(t), E(t), I(t), V(t) and R(t) as solutions of the system is proved. The uniformly boundedness of the solutions of the system under consideration is also proved. And finding the equilibrium points and studying their stability a fractional differential system orders are checked locally and globally. The basic reproduction number is used to prove the stability of all equilibrium points as well as the method of the nature of the eigen values of the Jacobian at each equilibrium point. And then studied the local bifurcation to the the asymptotically stable and stable equilibrium points. We modify an SEIVR model concerning COVID-19 from 1st order system into multi fractional order system of differential equations and finding an approximate solution by using Sumudu Adomian decomposition method because we try to give qualitative results rather than qualitative results.

## NOMENCLATURE

| Symbol | Description | Symbol | Description |
|---|---|---|---|
| COVID-19 | *Corona virus disease 2019* | G(u) | Sumudu transform |
| Γ(z) | *Gamma function* | ADM | Adomian decomposition method |
| $\beta$(n,m) | *Beta function* | JM | Jacobian matrix |
| F(s) | *Laplace transform* | SADM | Sumudu transform with Adomian decomposition method |

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## Credit Authorship Contribution Statement

Saad Naji & Zainab Mohammed has created an analysis solution by evaluated the positivity and the uniformly boundedness of the solutions of the system under consideration is also proved of the functions. While Z.M. found the equilibrium points and studying their stability a fractional differential system orders are checked locally .

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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# تحليل تقريبي لنموذج تفاضلي ذو رتب كسرية باستخدام طريقة سومودو ادومين ديكمبوسشن لنظام وبائي من النوع SEIVR

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**الخلاصـة**

كما تم اقتراح نموذج رياضي لتصنيف كوفيد–19، إذ يجب تقسيم السكان إلى عدد من المجموعات، ونموذجنا يحتوي على خمس مجموعات وهي السليمة و الملامسة والمتأثرة والملقحة والمتعافية. هذا النموذج هو نظام ديناميكي مستمر في حين يكون مشتقاً من النظام التفاضلي الكسري. للحصول على نتائج مقبولة. تم برهان ان النظام موجب و له محدودية محددة. كذلك تم إيجاد نقاط الاتزان و دراسة استقراريتها هل هي مستقرة ام مستقرة محلياً. فبعض نقاط الاتزان كانت قيم تقريبية و بعضها كانت على شكل دوال. و بعدها قمنا بدراسة التفرع المحلي لنقاط الاتزان المستقرة و النقاط المستقرة محليا, نحن نحتاج إلى الحلول. و بعض الحلول هي نقاط تسمى بالنقاط التوازن والبعض الآخر عبارة عن دوال. ويتم تقييمها تقريبًا. يجب أن تلبي هذه الحلول طبيعة المشكلة قيد الدراسة تحت ظروف معينة، على سبيل المثال في ظل ظروف معينة تكون بعض نقاط التوازن مستقرة. كما يجب أن يعطي الحل التقريبي نتائج قريبة من الوضع الحقيقي. يتم عرض كل هذه المطالب في هذا البحث. يتم تقديم المحاكاة التقريبية والعددية من خلال جدول ورسوم بيانية توضح كفاءة الطريقة، باستخدام برنامج الماتلاب لجميع الأشكال.

**الكلمات المفتاحية:** تفرع، حلول مقيدة، استقراريه، تفاضل كسري.