University of Baghdad
College of Engineering
JOURNAL OF ENGINEERING

Journal of Engineering

journal homepage: www.jcoeng.edu.iq

Volume 31 Number 6 June 2025

Oscillation in the Food Rations for Neutral Differential Equation with Piecewise Constant

Sora Ali Majeed [1,*], **Hussain Ali Mohamad** [2]

[1]Department of Computer Science, College of Computer Science, University of Technology, Baghdad, Iraq
[2]Department of Mathematics Science, College of Science for Women, University of Baghdad, Baghdad, Iraq

ABSTRACT

There are numerous real-world applications for delay differential equations, including engineering model systems with time delays, such as control systems and communication networks, time-limited meals, blood pressure, hemopoiesis, and others, especially when the oscillation in these equations is exploited. To fulfill the goal of this study, certain of the coefficients in the first-order logistic equation must be piecewise continuous. This can only be accomplished by using the delay differential equations with the piecewise constant argument to investigate the oscillation or nonoscillation property of all first-order logistic equation solutions. The solution's piecewise constant is the largest integer function. Using techniques such as transforming the non-linear delay differential equation to a linear delay differential equation and then using integral inequality, we provide adequate circumstances for all solutions to oscillate. To ensure all solutions, required and adequate conditions have been defined. After that, looking at an example shows how the oscillation of the food-limited equation. Also, the figures appearing at the end of examples show more explanation.

Keywords: Delay, Differential equation, Logistic equation, Oscillation, Piecewise constant.

1. INTRODUCTION

Due to their widespread application in a variety of fields, including biology, ecology, engineering, communications, and others, delay differential equations are regarded as one of the most significant categories of food limited **(Cooke, 1984; Mahmoud and Dheyaa, 2013; Ali and Al-Zughaibi, 2024)**. Oscillation of erythrocytosis and anemia were studied in females and males in equilibrium**(Hadeed and Mohamad, 2024; Hadeed and Mohamad, 2024)**. First-order nonlinear neutral differential equations with multiple non-monotonic delays and several variable coefficients with influential terms are discussed and dependent on many essential theorems in delay **(Yuan, 2001; Papaschinopoulos and Schinas, 2007; Qaraad et al., 2022; Abbas and Mohamad, 2023)**. The existence of a

*Corresponding author
Peer review under the responsibility of University of Baghdad.
https://doi.org/10.31026/j.eng.2025.06.05

Article received: 13/11/2024
Article revised: 20/01/2025
Article accepted: 15/03/2025
Article published: 01/06/2025

solution to the nonlinear differential-difference problem is discussed **(Jones, 1962)** . The speed of propagation of solutions is then explored using initial data supported by a compact form of the differential equation, $\dot{\varphi}(\omega) = r(\omega)\varphi(\omega)\left(1 - \frac{\varphi(\tau(\omega))}{k}\right), \tau(\omega) < \omega$, known as Hutchinson's equation, $r(\omega) \geq 0$ and K is positive constant. Hutchinson's equation has been investigated by several authors, who have provided appropriate conditions to ensure the oscillation of each solution as well as conditions to ensure the convergence of all non-oscillatory solutions of that equation **(Gopalsamy et al., 1988; Gopalsamy, 1992)**. **(Bazighifa et al., 2020)** introduced a new oscillation criteria for the solutions of even-order neutral differential equations with a p-Laplacian like operator. **(Chatzarakis and Logaarasi, 2023) i**nvestigated the forced oscillation of impulsive fractional partial differential equations. Sufficient conditions are established which guarantee the solutions' oscillation by applying the integral transformation technique and differential inequality method. Also, an example is shown to illustrate them. Some oscillation conditions for the solution of the non-autonomous food-limited equation with constant and variable delays were discussed in **(Berezensky and Braveman, 2003; Yuji and Weigao, 2003; Dou and Li, 2011; Abdulhamid, 2012; Tian and An, 2023)**. The oscillation criteria for the delay equation or neutral differential equations with the piecewise constant argument are discussed, where some conditions are set to ensure the oscillation for every solution of these equations **(Aftabizadeh et al., 1987; Partheniadis, 1988; Agwo, 1998; Yuan, 2001; Zhiguo and Jianhua, 2003; Wang and Cheng, 2009; Zhang and Hong-Xu, 2011; Muminov and Radjabov, 2024)**. The oscillation properties and asymptotic behavior of solutions of neutral differential equations under the influence of impulses have been investigated, and some appropriate conditions have been obtained to ensure the oscillation of each solution of these equations **(Mohamad and Jaddoa, 2020a; Mohamad and Jaddoa, 2020b)**.

In this paper, some conditions are produced to ensure the oscillation of the logistic equation, also when using some conditions for the function $r(\omega)$ to translate the logistic equation to time delay limited food and find the oscillation to it.

$$\varphi'(\omega) - r(\omega)\varphi(\omega)\left(\alpha(\omega) - \sum_{i=1}^{n} \beta_i \,\varphi(\omega - \tau_i) - \varphi[\omega - 1]\right) = 0. \tag{1}$$

Where $r \in C\big[[t_0, \infty); R^+\big]$, $\alpha \in PC\big[[t_0, \infty); R^+\big]$, $\beta_i, \tau_i \in (0, \infty)$, $[.]$ Is the greatest integer function, PC is the space of all piecewise continuous functions. For time, $\omega \in [k.k+1), k = 0,1,2, \ldots$, Eq. (1) becomes

$$\varphi'(\omega) = r(\omega)\varphi(\omega)(\alpha(\omega) - \sum_{i=1}^{n} \beta_i \,\varphi(\omega - \tau_i) - \varphi(k-1)). \tag{2}$$

A function $\varphi(\omega)$ is considered to be a solution of Eq. (1) if $\varphi(\omega) \in C^1[[\omega_0 - \tau, \infty); R]$, with the potential exception of the points $[\omega] \in [0, \infty)$ where one-sided derivatives exist, and Eq. (1) satisfied by $\varphi(\omega)$ as a solution **(Cooke, 1984)**.

The first-order impulsive neutral differential equations were explored, and some impulsive conditions were discovered to ensure the oscillation of all solutions to these equations **(Mohamad and Jaddoa, 2020a)**. The oscillation criteria were examined for all solutions to first-order linear neutral differential equations with positive and negative coefficients.

A solution $\varphi(\omega)$ Is said to oscillate if there exists a sequence $\{\omega_m\}, \omega_m \to \infty$ as $m \to \infty$ such that $\varphi(\omega_m) = 0$.

The graphing of the figures shown in this paper was used the mathway graph in the internet.

2. MATERIALS AND METHODS

In this section, three results are obtained for the oscillation of every solution of Eq. (1). The unique positive equilibrium, or what we can call the steady state of Eq. (2), can be calculated as follows: let at equilibrium, $\varphi(\omega) = \varphi(\omega - \tau_i) = \varphi(k-1) = \varphi^*$ and $\alpha(\omega) = \alpha^*$ hence

$$\varphi^* = \frac{\alpha^*}{1 + \sum_{i=1}^{n} \beta_i} . \tag{3}$$

Theorem 1. Assume that $0 < \alpha(\omega) \le \varphi^*(\sum_{i=1}^{n} \beta_i + 1)$, and

$$\lim_{k\to\infty} \sup \int_k^{k+1} r(s)\, ds > \frac{1}{\varphi^*} . \tag{4}$$

Then every solution of Eq. (2) oscillates about its equilibrium φ^*.

Proof: Suppose that Eq. (2) possesses a nonoscillatory solution $\varphi(\omega)$, let $\varphi(\omega) > \varphi^*$, $\varphi(\omega - \tau_i) > \varphi^*$, $\varphi(k-1) > \varphi^*$, for time, $\omega \in [k, k+1)$, $k = 0,1,2,\cdots$. (for the case $\varphi(\omega) < \varphi^*$, the proof can be treated similarly).

Let $x(\omega) = \ln \frac{\varphi(\omega)}{\varphi^*}$, then $x(\omega) > 0$, $e^{x(\omega)} \ge 1$, and $x(\omega) = 0$ if and only if $\varphi(\omega) = \varphi^*$, that is $x(\omega)$ is oscillating if and only if $\varphi(\omega)$ oscillates about equilibrium φ^*, from $\varphi(\omega) = \varphi^* e^{x(\omega)}$ yields $\varphi'(\omega) = \varphi^* x'(\omega) e^{x(\omega)}$, hence Eq. (2) leads to

$$x'(\omega) = r(\omega)(\alpha(\omega) - \sum_{i=1}^{n} \beta_i \varphi^* e^{x(\omega - \tau_i)} - \varphi^* e^{x(k-1)}). \tag{5}$$

By using $\alpha(\omega) - \sum_{i=1}^{n} \beta_i \varphi^* - \varphi^* \le 0$, Eq. (5) reduce to

$$x'(\omega) \le r(\omega)\left(\alpha(\omega) - \varphi^*\left(1 + \sum_{i=1}^{n} \beta_i\right)\right) \le 0.$$

Therefore, $x(\omega)$ is a nonincreasing function.

Integrating Eq. (5), from k to $k+1$ one can conclude.

$$x(k+1) - x(k) = \int_k^{k+1} \alpha(\omega) r(\omega)\, d\omega - \varphi^* \int_k^{k+1} r(\omega) \sum_{i=1}^{n} \beta_i\, e^{x(\omega - \tau_i)} d\omega - \varphi^* e^{x(k-1)} \int_k^{k+1} r(\omega) d\omega. \tag{6}$$

Since $\alpha(\omega) \le \varphi^*(\sum_{i=1}^{n} \beta_i + 1)$, $e^{x(\omega)} \ge 1$, $e^{x(\omega)} \ge 1 + x(\omega)$, then Eq. (6), leads to

$$\begin{aligned} x(k+1) - x(k) &\le \varphi^* \sum_{i=1}^{n} \beta_i \int_k^{k+1} r(\omega) d\omega + \varphi^* \sum_{i=1}^{n} \beta_i \int_k^{k+1} r(\omega) d\omega \\ &- \varphi^* \sum_{i=1}^{n} \beta_i \int_k^{k+1} r(\omega) d\omega - \varphi^* e^{x(k-1)} \int_k^{k+1} r(\omega) d\omega, \\ &\le \varphi^* \sum_{i=1}^{n} \beta_i \int_k^{k+1} r(\omega) d\omega - \varphi^*(1 + x(k-1)) \int_k^{k+1} r(\omega) d\omega, \end{aligned}$$

$$\leq -\varphi^* x(k-1)\int_k^{k+1} r(\omega)d\omega,$$

$$x(k+1) \leq x(k) - \varphi^* x(k)\int_k^{k+1} r(\omega)d\omega,$$

$$\leq x(k)\left(1 - \varphi^*\int_k^{k+1} r(\omega)d\omega\right).$$

Since $x(k+1) > 0$ and $x(k) > 0$, then

$$1 - \varphi^*\int_k^{k+1} r(\omega)d\omega > 0. \tag{7}$$

Letting $k \to \infty$, the inequality (7) leads to a contradiction with condition (4).

Theorem 2. Suppose that $\alpha(\omega) \geq \varphi^*(\sum_{i=1}^{n}\beta_i + 1)$, and

$$\limsup_{k\to\infty}\int_k^{k+1} r(s)\,ds > \frac{\delta}{\varphi^*}. \tag{8}$$

For some $\delta > 0$. Then every solution of Eq. (2) oscillates about its equilibrium φ^*.

Proof: Suppose that Eq. (2) possesses a nonoscillatory solution $\varphi(\omega)$. Let $x(\omega) = \ln\frac{\varphi(\omega)}{\varphi^*}$, so $x(\omega) < 0$, if $\varphi(\omega) < \varphi^*$ or $x(\omega) > 0$, if $\varphi(\omega) > \varphi^*$, and $x(\omega) = 0$ if and only if $\varphi(\omega) = \varphi^*$, this means that $x(\omega)$ is oscillatory if and only if $\varphi(\omega)$ oscillates about equilibrium φ^*. Let $\varphi(\omega) < \varphi^*$, $\varphi(\omega - \tau_i) < \varphi^*$, $\varphi(k-1) < \varphi^*$, for time, $\omega \in [k, k+1)$, $k = 0,1,2,\cdots$, (for the case $\varphi(\omega) > \varphi^*$, the proof can be treated similarly). It follows that $e^{x(\omega)} \leq 1$, and from

$\varphi(\omega) = \varphi^* e^{x(\omega)}$ yields $\varphi'(\omega) = \varphi^* x'(\omega)e^{x(\omega)}$, hence Eq. (2) leads to

$$x'(\omega) = r(\omega)\left(\alpha(\omega) - \sum_{i=1}^{n}\beta_i\varphi^* e^{x(\omega-\tau_i)} - \varphi^* e^{x(k-1)}\right), \tag{9}$$

Or

$$x'(\omega) \geq r(\omega)(\alpha(\omega) - \sum_{i=1}^{n}\beta_i\varphi^* - \varphi^*). \tag{10}$$

Since $\alpha(\omega) - \sum_{i=1}^{n}\beta_i\varphi^* - \varphi^* \geq 0$, it can be concluded from (9) that $x(\omega)$ is a non-decreasing function. Integrating Eq. (9), from k to $k+1$ one can conclude.

$x(k+1) - x(k) = \int_k^{k+1}\alpha(\omega)r(\omega)\,d\omega - \varphi^*\int_k^{k+1} r(\omega)\sum_{i=1}^{n}\beta_i\, e^{x(\omega-\tau_i)}d\omega -$
$\varphi^* e^{x(k-1)}\int_k^{k+1} r(\omega)d\omega.$

Since $\alpha(\omega) \geq \varphi^*(\sum_{i=1}^{n}\beta_i + 1)$, $e^{x(\omega)} \leq 1$, $e^{x(\omega)} \leq 1 + \frac{1}{x(\omega)}$ Then Eq. (10) leads to (11)

$$x(k+1)-x(k)$$
$$\geq \varphi^*(\sum_{i=1}^{n}\beta_i + 1)\int_k^{k+1} r(\omega)d\omega - \varphi^*\sum_{i=1}^{n}\beta_i\int_k^{k+1} r(\omega)d\omega$$
$$-\varphi^* e^{x(k-1)}\int_k^{k+1} r(\omega)d\omega,$$

$$\geq \varphi^*\int_k^{k+1} r(\omega)d\omega - \varphi^*(1+\frac{1}{x(k-1)})\int_k^{k+1} r(\omega)d\omega,$$

$$= -\frac{\varphi^*}{x(k-1)}\int_k^{k+1} r(\omega)d\omega,$$

$$x(k+1) = x(k) - \frac{\varphi^*}{x(k)}\int_k^{k+1} r(\omega)d\omega,$$

Since $x(\omega) < 0$, and nondecreasing then there is a time, $\omega_1 \in [k, k+1) > 0$ such that for any $L \geq -x(\omega_1)$ or $x(\omega) \geq -L, \omega \geq \omega_1$ thus

$$x(k+1) \geq L\left(-1 + \frac{\varphi^*}{L^2}\int_k^{k+1} r(\omega)d\omega\right).$$

Since $x(k+1) < 0$ then

$$-1 + \frac{\varphi^*}{L^2}\int_k^{k+1} r(\omega)d\omega < 0.$$

When $k \to \infty$, the last inequality leads to a contradiction with condition (8).

Remark 1. Let $r^*, c \in (0, \infty)$ exists such that

$$r(\omega) = \frac{r^*}{c\sum_{i=1}^{n}\beta_i}. \quad (12)$$

For time, $\omega \in [k.k+1)$, $k = 0,1,2, \ldots$ then Eq.(2) becomes

$$x'(\omega) = r(\omega)\left(\alpha(\omega) - \sum_{i=1}^{n}\beta_i\varphi^* e^{x(\omega-\tau_i)} - \varphi^* e^{x(k-1)}\right). \quad (13)$$

The following results concern with Eq. (13).

Theorem 3. Suppose that $\alpha(\omega) \geq \varphi^*(\sum_{i=1}^{n}\beta_i + 1)$. Then every solution of Eq. (13) oscillates about its equilibrium φ^* if and only if

$$\frac{r^*\varphi^*\frac{1}{L^2}}{c\sum_{i=1}^{k}\beta_i} \geq 1. \quad (14)$$

Proof: Suppose that Eq. (2) possesses a nonoscillatory solution $\varphi(\omega)$. Let $x(\omega) = \ln\frac{\varphi(\omega)}{\varphi^*}$, so $x(\omega) < 0$, if $\varphi(\omega) < \varphi^*$ or $x(\omega) > 0$, if $\varphi(\omega) > \varphi^*$, and $x(\omega) = 0$ if and only if $\varphi(\omega) =$

φ^*, this means that $x(\omega)$ is oscillatory if and only if $\varphi(\omega)$ oscillates about equilibrium φ^*. Let $\varphi(\omega) < \varphi^*$, $\varphi(\omega - \tau_i) < \varphi^*$, $\varphi(k-1) < \varphi^*$, for time, $\omega \in [k, k+1)$, $k = 0,1,2,\cdots$ (for the case $\varphi(\omega) > \varphi^*$, the proof can be treated similarly). It follows that $e^{x(\omega)} \leq 1$, and from $\varphi(\omega) = \varphi^* e^{x(\omega)}$ yields $\varphi'(\omega) = \varphi^* x'(\omega) e^{x(\omega)}$, hence Eq. (2) leads to

$$x'(\omega) = r(\omega)\left(\alpha(\omega) - \sum_{i=1}^{n} \beta_i \varphi^* e^{x(\omega-\tau_i)} - \varphi^* e^{x(k-1)}\right), \tag{15}$$

Or

$$x'(\omega) \geq r(\omega)\left(\alpha(\omega) - \sum_{i=1}^{n} \beta_i \varphi^* - \varphi^*\right).$$

Since $\alpha(\omega) - \sum_{i=1}^{n} \beta_i \varphi^* - \varphi^* \geq 0$, it can be concluded from (15) that $x(\omega)$ is a non-decreasing, Eq. (15) reduced to

$$x'(\omega) \geq r^* \left(\frac{\alpha(\omega) - \varphi^*(1 + \sum_{i=1}^{n} \beta_i)}{c \sum_{i=1}^{k} \beta_i}\right) \geq 0.$$

Therefore, $x(\omega)$ is a non-decreasing function on $\omega \in [k, k+1)$.
Integrating Eq. (15), from k to $k+1$ one can conclude.

$$x(k+1) - x(k) = r^* \int_k^{k+1} \frac{\alpha(\omega) - \sum_{i=1}^{n} \beta_i \varphi^* e^{x(\omega-\tau_i)} - \varphi^* e^{x(k-1)}}{c \sum_{i=1}^{k} \beta_i} d\omega. \tag{16}$$

Since $\alpha(\omega) \geq \varphi^*(\sum_{i=1}^{n} \beta_i + 1)$, $e^{x(\omega)} \leq 1$,, then Eq. (15) leads to

$$x(k+1) - x(k) \geq r^* \int_k^{k+1} \frac{\varphi^*(\sum_{i=1}^{n} \beta_i + 1) - \varphi^* \sum_{i=1}^{n} \beta_i - \varphi^* e^{x(k-1)}}{c \sum_{i=1}^{k} \beta_i} d\omega,$$

$$\geq r^* \varphi^* \int_k^{k+1} \frac{1 - e^{x(k-1)}}{cr \sum_{i=1}^{k} \beta_i} d\omega \geq -\frac{r^* \varphi^* \frac{1}{x(k-1)}}{c \sum_{i=1}^{k} \beta_i}.$$

$$x(k+1) - x(k) \geq -\frac{r^* \varphi^* \frac{1}{x(k-1)}}{c \sum_{i=1}^{k} \beta_i}.$$

Since $x(\omega)$ is a non-decreasing function, r^* and $c \sum_{i=1}^{k} \beta_i$ Are positive, which yields

$$x(k+1) - x(k-1) \geq -\frac{r^* \varphi^* \frac{1}{x(k-1)}}{cr \sum_{i=1}^{k} \beta_i}.$$

$$x(k+1) \geq x(k-1)\left(1 - \frac{r^* \varphi^* \frac{1}{x^2(k-1)}}{c \sum_{i=1}^{k} \beta_i}\right).$$

Since $x(\omega) < 0$, and nondecreasing then there is a time, $\omega_1 \in [k, k+1) > 0$ such that for any $L \geq -x(\omega_1)$ or $x(\omega) \geq -L, \omega \geq \omega_1$ thus

$$x(k+1) \geq L\left(-1 + \frac{r^* \varphi^* \frac{1}{L^2}}{c \sum_{i=1}^{k} \beta_i}\right)$$

Since $x(k+1) < 0$ then

$$-1+\frac{r^*\varphi^*\frac{1}{L^2}}{c\sum_{i=1}^{k}\beta_i}\leq 0$$

which contradicts the presumption.

3. RESULTS AND DISCUSSION

In this section, finding sufficient conditions for the logistic equation solutions of first-order nonlinear delay differential equations with piecewise constant argument—whether these oscillatory solutions are convergent or non-convergent—was necessary to arrive at the results.

The following example satisfies the conditions of theorem1.

Example 1. Consider the first-order neutral differential equation with a piecewise constant

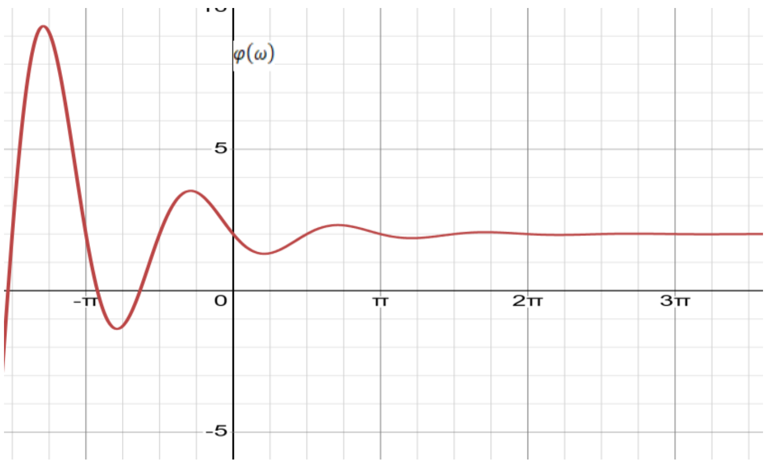
$$\varphi'(\omega)-r(\omega)\varphi(\omega)\left[\alpha(\omega)-\frac{1}{2}e^{-\frac{\pi}{2}}\varphi(\omega-\pi)-\varphi(k-1)\right]=0,\ \ k=1,2,\dots,\omega_0=0\,. \qquad (17)$$

Where $r(\omega)=\frac{3}{2-\delta e^{-\frac{\omega}{2}}\sin 2\omega}$, $\alpha(\omega)=e^{-\frac{\pi}{2}}+2-\delta e^{-\frac{k-1}{2}}\sin(2k-2)+\frac{2}{3}\delta e^{-\frac{\omega}{2}}\cos 2\omega$, $\beta=\frac{1}{6}e^{-\frac{\pi}{2}}$, $\delta\in[0,3)$.

$$r(\omega)\geq\frac{5}{4},\qquad \omega_0=0,$$

$$\int_k^{k+1}r(\omega)\,d\omega\geq\int_k^{k+1}\frac{5}{4}d\omega=\frac{5}{4}>\frac{1}{2}.$$

All conditions of theorem 1 are met, hence according to theorem 1, every solution of Eq. (17) oscillates about equilibrium and the solution $\varphi(\omega)=2-\delta e^{-\frac{\omega}{2}}\sin 2\omega$ oscillates about $\varphi^*=2$. **Fig. 1** show the solution $\varphi(\omega)$ oscillates about 2 when $\delta=1$ and $\delta=5$.



(a)

(b)

Figure 1. (a) $\delta = 1$, the solution oscillates about $\varphi^* = \frac{-7\pi}{12}$, **(b)** $\delta = 5$, the solution oscillates about $\varphi^* = \frac{5\pi}{12}$.

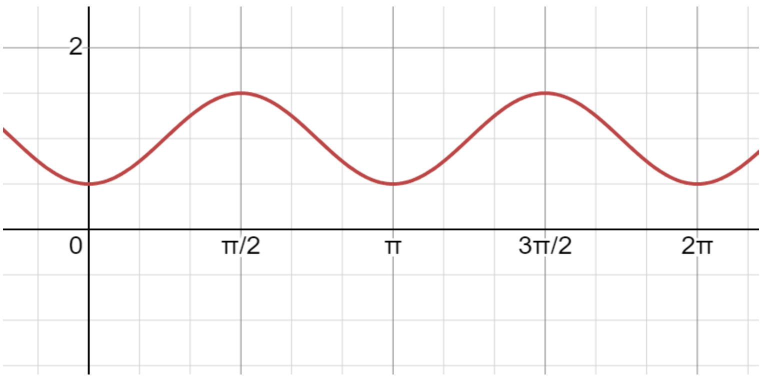
The following second example satisfies the conditions of theorem 2.

Example 2. Suppose that the first-order neutral differential equation with piecewise constant given by

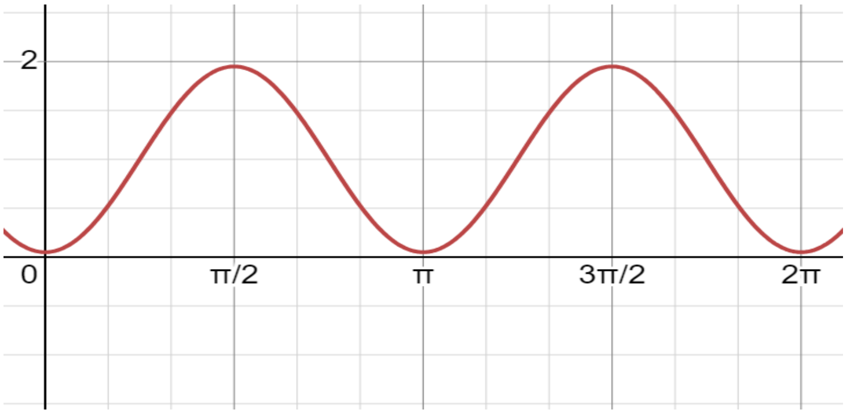
$$\varphi'(\omega) - r(\omega)\varphi(\omega)\left[\alpha(\omega) - (e^{\frac{\pi}{2}} - 1)\varphi(\omega - \pi) - (e^{\frac{\pi}{2}} + 1)\varphi(\omega - 2\pi) - \varphi(k-1)\right] = 0, \ k = 1,2,\dots, \omega_0 = 0 . \quad (18)$$

While, $r(\omega) = \frac{3}{1 - \frac{\delta}{2}\cos 2\omega}$, $\alpha(\omega) = 1 + 2e^{\frac{\pi}{2}}\left(1 - \frac{\delta}{2}\cos 2\omega\right) + \frac{\delta}{3}\sin 2\omega - \frac{\delta}{2}\cos(2k-2)$, $\beta_1 = e^{\frac{\pi}{2}} - 1$, $\beta_2 = e^{\frac{\pi}{2}} + 1$, $\delta \in [-2,2)$. $r(\omega) \geq \frac{6}{4}$, $\omega_0 = 0$, $\varphi^* = 1$.

$$\int_k^{k+1} r(\omega)\, d\omega \geq \int_k^{k+1} \frac{6}{4}\, d\omega = 1.25 > 1.$$



(a)



(b)

Figure 2. (a) $\delta = 1$, the solution oscillates about $\varphi^* = 1$.**(b)** $\delta = 1.9$, the solution oscillates about $\varphi^* = 1$.

All conditions of theorem 1 are met, hence according to theorem 1, every solution of Eq. (18) oscillates about equilibrium, the solution $\varphi(\omega) = 1 - \frac{\delta}{2}\cos 2\omega$ oscillate about $\varphi^* = 1$. **Fig. 2** shows the solution $\varphi(\omega)$ oscillates about 1 when $\delta = 1$.

Note that, the solution approach is more equilibrium when $\delta \rightarrow 0$.

The following example satisfies the conditions of theorem 3.

Example 3. Suppose that the first-order neutral differential equation with piecewise constant given by

$$\varphi'(\omega) - r(\omega)\varphi(\omega)\left[\alpha(\omega) - \varphi\left(\omega - \frac{\pi}{2}\right) - \varphi(k-1)\right] = 0, \quad k = 1,2, \dots, \omega_0 = 0. \tag{19}$$

$\varphi(\omega) = 2 - sin\omega$, While,$r(\omega) = 4, \alpha(\omega) = \frac{-0.25cos\omega}{2-sinw} + 4 - \sin(k-1) + cos\omega,\ \beta_1 = 1, c = 2, L = -1, r^* = 8, \omega_0 = 0, \varphi^* = 2$.

Clear that $\frac{r^*\varphi^*\frac{1}{L^2}}{c\sum_{i=1}^{k}\beta_i} \geq 1$.

So that, by using theorem 3, one can conclude, the function $\varphi(\omega)$ Is oscillatory, see **Fig.3**

Figure 3. The solution oscillates about $\varphi^* = 2$.

4. CONCLUSIONS

Understanding and computing all possible solutions to the first-order nonlinear logistic neutral differential equation using the piecewise constant argument. Furthermore, the oscillation trend seeks to investigate and determine the necessary and sufficient conditions for the oscillation of solutions to the first-order neutral differential equation with piecewise constant argument. In this work, we used a neutral differential equation model with piecewise constant arguments to examine the oscillatory behavior of food rations. According to our research, the model displays oscillatory behavior under specific circumstances, indicating variations in the availability and consumption of food. These findings emphasize how crucial it is to model food systems while taking temporal delays and discrete changes in the food supply into account. The results of this study can help guide resource management and food security policy decisions. Future studies should examine how other variables. Adequate conditions were drawn to create the convergence or divergence of all solutions for the first-order neutral differential equation with a piecewise constant argument for time. $\omega \to \infty$. All obtained results are presented with illustrative examples.

NOMENCLATURE

Symbol	Description	Symbol	Description
ω	The time	δ	Positive constant
$r(\omega)$	Growth rate of people at time ω	L	Positive constant
$\varphi(\omega)$	Density of people at time ω	$\varphi[\omega - 1]$	Density of people at discontinues time $\omega - 1$
$\alpha(\omega)$	Positive function at time ω	$e^{x(\omega)}$	Exponential function
$\sum_{i=1}^{n} \beta_i$	Positive constants	C	Positive constant
$\varphi(\omega - \tau_i)$	Density of people at delay time $\omega - \tau_i$	$x(\omega)$	Logarithmic function
$[.]$	Is the greatest integer function		

Acknowledgements

Appreciative of the academic assistance, College of Engineering, University of Baghdad.

Credit Authorship Contribution Statement

The authors have been read and approved the manuscript, Sora A. Majeed, writing the original draft of the manuscript, Hussain A. Mohamad, reviewed and edited the manuscript.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

REFERENCES

Abbas, N. and Mohamad, H.A., 2023. Oscillation criteria of neutral first order differential equations. *AIP Conference Proc*eeding., 2834, 080122. https://doi.org/10.1063/5.0161728.

Abdulhamid, B.M., Haruna, Y. and Muhammad, B.M., 2012. On the oscillation of the generalized food-limited equations with delay. *Aceh International Journal of Science and Technology*, *1*(3), pp. 67-72. http://doi.org/10.13170/aijst.1.3.132.

Aftabizadeh, A., Wiener, R.J. and J-M. Xu, 1987. Oscillatory and periodic solutions of delay differential equations with piecewise constant argument. *American Mathematical Society,* 99(4), pp. 673-679, https://doi.org/10.2307/2046474.

Agwo, H.A., 1998. Necessary and sufficient conditions for the oscillation of delay differential equation with piecewise constant argument. *International Journal of Math & Mathematic Science,* 21(3), pp. 493-498. http://doi.org/10.1155/S0161171298000702.

Ali, L.M. and Al-Zughaibi, A., 2024. Investigation of control behavior via active suspension system considering time-delay and variable masses. *Journal of Engineering,* 30(11), pp. 108–127. https://doi.org/10.31026/j.eng.2024.11.07.

Bazighifan, O., Grace, S.R., Alzabut. J. and Özbekler, A., 2020. New results for oscillatory properties of neutral differential equations with a p-Laplacian-like operator*. Miskolc Math.* Notes, 21, pp. 631–640, https://doi.org/10.18514/MMN.2020.3322.

Berezensky, L. N. and Braveman, E., 2003. On oscillation of a food-limited population model with time delay. *Abstract and Applied Analysis*. 2003(1), pp. 1-12. https://doi.org/10.1155/S1085337503209040.

Chatzarakis, G.E. and Logaarasi, K., 2023. Forced oscillation of impulsive fractional partial differential equations, *Partial Differential Equations in Applied Mathematics* 7, 100478.

Cooke, K.L., 1984. Retarded differential equations with piecewise constant delays*. Journal of Mathematical Analysis and Applications*, 99, pp. 265-297, https://doi.org/10.1016/0022-247X(84)90248-8.

Dou, X. and Li, Y., 2011. Almost periodic solution for a food-limited population model with delay and feedback control. *International Journal of Mathematical and Computational Sciences*, 5(8).

Gopalsamy, K., Kulenovi, M. R. S. and Ladas, G.,1988. Environmental periodicity and time delays in a ''Food-Limited'' population model. *Journal of Mathematical Analysis and Applications*, 147, pp. 545-555, https://doi.org/10.1016/0022-247X(90)90369-Q.

Gopalsamy, K., 1992. Stability and oscillations in delay differential equations of population dynamics. *Mathematics and its Applications,* 74, Kluwer Academic Publishers Group, Dordrecht.

Hadeed, I. S. and Mohamad, H. A., 2024, Oscillation of the impulsive hematopoiesis model with positive and negative coefficients. *Baghdad Science Journal*, 21(7), pp. 2403-2412 https://doi.org/10.21123/bsj.2023.8796.

Hadeed, I.S., Mohamad, H.A., 2024. Asymptotic properties and oscillation of the solutions in a periodic impulsive hematopoiesis model. *Journal of University of Babylon for Pure and Applied Sciences* (JUBPAS) 3(1).

Hasik, K., Kopfov*a,* J., Abelkov*a,* P. N. and S. Trofimchuk, 2020. On the geometric diversity of wavefronts for the scalar Kolmogorov ecological equation. *Journal Nonlinear Science,* 30, pp. 2989-3026. https://doi.org/10.1007/s00332-020-09642-9.

Jones, G.S., 1962. On the nonlinear differential-difference equation $f(x) = -\alpha f(x-1)\big(1+f(x)\big)$. *Math. Anal. Appl.* 4, pp. 440–469, https://doi.org/10.1016/0022-247X(62)90041-0.

Mahmoud, W. A. and Dheyaa, J. K., 2013. A proposal algorithm to solve delay constraint least cost optimization problem. *Journal of Engineering*, 19(1), pp. 155–160. https://doi.org/10.31026/j.eng.2013.01.09.

Moaaz, O., Albalawi, W., 2023. Differential equations of the neutral delay type: More efficient conditions for oscillation. *AIMS Math.*, *8*, pp. 2729–12750. https://doi.org/10.3934/math.2023641.

Mohamad, H.A. and Jaddoa, A. F., 2020a. Oscillation criteria for solutions of neutral differential equations of impulses effect with positive and negative coefficients. *Baghdad Science Journal,* 17(2), pp. 537-544. http://dx.doi.org/10.21123/bsj.2020.17.2.0537.

Mohamad, H.A. and Jaddoa, A. F., 2020b. Asymptotic criteria of neutral differential equations with positive and negative coefficients and impulsive integral term. *Iraqi Journal of Science*, 2020, 61(9), pp. 2315-2323. https://doi.10.24996/ijs.2020.61.9.18.

Muminov, M. I. and Radjabov, T. A., 2024. Existence conditions for 2-periodic solutions to nonhomogeneous differential equations with piecewise constant argument. *Examples and Counterexamples*, 5, pp. 100145. https://doi.org/10.1016/j.exco.2024.100145.

Papaschinopoulos, G. and Schinas, J., online: 02 May 2007. Existence stability and oscillation of the solutions of first-order neutral delay differential equations with piecewise constant argument. *Applicable Analysis*, 44 (1-2), pp. 99-111. https://doi.org/10.1080/00036819208840070.

Partheniadis, E.C., 1988. Stability and oscillation of neutral delay differential equations with piecewise constant argument. *Differential and Integral Equations*, 1(4), pp. 459-472, https://typeset.io/pdf/stability-and-oscillation-of-neutral-delay-differential-25if1m25lu.pdf.

Qaraad, B., Bazighifan, O., Nofal, T.A. and Ali, A.H., 2022. Neutral differential equations with distribution deviating arguments: Oscillation conditions. *Journal of Ocean Engineering and Science,* 2022, pp. 19-35. https://doi.org/10.1016/j.joes.2022.06.032.

Tian, G. and An, Rf., 2023. Spreading speed of a food-limited population model with delay. *Applied Mathmatic Journal.,* 38(2), pp. 264-273, https://doi.org/10.1007/s11766-023-4232-8.

Wang Gen-q. and Cheng, S.S., 2009. Existence and uniqueness of periodic solutions for a second-order nonlinear differential equation with piecewise constant argument. *International Journal of Mathematics and Mathematical Sciences*, Article ID 950797. https://doi.org/10.1155/2009/950797.

Yuan, R.A., 2001. New almost periodic type solution of first order neutral delay differential equations with piecewise constant argument. *Acta Mathematica Sinica, English Series,* 17(3), pp. 403–412. https://doi.org/10.1007/s101149900003.

Yuji, L. and Weigao, G., 2003. Global attractively in delay ''Food Limited'' models with the exponential impulses. *Journal Of Mathematical Analysis And Applications*, 287(1), pp. 200-216. https://doi.org/10.1016/S0022-247X(03)00543-2.

Zhang L. L. and Hong-Xu L., 2011. Weighted pseudo almost periodic solutions of second-order neutral-delay differential equations with piecewise constant argument. *Computers & Mathematics with Applications*, 62(12), pp. 4362-4376. https://doi.org/10.1016/j.camwa.2011.10.004.

Zhiguo, L. and Jianhua, S., 2003. New results on oscillation for delay differential equations with piecewise constant argument. *Computers and Mathematics with Applications*, 45, pp. 1841-1848. https://doi.org/10.1016/S0898-1221(03)90005-8.

التذبذب في حصص الطعام لمعادلة تفاضلية محايدة ذات ثابت مجزأ

سرى علي مجيد[1,*]، حسين علي محمد[2]

[1]قسم علوم الحاسوب، كلية العلوم، الجامعة التكنولوجية، بغداد، العراق.

[2]قسم علوم الرياضيات، كلية علوم بنات، جامعة بغداد، بغداد، العراق.

الخلاصة

تحتوي معادلات التفاضل التأخيرية على العديد من التطبيقات في العالم الحقيقي، مثل أنظمة النماذج الهندسية ذات التأخيرات الزمنية، مثل شبكات الاتصالات وأنظمة التحكم، والوجبات المحدودة الوقت، وضغط الدم، وتكوين الدم، وغيرها، وخاصة عندما تستفيد من التذبذب في هذه المعادلات. يجب أن تكون بعض معاملات معادلة اللوجستيك من الدرجة الأولى متصلة بشكل متقطع من أجل تحقيق هدف الدراسة. للقيام بذلك، يجب فحص خاصية التذبذب أو عدم التذبذب لجميع حلول معادلات اللوجستيك من الدرجة الأولى باستخدام معادلات التفاضل التأخيرية مع وسيطة الثابت المتقطع. الثابت المتقطع الخاص بها يعبر عن أكبر دالة صحيحة في الحل. لتقديم الظروف المناسبة لجميع الحلول، نستخدم طرقًا مثل تحويل معادلة التفاضل التأخيرية غير الخطية إلى معادلة تفاضلية تأخير خطية ثم تطبيق المتباينة التكاملية.

الكلمات المفتاحية: المعادلة اللوجستية، المعادلة التفاضلية، التأخير، التذبذب، الثابت القطعي.