University of Baghdad
College of Engineering
JE JOURNAL OF ENGINEERING

Journal of Engineering

journal homepage: www.jcoeng.edu.iq

Volume 31 Number 4 April 2025

DCKTKT: A New Discrete Cosine-Krawtchouk-Tchebichef Transform

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ABSTRACT

Real-world signals are often intricate and difficult to analyze. Therefore, to facilitate the analysis of signal components, the researchers represent the signal in different domains (transform domain), providing a new perspective and offering significant advantages in understanding the various components of signals. Therefore, discrete transforms have been the subject of extensive study. In this paper, a new hybrid form of orthogonal polynomial is introduced named discrete Cosine-Krawtchouk–Tchebichef transform (DCKTKT). Which is based on combining discrete Cosine transform with krawtchouk and tchebichef polynomials. The mathematical and theoretical formulations of DCKTKT are presented, followed by an evaluation of its performance against other hybrid forms. The results demonstrate that DCKTKT along with their corresponding moments. Surpasses existing hybrid polynomials regarding energy compaction. Additionally, a face recognition application is performed and by using a well-known database with clean and noisy environments, DCKTKT is used to transform face images into the moment domain to facilitate feature extraction. illustrating the proposed polynomial's robustness against different types of noise and its superior feature extraction capabilities compared to the latest hybrid forms.

Keywords: Discrete COS transform, Discrete Tchebichef transform, Discrete Krawtchouk transform, Face recognition, Hybrid form.

1. INTRODUCTION

Signals are considered an essential carrier of information. These signals can be either deterministic or stochastic **(Pachori, 2023)**. Also, the conveyed signal can be classified as one-dimensional (1D), like a speech signal, two-dimensional (2D), like an image, three-dimensional (3D), like a video, and four-dimensional (4D), like volume data over time. The study of signals is involved in many disciplines, including signal processing, communications theory, and control systems. This study aroused the interest of many researchers in the field of engineering, science, and many specializations, and they used different methods to analyze, transmit, and process that signal.

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Peer review under the responsibility of University of Baghdad.
https://doi.org/10.31026/j.eng.2025.04.09

Article received: 10/09/2024
Article revised: 02/11/2024
Article accepted: 13/01/2025
Article published: 01/04/2025

In general, signals are complex in nature, making them difficult to deal with and understand. Therefore, to facilitate their handling, we must simplify or represent them in an understandable mathematical formula, such as simple signals, general basis functions, complex exponential functions, Bessel functions, or orthogonal functions.

Representation using orthogonal functions is one of the most preferred signal representations **(Pachori, 2023)** because of its properties, such as orthogonality in its basis function that gives an efficient analysis of the signal's components by isolating each feature in a specific polynomial, making it easier to process. Also, it reduces noise **(Yaru and Xiaohong, 2009)** by analyzing the behavior of each type of noise and separating the signal from the noise. Also, the orthogonality property provides a high compression without losing essential information; and the discrete cosine transform undeniably exhibits this property to a significant extent **(Walmsley et al., 1994)**. Lastly, it has a near-zero redundancy **(Mukundan et al., 2001)** and numerical stability **(Mahmmod et al., 2022)**.

Other unique properties of orthogonal functions that determine their performance and capabilities to extract features are localization and energy compaction (EC) properties **(Mahmmod et al., 2018; Abdulhussain et al., 2021)**. Energy compaction facilitates data transmission and storage by concentrating the majority of the signal's energy into a limited number of transform coefficients **(Wang et al., 2000)**. The localization property defines the orthogonal function's ability to extract features by creating a relation between the transform coefficients and their specific locations within the signal **(Abdulhussain et al., 2021)**.

Given the urgent need to extract more features and streamline data processing and storage for optimal analysis and transmission, combining multiple polynomials for example Tchebichef polynomial **(Zhu et al., 2007)**, Krawtchouk polynomial **(Feinsilver and Kocik, 2005; Asli and Flusser, 2014)**, Charlier polynomial **(Abdul-Hadi et al., 2020)**, wavelet polynomial **(Abood, 2013)** and Hahn polynomial **(Yap et al., 2007)** to leverage their unique features and introducing a new polynomial with more robust features than the individual one is imperative. Hence, the proposed polynomial combines the strengths of several polynomials, including Krawchouk, Chebyshev, and the Discrete Cosine Transform (DCT), to create a new hybrid transform that offers distinct advantages, particularly in terms of energy compaction and feature extraction. This elevates performance in various applications, such as image processing, compression, and pattern recognition. For instance, Krawchouk polynomials are effective at extracting features from signals, while Chebyshev polynomials and the Discrete Cosine Transform (DCT) are commonly used in compression algorithms. DCT achieves robust energy compaction by concentrating the signal's energy into a few low-frequency coefficients. By combining these polynomials, the resulting transform leverages the strengths of each component, thereby enriching the overall transform's capability for efficient data representation and analysis.

2. MATERIALS

In this section, the mathematical definitions and fundamentals of the materials utilized in this paper is presented, which include the preliminaries of discrete orthogonal polynomials and the definitions of orthogonal moments.

2.1 Tchebichef Moments

In general, orthogonal moments are coefficients that provide a concise representation that captures an image's global information (features) using orthogonal polynomials as basis functions **(Hu, 1962; Markandey and deFigueiredo, 1992)**. Let $\emptyset_{mn}$ be the definition of Tchebichef moments based on a discrete Tchebichef polynomial **(Mukundan, 2004)**

$$\emptyset_{mn} = \frac{1}{\rho(m,N)\rho(n,N)} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} T_n(x)\, T_m(y)\, f(i,j) \tag{1}$$

Where *f(i,j)* denotes the pixel value of *(i,j)* position in the image, $\rho(\cdot)$ is the squared-norm which is given by **(Wu and Yan, 2016; Hussein et al., 2023)**:

$$\rho(n,N) = (2n)! \binom{N+n}{2n+1} \tag{2}$$

Where N is the polynomial size $T_n(x)$ is the Tchebichef polynomial (TP) function, with order n and length N, which is determined by **(Pee, 2017; Lu and Asli, 2023)**:

$$T_n(x) = \sqrt{\frac{\omega_T(x)}{\rho_T(n)}} (1-N)_n\ {}_3F_2(-n,-x,1+n;1,1-N;1), n, x = 0,1,\dots,N-1 \tag{3}$$

where n is the polynomial order , x is the signal length and $\omega_T(x)$ is the wight function and ${}_3F_2$ is the hypergeometric function **(Idan et al., 2020)**, they are expressed as follows:

$$\omega_T\,(x) = 1 \tag{4}$$

$$3F_2(-n,-x,1,+n;1,1-N;1) = \sum_{k=0}^{\infty} \frac{(-n)_k(-x)_k(1+n)_k}{(1)_k(1-N)_k k!} \tag{5}$$

$$(\mathrm{a})_\mathrm{k} = a(a+1)\,(a+2),\dots,(a+k+1) = \frac{\Gamma(\mathrm{a+k})}{\Gamma(\mathrm{a})} \tag{6}$$

therefore, $T_n(x)$ can be rewritten as follows:

$$T_n(x) = \frac{(1-N)_n\ {}_3F_2(-n,-x,1+n;1,1-N;1)}{\sqrt{(2\mathrm{n})! \binom{N+n}{2n+1}}} \tag{7}$$

Using of hypergeometric and gamma formulas to calculate the polynomial values is highly time-consuming and it causes numerical propagation **(Radeaf et al., 2019)**. On the other hand, calculating the higher order of polynomial values using recurrence relations concerning order n and variable x is limited by numerical instabilities, which results in significant information loss in reconstructing large images. Thus, the algorithm in **(Abdulhussain et al., 2017)** is used to mitigate this issue, where the $T_n(x)$ is equal to:

$$T_n(x) = \begin{cases} a_1 T_n(x-1) + a_2 T_n(x-2), \\ 0 < n < {}^N\!/_2, 2 < x < {}^N\!/_2 - 1 \\ b_1 T_{n-1}(x) + b_2 T_{n-2}(x), \\ {}^N\!/_2 < n < N-1, L_x < x < {}^N\!/_2 - 1 \\ {}^1\!/_{a_2}\, T_n(x+1) + {}^{a_1}\!/_{a_2}\, T_n(x+2), \\ {}^N\!/_2 < n < N-1, L_x < x < L_x - 12 \end{cases} \tag{8}$$

where

$a_1 = \frac{-n(n+1)-(2x-1)(x-N-1)-x}{x(N-x)}$

$a_2 = \frac{(x-1)(x-N-1)}{x(N-x)}$

$b_{1=} \frac{2x+1\_N}{n}\sqrt{\frac{(4n^2-1)}{(n^2-N^2)}}$

$b_{2=} \frac{1-n}{n}\sqrt{\frac{(2n+1)}{(2n-3)}}\sqrt{\frac{N^2-(n-1)^2}{(n^2-N^2)}}$

$L_x = 0.5N - \sqrt{(0.5N)^2 - (0.5n)^2}$

For signal reconstruction the following formula is used **(Zhu et al., 2010)**:

$$\hat{f}(i,j) = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} T_n(i;N)\,T_m\,(j;\;N)\emptyset_{nm}\;,\; i,j\; =\; 0,1,\dots,N-1; \tag{9}$$

2.2 Krawtchouk Moments

Krawtchouk moments are constructed using Krawtchouk polynomials, which serve as the basis function set **(Yap et al., 2003)**; these polynomials are linked to the binomial distribution and satisfy the orthogonality property that introduced in **(Zhu et al., 2010)**, and which states that:

$$\sum_{x=0}^{N-1} K_n^p(x)K_m^p(x)w_k(x) = \rho_k(n)\emptyset_{nm} \tag{10}$$

where $\emptyset_{nm}$ is the krawtchouk moment, $\rho_k(n)$ is the square norm of krawtchouk polynomial which is given by **(Abdulhussain et al., 2018)**:

$$\rho_k(n) = (-1)^n\left(\frac{1-p}{p}\right)^n \frac{n!}{(-N+1)_n} \tag{11}$$

$w_k(x)$ is the weight function which is given by **(Abdulhussain et al., 2018)** and define as:

$$\omega_k(x) = \binom{N-1}{x}p^x\,(1\;-\;p)^{N-x-1} \tag{12}$$

and $K_n^p(x)$ is a classical krawtchouk polynomial with nth order and parameter p, which is define by **(Yap et al., 2003)**:

$$K_n^p(x) = \sum_{K=0}^{N} a_{k,n,p}x^k = {}_2F_{\,1}(-n,-x;-N;\tfrac{1}{p}) \tag{13}$$

Where x, n=0,1,2, …, N, N > 0, p ∈ {0, 1}, and ${}_2F_{\,1}$ named the hypergeometric function of krawtchouk polynomial and is given by **(Yap et al., 2003)**:

$${}_2F_{\,1}(a,b;c;z) = \sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{(c)_k}\frac{z^k}{k!} \tag{14}$$

And $(a)_k$ is the rising factorial which is given by:

$$(a)_k = a(a+1)(a+2),\dots,(a+k+1) = \frac{\Gamma(a+k)}{\Gamma(a)} \tag{15}$$

To avoid numerical fluctuations in Krawtchouk polynomial computations, the traditional method of normalizing by the norm is used **(Yap et al., 2003)**:

$$\tilde{K}_n^p(x) = \frac{K_n^p(x)}{\sqrt{\rho_k(n)}} \tag{16}$$

But when dealing with higher values of N (e.g., 100), the above equation alone fails to guarantee the stability of the Krawtchouk polynomials. Therefore, a weighted Krawtchouk polynomials is introduced to achieve numerical stability **(Tahiri et al., 2022)**:

$$\tilde{K}_n^p(x) = K_n^p(x)\sqrt{\frac{w_k(x)}{\rho_k(n)}} \tag{17}$$

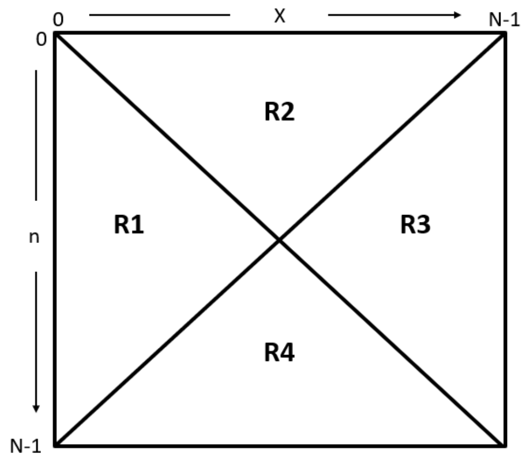

Figure 1. Four parts partitioning of krawtchouk polynomial plain.

The calculation of polynomial values using the hypergeometric and gamma functions is highly demanding in terms of time, requiring high mathematic operations (factorials, powers, sums, etc.) for each polynomial order, therefor, to mitigate this problem, the algorithm described in **(Mahmmod et al., 2020)** is used. First the coefficients $K_n(0)$ and $K_n(1)$ are computed, using the following formula:

$$\tilde{K}_n^p(0) = \sqrt{\frac{(N-n)p}{n(1-p)}} \times \tilde{K}_{n-1}^p(0) \tag{18}$$

$$\tilde{K}_0^p(0) = \sqrt{(1-p)^{N-1}} \tag{19}$$

$$\tilde{K}_n^p(1) = \frac{-n+p(N-1)}{p(N-1)}\sqrt{\frac{(N-1)\,p}{(1-p)}}\,\tilde{K}_n^p(0),\ \mathrm{n} = 0, 1, \ldots, \mathrm{N}-2 \tag{20}$$

For the R1 coefficients as **Fig. 1** shows the following equations are used:

$$\tilde{K}_n^p(x+1) = E \times K_n^p(x) - F \times K_n^p(x-1), n = 0, 1, \ldots, N-1 \tag{21}$$

$$E = \frac{p(N-x-1)+x(1-p)-n}{\sqrt{p(1-p)(x+1)(N-1-x)}} \tag{22}$$

$$F = \frac{\sqrt{p(1-p)x(N-x)}}{\sqrt{p(1-p)(x+1)(N-1-x)}} \tag{23}$$

Third, to compute the R2 coefficients using the symmetry relation of Krawtchouk polynomial about the primary diagonal (n=x) the following equation is used:

$$\widetilde{K}_n^p(x) = \widetilde{K}_x^p(n) \tag{24}$$

And for the R3 and R4 coefficients the symmetry relation about secondary diagonal is used:

$$\widetilde{K}_{(N-x-1)}^p(N-n-1) = (-1)^{N-n-x-1}\widetilde{K}_n^p(x) \tag{25}$$

At the end, the (p > 0.5) coefficients are computed using the following relation:

$$K_n^{1-p}(x) = (-1)^n K_n^p(N-x-1) \tag{26}$$

To compute krawtchouk moments the following equation is used:

$$\emptyset_{nm} = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} K_n^p(x)K_m^p(x)\, f(i,j)\ ,\ \mathrm{i,j} = 0,1,\dots,\mathrm{N}-1; \tag{27}$$

While, for signal reconstruction the following formula is used:

$$\hat{f}(\mathrm{i,j}) = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} K_n^p(x)K_m^p(x)\, \emptyset_{nm}\ ,\ \mathrm{i,j} = 0,1,\dots,\mathrm{N}-1; \tag{28}$$

2.3 Discrete Cosine Transform (DCT)

In image processing, the Discrete Cosine Transform (DCT) is highly regarded by researchers. Large DCT coefficients are mainly found in the low-frequency region, leading to excellent energy compaction **(Abbas, 2005; Wang and Shang, 2020)**. The discrete cosine transform a(k) function is defined using the following Eq. given by **(Jain, 1989)**:

$$\alpha(\mathrm{k}) = \begin{cases} \sqrt{\frac{1}{N}}\cos(\frac{\pi n}{2N}(2x+1)) & for\ k = 0 \\ \sqrt{\frac{2}{N}}\cos(\frac{\pi n}{2N}(2x+1)) & for\ k > 0 \end{cases} \tag{29}$$

the moments computation of 2D signal for the DCT are given by the following equation **(Wang and Shang, 2020)**:

$$\emptyset_{nm} = \beta_n\beta_m \sum_{i=0}^{M-1}\sum_{j=0}^{N-1} \mathrm{f(i,j)}\cos(\frac{\pi n}{2M}(2x+1))\cos(\frac{\pi n}{2N}(2x+1)) \tag{30}$$

Where $0 \le n \le M-1$ and $0 \le m \le N-1$
While to reconstruct the 2D signal, the following equation is used:

$$\hat{f}(i,j) = \beta_n \beta_m \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} \emptyset_{nm} \cos(\frac{\pi n}{2M}(2x+1)) \cos(\frac{\pi n}{2N}(2x+1)) \tag{31}$$

3. THE PROPOSED HYBRID POLYNOMIAL

For effective signal analysis and enhanced efficiency in signal transmission and storage, the combination of various polynomials exploits and strengthens their individual characteristics (localization and energy compaction). This combination ensures that the strengths of each polynomial are utilized to their fullest potential, resulting in more effective and reliable signal processing. Therefore, this section presents the proposed hybrid form of orthogonal polynomial namely DCKTKT, which is based on a combination of well-known OPs such as Cosine transform with krawtchouk and tchebichef polynomials. The nth order hybrid form $R_n(x)$, is defined by the following formula:

$$Rn(x;N) = \sum_{j=0}^{N-1} Z_j(N) X_j(x;N) Y_j(n;N) X_j(x;N) Y_j(n;N), \ n,x = 0,1,\dots,N-1 \tag{32}$$

where $X_j(x;N)$, $Y_j(n;N)$, and $Z_j(N)$ are krawtchouk polynomial (KP), tchebichef polynomial (TP) and discrete Cosine transform (DCT), respectively. The matrix representation is defined as follows:

$$R = Q_c Q_k Q_T Q_k Q_T \tag{33}$$

$$R = Q_c (Q_k Q_T)^2 = R_{DCKTKT} \tag{34}$$

where Q_K, Q_c and Q_T are matrix form of KP, DCT, and TP, respectively. **Fig. 2** shows the plot of the DCKTKT of a cameraman image of (128×128) image size, and using control parameter p = 0.5, and N = 128. Observably, the first quarter accumulates all the signal information by containing the high energies, whereas the low energies are dispersed among the other transform coefficients; therefore, this hybrid form can achieve high energy compaction as compared to other hybrid forms.

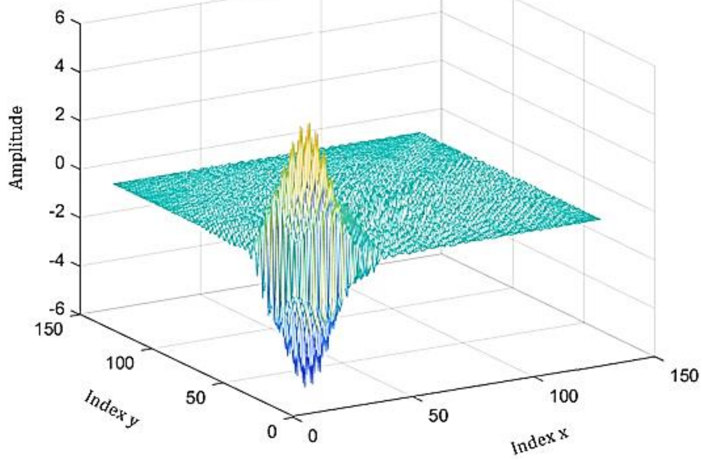

Figure 2. The representation of cameraman image in DCKTKT domain.

Fig. 3 demonstrates the process for generating DCKTKT coefficients based on DCT, KP and TP. It is essential to mention that DCKTKT has superior EC property over previous hybrid forms and does not localize in space.

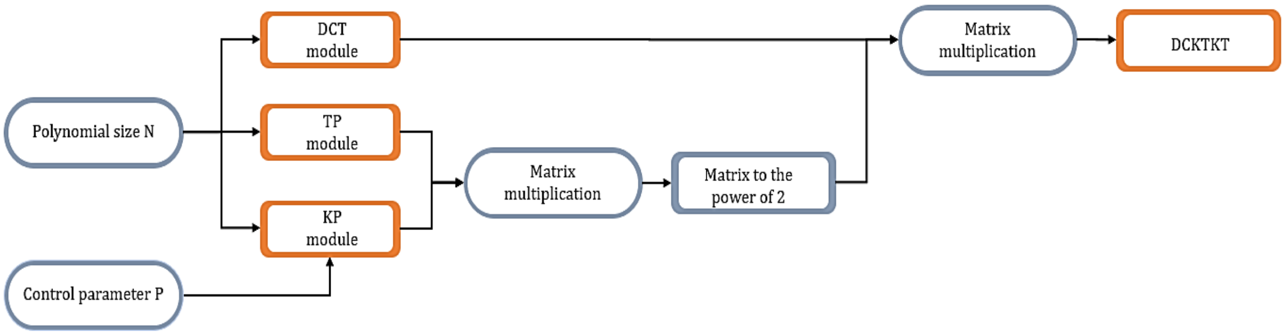

Figure 3. The flow chart of DCKTKT moments generation.

4. METHODS

In this section, the methods used to evaluate the performance of the DCKTKT hybrid form is presented.

4.1 Energy Compaction (EC)

Energy compaction is a cornerstone of any orthogonal polynomial. It is defined as the ability of a transformation to represent the signal with a small number of transform coefficients while maintaining the accuracy and quality of the signal. This property is particularly useful for data compression, noise reduction, efficient storage and transmission and feature extraction.
Using a Markov sequence procedure of the first-order, zero mean and length N to find the distribution of moment energies for DCKTKT which is based on conversion the M matrix (covariance matrix with different covariance coefficient, ρ) into a transform domain using the following equation from **(Abdulhussain et al., 2019)**:

$$T_m = R * M * R^T \tag{35}$$

Where R is any orthogonal polynomial matrix and T_m is the transform coefficients' matrix. ρ=0.8 and ρ=0.9 are two covariance coefficients that used in this study with N=8 to make a comparison between the existing polynomials (DKTK **(Mahmmod et al. 2018)**, DTKT **(Jassim and Raveendran, 2012)**, SKTP **(Abdulhussain et al., 2019)**, STKP **(Idan, 2020)**) and the proposed one in terms of the transform coefficient variance as shown in **Table 1**, for DKTT and SKTP.
The minimum values start from the edges and then gradually increase until reaching the middle, where the maximum value settles and vice versa for DTKT, STKP and DCKTKT where the maximum value settle in the middle.

Whilst, to check the EC capabilities of the existing polynomials and DCKTKT, the normalised basis restriction error J_n is used **(Zhu et al., 2010)**:

$$J_n = \frac{\sum_{q=n}^{N-1} \sigma_q^2}{\sum_{q=0}^{N-1} \sigma_q^2}, \mathrm{n} = 0, 1, \dots, \mathrm{N} - 1 \tag{36}$$

Table 1. The variance distribution of the transform coefficient when N = 8 and ρ = (0.8, 0.9).

D	ρ=0.8					ρ=0.9				
	DTKT	DKTT	STKT	SKTT	DCKTKT	DTKT	DKTT	STKT	SKTT	DCKTKT
1	2.336	0.254	3.031	0.188	2.038	2.585	0.160	3.402	0.108	2.616
2	0.659	0.676	0.457	0.349	1.246	0.571	0.568	0.257	0.187	1.248
3	0.526	1.295	0.269	1.016	2.515	0.446	1.309	0.183	0.869	2.852
4	0.479	1.774	0.242	2.446	1.087	0.398	1.964	0.158	2.836	0.677
5	0.479	1.774	0.242	2.446	0.577	0.398	1.964	0.158	2.836	0.348
6	0.526	1.295	0.269	1.016	0.260	0.446	1.309	0.183	0.869	0.126
7	0.659	0.676	0.457	0.349	0.155	0.571	0.568	0.257	0.187	0.074
8	2.336	0.254	3.031	0.188	0.122	2.585	0.160	3.402	0.108	0.058

4.2 Face Recognition Application

Face recognition technology plays a crucial role in automatically identifying or verifying individuals from images or videos. It operates in two main modes: face authentication**(Jassim and Raveendran, 2012)**, which is a one-to-one matching process, comparing a query face image with a specific template image to verify identity, and face identification **(Jassim and Raveendran, 2012)**, which involves a one-to-many comparison between the query face against multiple images in a database to determine the correct identity.

Historically, various approaches have been developed for face recognition, starting in the 1990s where the entire face is used as input for recognition and included both linear and non-linear techniques such as PCA, LDA, DCT, KPCA, and CNNs; these approaches are called holistic approaches **(Zafaruddin and Fadewar, 2014)**. While in the early 21st century, Feature-based approaches **(Chellappa et al., 1992)** gained traction by focusing on key facial features (e.g., nose, mouth, eyes) or geometric properties, employing tools like LBP, HOG, SIFT, and SURF. More recently, Hybrid approaches **(Benradi et al., 2023)** were developed, and blended these methods to leverage the strengths of both and enhance recognition performance **(Ameen et al., 2023)**.

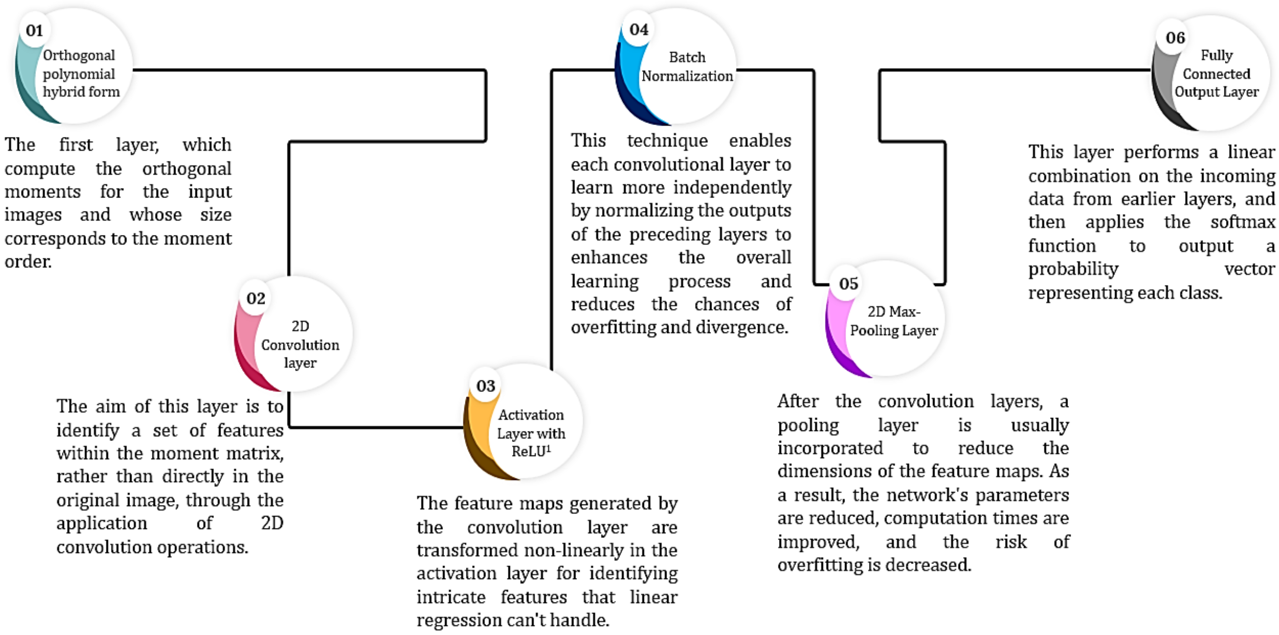

Figure 4. Flow chart of face recognition system.

On the other hand, to extract more features efficiently from facial images, orthogonal moments are used, particularly those involving Krawtchouk polynomials, due to their robustness against noise and their ability to capture both global and local features. In this work a comparison between the recent hybrid forms and the proposed one in terms of accuracy in noisy and noise free environments using ORL database. **Fig. 4** illustrates the flow chart of the face recognition implementation process that depends on CNN hybrid approach with orthogonal polynomials, which follows a similar structure to the face recognition workflow detailed in **(El Madmoune et al., 2023)**.

4.2.1 Dataset

The ORL Faces database, formerly titled 'The ORL Database of Faces,' comprises a collection of face images captured between April 1992 and April 1994 at the laboratory. It includes ten images per subject for 40 distinct individuals, with variations in lighting, facial expressions (such as open/closed eyes or smiling/not smiling), and facial details (such as wearing glasses or not). The images were consistently taken against a dark, uniform background, with subjects positioned upright and facing forward, allowing for slight lateral movement. Each image is 92x112 pixels in size, with 256 levels of gray per pixel. **Fig. 5** depicts an example of the used images.
The database is randomly divided into 70% training images and the rest for testing in the case of a noisy and noise-free environment, except that in the case of a noisy environment, the (Speckle, Poisson, Salt and Pepper, and Gaussian) types of noise are added to the testing images.

Figure 5. image examples of ORL database.

5. RESULTS AND DISCUSSION

5.1 Energy Compaction

The fewer polynomial coefficients used in signal reconstruction, the better, and this is achieved through the EC property using the restriction error equation (36). **Figs. 6 and 7** compare the existing polynomials and the proposed polynomial in terms of Jn against n, where the covariance coefficient ρ is 0.8 and 0.9. This shows that DCKTKT exceeds the recent hybrid form in performance, achieving the minimum Jn value faster. As a result, DCKTKT's superior ability to concentrate signal energy into fewer coefficients leads to higher efficiency and reduced computational complexity. Adequate energy compaction guarantees that the majority of signal information is captured in a smaller subset of moments, leading to faster reconstruction and better preservation of essential features with minimal loss.

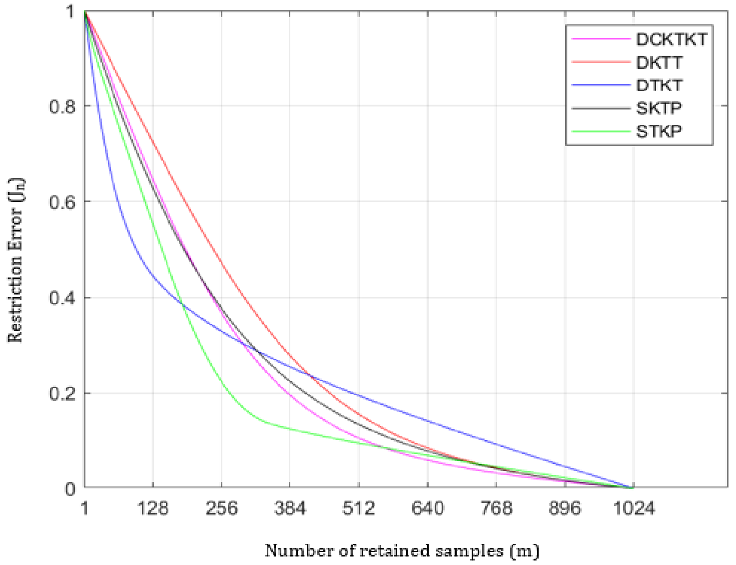

Figure 6. Restriction error comparison of the proposed polynomial and the recent polynomials (ρ = 0.8).

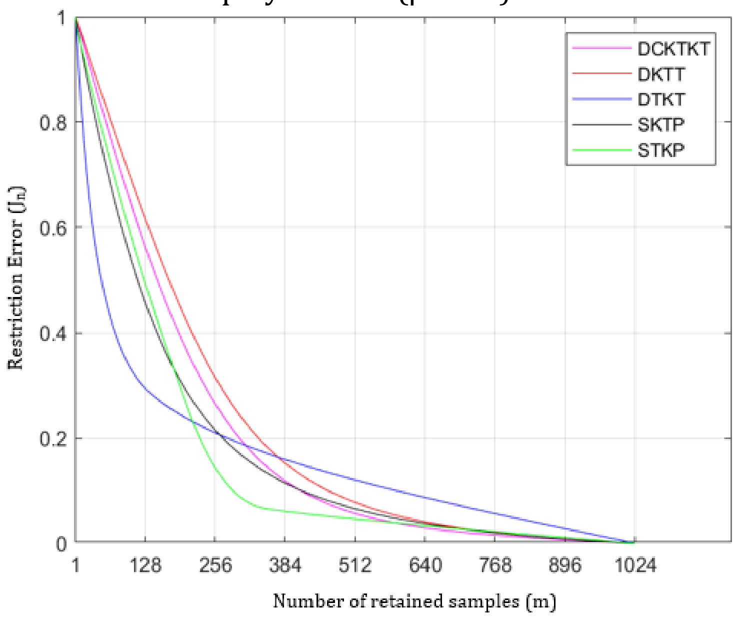

Figure 7. Restriction error comparison of the proposed polynomial and the recent polynomials (ρ = 0.9).

5.2 Face Recognition Application

Two benefits will be obtained by combining orthogonal polynomials with the CNN module in the face recognition application. The first is the reduction in the processing complexity, and the second is an increase in the computational speed **(El Madmoune et al., 2023)**.

For noise-free environment and with different orders of moments' selection, as shown in **Table 2**, which illustrates the accuracies of hybrid forms in different orders of moments' selection, The accuracy can be computed using the following formula:

$$Accuracy = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Predictions}} \tag{37}$$

the DCKTKT shows improved results, especially when the moments' orders become smaller; At order (30×30) the DCKTKT achieve 90.00% also at order (40×40) the accuracy becomes 95.00%, which lead to fewer data to be processed and less execution time.

On the other hand and as shown in **Fig. 8**, with the addition of noise to the images, for example (Speckle, Poisson, Salt and Pepper, and Gaussian) at different rates (0.05 and 0.01), the DCKTKT polynomial shows remarkable accuracy results at order (30×30) as shown in **Table 3**, and especially with speckle noise, whilst at order (40×40) and as shown in **Table 4**, the DCKTKT is superior to other polynomials in terms of salt and pepper noise the accuracy is 95.00% Which is similar to the accuracy result of noise-free environment. Also, when adding a Poisson noise, the DCKTKT polynomial achieved high results, 93.75%. This confirms that DCKTKT is highly noise-tolerant and successfully captures the essential features needed for face recognition. It also achieves the most remarkable recognition performance, even under the effect of noise.

Table 2. Classification accuracies of recent hybrid forms with DCKTKT in different orders.

Order	**SKTP**	**STKP**	**DTKT**	**DKTT**	**DCKTKT**
30	81.25%	78.75%	57.50%	83.75%	90.00%
40	82.50%	80.00%	63.75%	88.75%	95.00%
50	86.25%	86.25%	62.50%	92.50%	88.75%

Table 3. Classification accuracies of recent hybrid forms with DCKTKT in (order = 30).

	SKTP	**STKP**	**DTKT**	**DKTT**	**DCKTKT**
Speckle 6^2 = 0.05	86.25%	77.50%	38.75%	77.50%	90.00%
Speckle 6^2 = 0.01	85.00%	75.00%	53.75%	82.50%	91.25%
Poisson	78.75%	82.50%	58.75%	85.00%	88.75%
Salt and Pepper-d = 0.05	77.50%	71.25%	27.50%	76.25%	81.25%
Salt and Pepper-d = 0.01	80.00%	73.75%	53.75%	82.50%	88.75%
Gaussian = 0.05	72.50%	51.25%	7.50%	58.75%	60.00%
Gaussian = 0.01	81.25%	71.25%	36.25%	82.50%	85.00%

Table 4. Classification accuracies of recent hybrid forms with DCKTKT in (order = 40).

	SKTP	**STKP**	**DTKT**	**DKTT**	**DCKTKT**
Speckle 6^2 = 0.05	80.00%	73.75%	30.00%	82.50%	90.00%
Speckle 6^2 = 0.01	80.00%	81.25%	61.25%	88.75%	91.25%
Poisson	83.75%	76.25%	58.75%	85.00%	93.75%
Salt and Pepper-d = 0.05	83.75%	63.75%	23.75%	87.50%	85.00%
Salt and Pepper-d = 0.01	81.25%	80.00%	56.25%	90.00%	95.00%
Gaussian = 0.05	76.25%	40.00%	7.50%	77.50%	72.50%
Gaussian = 0.01	82.50%	66.25%	35.00%	90.00%	91.25%

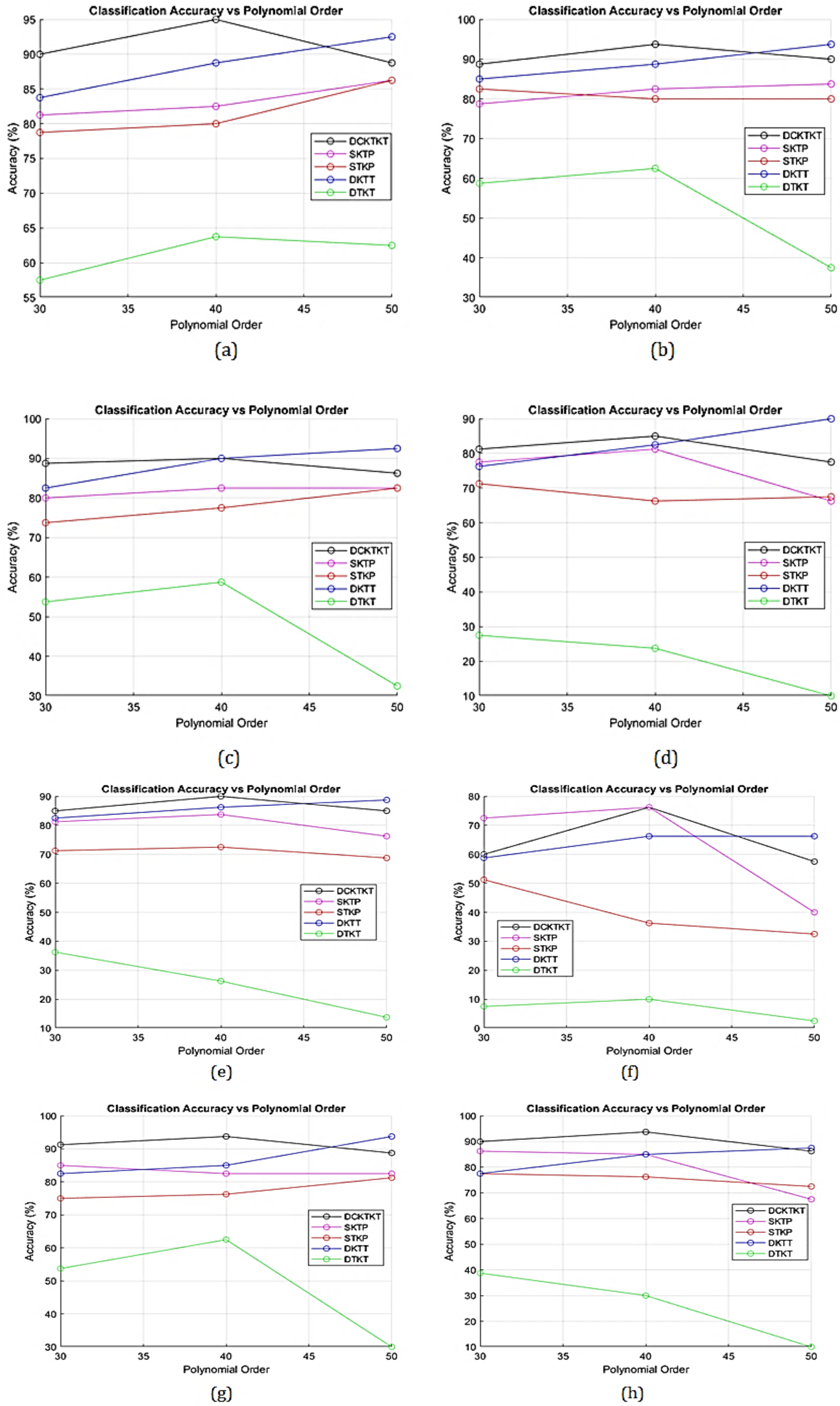

Figure 8. Classification accuracy for different orders of different hybrid forms with DCKTKT. (a) noise free environment, ((b) poisson, (c) 1% salt and pepper, (d) 5% salt and pepper, (e) 1% Gaussian, (f) 5% Gaussian, (g) 1% spacle, (h) 5% spacle) noises.

In summary, the study reveals that by combining different polynomials such as TP, KP, and DCT, the resulting hybrid form can leverage the strengths of each polynomial. For instance, the discrete cosine transform has a superior energy compaction property, distinguishing it from other polynomials, and the krawtchouk polynomial has a high localization, giving it a high ability to extract features. Therefore, the proposed hybrid form has superior energy compaction, and with improved feature extraction property, the proposed polynomial can more effectively focus on relevant information, resulting in a more precise and resilient face recognition system capable of identifying subtle variations in facial features under various conditions, whether noise-free or noisy. Furthermore, this hybrid form minimizes redundancy in data representation, optimizing resource utilization and improving the overall recognition performance.

6. CONCLUSIONS

This paper proposes a new hybrid form of orthogonal polynomials along with their corresponding moments. The proposed hybrid form, named DCKTKT, is derived from three OPs: DKP, DTP, and DCT. The results show that DCKTKT excels in energy compaction compared to existing hybrid forms. To assess its effectiveness, a Face Recognition system was implemented as an application. The integration of orthogonal polynomials with a CNN module in the Face Recognition system achieved remarkable accuracy, surpassing SKTP, STKP, DTKT, and DKTT in both clean and noisy environments. Thus, the proposed DCKTKT hybrid OP exhibits superior performance and holds significant promise in signal feature extraction. Future work will focus on applying DCKTKT and its transform domain to various computer vision fields, particularly image compression, due to its high energy compaction properties.

NOMENCLATURE

Symbol	Description	Symbol	Description
DCKTKT	Discrete Cosine-Krawtchouk–Tchebichef transform	KP	Krawtchouk Polynomial
DCT	Discrete Cosine Transform	TP	Tchebichef Polynomial
EC	Energy Compaction	STKP	Squared Tchebichef Krawtchouk Polynomial
DKT	Discrete Krawtchouk Transform	DTKT	Discrete Tchebichef Krawtchouk Transform
DTT	Discrete Tchebichef Transforms	CNN	Convolutional Neural Network
OP	Orthogonal Polynomial	TTR	Three Term Recurance Relation

Acknowledgements

The authors are grateful to the Department of Computer Engineering, College of Engineering, University of Baghdad, for their unwavering academic support and provision of necessary resources.

Credit Authorship Contribution Statement

Mariam Taha Yaseen: Writing –review & editing, Writing –original draft, Methodology, Validation, Software. Sadiq H. Abdulhussain: Writing –review & editing, Supervision, Methodology, Reviewing & support.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

Abbas, N. H.,2005. Implementations of 8x8 DCT and IDCT on different FPGA technologies using the modified loeffler algorithm. *Journal of Engineering*, 11(4), pp. 707-714. https://doi.org/10.31026/j.eng.2005.04.08.

Abdul-Hadi, A. M., Abdulhussain, S. H., and Mahmmod, B. M., 2020. On the computational aspects of Charlier polynomials. *Cogent Engineering*, 7(1). https://doi.org/10.1080/23311916.2020.1763553.

Abdulhussain, S. H., Mahmmod, B. M., Naser, M.A., Alsabah, M. Q., Ali, A., and Al-Haddad, S. A. R., 2021. A robust handwritten numeral recognition using hybrid orthogonal polynomials and moments. *Sensors*, 21(6), pp. 1-18. https://doi.org/10.3390/s21061999.

Abdulhussain, S. H., Ramli, A. R., Al-Haddad, S. A. R., Mahmmod, B. M., and Jassim, W. A., 2017. On computational aspects of Tchebichef polynomials for higher polynomial order. *IEEE Access*, 5(c), pp. 2470-2478. https://doi.org/10.1109/ACCESS.2017.2669218.

Abdulhussain, S. H., Ramli, A. R., Al-Haddad, S. A. R., Mahmmod, B. M., and Jassim, W. A., 2018. Fast recursive computation of Krawtchouk polynomials. *Journal of Mathematical Imaging and Vision*, 60(3), pp. 285–303. https://doi.org/10.1007/s10851-017-0758-9.

Abdulhussain, S. H., Ramli, A. R., Mahmmod, B. M., Saripan, M. I., Al-Haddad, S. A. R., and Jassim, W. A., 2019. A new hybrid form of Krawtchouk and Tchebichef polynomials: Design and application. *Journal of Mathematical Imaging and Vision*, 61(4), pp. 555–70. https://doi.org/10.1007/s10851-018-0863-4.

Abood, Z. I., 2013. Image compression using 3-D two-level technique. *Journal of Engineering*, 19(11 SE-Articles), pp. 1407–24. https://doi.org/10.31026/j.eng.2013.11.05.

Ameen, M. W., Abbas, A. I., and Alwan, N. A. S., 2023. Face identification using back-propagation adaptive multiwavenet. *Journal of Engineering*, 18(03), pp. (392–402). https://doi.org/10.31026/j.eng.2012.03.12.

Asli, B. H. S., and Flusser, J., 2014. Fast computation of Krawtchouk moments. *Information Sciences*, 288(1), pp. 73–86. https://doi.org/10.1016/j.ins.2014.07.046.

Benradi, H., Chater, A., and Lasfar, A., 2023. A hybrid approach for face recognition using a convolutional neural network combined with feature extraction techniques. *IAES International Journal of Artificial Intelligence*, 12(2), pp. 627–40. https://doi.org/10.11591/ijai.v12.i2.pp627-640.

Chellappa, R., Manjunath, B. S., and Malsburg, C. V. D., 1992. A feature based approach to face recognition. In *Proceeding of 1992 IEEE Conference on Computer Vision and Pattern Recognition*. Champaign, IL, USA. (pp. 373-378). IEEE, 373–78. https://doi.org/ 10.1109/CVPR.1992.223162.

El Madmoune, Y., El Ouariachi, I., Zenkouar, K., and Zahi, A., 2023. Robust face recognition using convolutional neural networks combined with Krawtchouk moments. *International Journal of Electrical and Computer Engineering*, 13(4), pp. (4052–67).

https://doi.org/10.11591/ijece.v13i4.pp4052-4067.

Feinsilver, P., and Kocik, J., 2005. Krawtchouk polynomials and Krawtchouk matrices. Recent Advances in Applied Probability, Springer, Boston, *MA Publisher*, pp. 115–41. https://doi.org/10.1007/0-387-23394-6_5.

Hu, M. K., 1962. Visual pattern recognition by moment invariants. *IEEE Transactions on Information Theory*, 8(2), pp. (179–87). https://doi.org/10.1109/TIT.1962.1057692.

Hussein, H. A., Hameed, S. M., Mahmmod, B. M., Abdulhussain, S. H., and Hussain, A. J., 2023. Dual stages of speech enhancement algorithm based on super gaussian speech models. *Journal of Engineering*, 29(09), pp. 1–13. https://doi.org/10.31026/j.eng.2023.09.01.

Idan, Z. N., Abdulhussain, S. H., and Al-Haddad, S. A. R., 2020. A new separable moments based on Tchebichef-Krawtchouk polynomials. *IEEE Access* 8, pp. 41013–25. https://doi.org/10.1109/ACCESS.2020.2977305.

Jain, A. K., 1989. Fundamentals of digital image processing. *Prentice-Hall*, Inc. ISBN- 0133361659.

Jassim, W. A., Raveendran, P., and Mukundan, R., 2012. New orthogonal polynomials for speech signal and image processing. *IET Signal Processing*, 6(8), pp.713–23. https://doi.org/10.1049/iet-spr.2011.0004.

Jassim, W. A., and Raveendran, P., 2012. Face recognition using discrete Tchebichef-Krawtchouk transform. In *Proceedings of 2012 IEEE International Symposium on Multimedia*. Irvine, CA, USA. (pp. 120–27). IEEE. https://doi.org/10.1109/ISM.2012.31.

Lu, A., and Asli, B. H. S., 2023. Seismic image identification and detection based on Tchebichef moment invariant. *Electronics*, 12(17), pp. (3692). https://doi.org/10.3390/electronics12173692.

Mahmmod, B. M., Abdul-Hadi, A. M., Abdulhussain, S. H., and Hussien, A., 2020. On computational aspects of krawtchouk polynomials for high orders. *Journal of Imaging*. 6(8), pp. 81. https://doi.org/10.3390/jimaging6080081.

Mahmmod, B. M., Abdulhussain, S. H., Suk, T., and Hussain, A., 2022. Fast computation of Hahn polynomials for high order moments. *IEEE Access*, 10, pp. (48719–32). https://doi.org/10.1109/ACCESS.2022.3170893.

Mahmmod, B. M., Ramli, A. R. B., Abdulhussain, S. H., Al-Haddad, S. A. R., and Jassim, W. A., 2018. Signal compression and enhancement using a new orthogonal-polynomial-based discrete transform. *IET Signal Processing*, 12(1), pp. (129–42). https://doi.org/10.1049/iet-spr.2016.0449.

Markandey, V., and DeFigueiredo, R. J. P., 1992. Robot sensing techniques based on high-dimensional moment invariants and tensors. *IEEE Transactions on Robotics and Automation*, 8(2), pp. (186–95). https://doi.org/10.1109/70.134273.

Mukundan, R., 2004. Some computational aspects of discrete orthonormal moments. *IEEE Transactions on Image Processing*, 13(8), pp. (1055–59). https://doi.org/10.1109/TIP.2004.828430.

Mukundan, R., Ong, S. H., and Lee, P. A., 2001. Image analysis by Tchebichef moments. *IEEE Transactions on Image Processing*, 10(9), pp. (1357–64). https://doi.org/10.1109/83.941859.

Pachori, R. B., 2023. Time-Frequency analysis techniques and their applications time-frequency analysis techniques and their applications. ISBN-9781003367987.

Pee, C. Y., Ong, S. H., and Raveendran, P., 2017. Numerically efficient algorithms for anisotropic scale and translation Tchebichef moment invariants. *Pattern Recognition Letters*, 92, pp. (68–74). https://doi.org/10.1016/j.patrec.2017.04.008.

Radeaf, H. S., Mahmmod, B. M., Abdulhussain, S. H., and Al-Jumaeily, D., 2019. A steganography based on orthogonal moments. In *Proceedings of the International Conference on Information and Communication Technology*. Baghdad, Iraq. (pp. 147–153). Association for Computing Machinery. https://doi.org/10.1145/3321289.3321324.

Tahiri, M. A., Karmouni, H., Sayyouri, M., and Qjidaa, H., 2022. 2D and 3D image localization, compression and reconstruction using new hybrid moments. *Multidimensional Systems and Signal Processing*, 33(3), pp. (769-806). https://doi.org/10.1007/s11045-021-00810-y.

Walmsley, N. P., Skodras, A. N., and Curtis, K. M., 1994. A fast picture compression technique. *IEEE Transactions on Consumer Electronics,* 40(1), pp. (11–19). https://doi.org/10.1109/30.273656.

Wang, M., and Shang, X., 2020. A fast image fusion with discrete Cosine transform. *IEEE Signal Processing Letters*, 27(8), pp. (990–94). https://doi.org/10.1109/LSP.2020.2999788.

Wang, Y., Vilermo, M., and Leonid, Y., 2000. Energy compaction property of the MDCT in comparison with other transforms. *journal of the audio engineering society,* (5178). https://aes2.org/publications/elibrary-page/?id=9160

Wu, H., and Yan, S., 2016. Computing invariants of Tchebichef moments for shape based image retrieval. *Neurocomputing*, 215, pp. (110-117). https://dx.doi.org/10.1016/j.neucom.2015.05.147.

Yap, P. T., Paramesran, R., and Ong, S. H., 2003. Image analysis by Krawtchouk moments. *IEEE Transactions on Image Processing,* 12(11), pp. (1367–77). https://dx.doi.org/10.1109/TIP.2003.818019.

Yap, P. T., Paramesran, R., and Ong, S. H., 2007. Image analysis using Hahn moments. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 29(11), pp. (2057–62). https://doi.org/10.1109/TPAMI.2007.70709.

Yaru, X., and Xiaohong, C., 2009. Random noise attenuation based on orthogonal polynomials transform. *SEG Technical Program Expanded Abstracts*, 28(1), pp. (3377–80). https://doi.org/10.1190/1.3255562.

Zafaruddin, G. M., and Fadewar, H. S., 2014. Face recognition: a holistic approach review. In *Proceedings of 2014 International Conference on Contemporary Computing and Informatics. (pp.*175-178*). IEEE.IC3I 2014*: 175–78. https://doi.org/10.1109/IC3I.2014.7019610.

Zhu, H., Liu, M., Shu, H., Zhang, H., and Luo, L., 2010. General form for obtaining discrete orthogonal moments. *IET Image Processing,* 4(5), pp. (335–52). https://doi.org/10.1049/iet-ipr.2009.0195.

Zhu, H., Shu, H., Xia, T., Luo, L., and Coatrieux, J. L., 2007. Translation and scale invariants of Tchebichef moments. *pattern recognition*, 40(9), pp. (2530–42). https://doi.org/10.1016/j.patcog.2006.12.003.

تحويل متقطع جديد يجمع بين جيب التمام وكراوتشوك وتشيبشيف :DCKTKT

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الخلاصة

في مجال معالجة الصــور الرقمية ومعالجة إشــارات الصــوت، تتميز كثيرات الحدود المتعامدة بدور مهم في تحليل واســتخراج المميزات من الإشارات بفضل خصائصها الفريدة وقدراتها العالية. لفهم وتحليل أكثر لخصائص الإشارات توجه الباحثون لأيجاد معادلات جديده لكثيرات الحدود المتعامده تحمل مواصـفات أكثر تطور، حيث يعرض هذا البحث نوعًا هجينًا جديدًا (Hybrid form) من كثيرات الحدود المتعامدة يرمز له ب (DCKTKT) يعتمد بشكل رئيسي على دالة الجيب التمام (DCT) بالإضـافة الى متعددة الحدود كروتشـوك (KrP) ومتعددة الحدود تشـيبشـيف (TcP). يتم في هذا البحث تقديم الأطر الرياضـية والنظرية للمعادلة الهجينة المقترحة. علاوة على ذلك، تم التركيز على تقييم خصــــائص المعادلة المقترحه من ناحية تجميع الطاقات (Energy compaction EC) والقدرة على اســتخراج مميزات أكثر من الاشـــارة (صـــوت وصـــورة)، حيث اثبتت المعادلة المقترحة كفائتها مقارنةً مع معادلات هجينة مقترحة سابقاً في استخراج المميزات وهي الأفضل في تجميع الطاقة. بالإضافة الى ذلك تم تطبيق تقنية التعرف على الوجوه (Face recognition application) كدراسـة حالة لأظهار فعالية المعادلة المقترحة واسـتخدامها كأحد أدوات اسـتخراج الميزات والتعرف على الوجوه، وأثبتت المعادلة المقترحة انها الأفضـل في بيئة مشـوشـة او خالية من التشويش (الضوضاء).

الكلمات المفتاحية: اكتب دالة الجيب تمام، متعددة الحدود الهجينه، تقنية التعرف على الوجوه، كثيرات الحدود المتعامدة.