Free Vibration Analysis of Laminated Composite plates with General Elastic Boundary Supports

Dr.Wedad Majed Ibrahim
Assisant proffesor
College of Engineering-University of Baghdad
email: wedad_majeed@yahoo.com

Rafat Assad Ghani
Msc student
College of Engineering-University of Baghdad
email-rafat_eng90@yahoo.com

ABSTRACT

In this investigation, Rayleigh–Ritz method is used to calculate the natural frequencies of rectangular isotropic and laminated symmetric and anti-symmetric cross and angle ply composite plate with general elastic supports along its edges. Each of the admissible functions here is composed of a trigonometric function and an arbitrary continuous function that is introduced to ensure the sufficient smoothness of the so-called residual displacement function at the edges. Perhaps more importantly, this study has developed a general approach for deriving a complete set of admissible functions that can be applied to various boundary conditions. Several numerical examples are studied to demonstrate the accuracy and convergence of the current solution with considering some design parameters such as boundary conditions, aspect ratio, lamination angle, thickness ratio, orthotropy ratio, also these results are compared with other researchers and give a good agreement .

Key words: free vibration, Rayleigh–Ritz method, general boundary condition, composite laminated plate

تحليل الاهتزازالحر لصفيحة مركبة مع اسناد حافات مرنة وعامة

أ.م.د وداد مجيد ابراهيم
مساعد بروفيسور
جامعة بغداد-كلية الهندسة
قسم الهندسة الميكانيكية

رأفت أسعد غني
طالب ماجستير
جامعة بغداد-كلية الهندسة
قسم الهندسة الميكانيكية

الخلاصة

في هذة الدراسة ، تستخدم طريقة Rayleigh–Ritz لأيجاد التردد الطبيعي للصفيحة الموحدة الخواص والمركبة المستطيلة ذات الزوايا المتعامدة والمائلة ،المتماثلة والغير متماثلة مع ظروف اسناد لحافاتها مختلفة . والدوال المستخدمة في هذا البحث يمكن ان تمثل بدوال مثلثية و دوال عشوائية مستمرة و ذلك لضمان السلاسة المطلوبة لعمل الدالة الرئيسية . ولعل الأهم من ذلك، ان هذه الدراسة قد طورت اسلوب عام لاشتقاق مجموعة كاملة من الدوال المقبولة التي يمكن تطبيقها لشروط اسناد الحافات المختلفة. لقد تم دراسة عدة امثلة عددية لاثبات دقة وتقارب نتائج الحل الحالي مع الاخذ بنظر الاعتبار تغيير في بعض معايير التصميم مثل شروط الحدود، نسبة الارتفاع، وزاوية التصفيح، ونسبة سماكة، ونسبة الـ orthotropy ،حيث تم مقارنة النتائج مع باحثين اخرين واعطت تقارب جيد جدا .

الكلمات الرئيسية: نظرية القص ذات الرتبة العالية ، الالواح الطبقية المركبة ، التحليل الاستاتيكي .

1. INTRODUCTION:

Composite materials are so necessary in many engineering applications, as vehicles parts industry, aero structures industry and medical devices industry. With the wide use of composite plate in the modern industry, static and dynamic analysis of plate structure under different types of loads and different boundary condition become a main part in design procedure. In the past few years, many researchers resorted to the development of many theories to clearly predict the response of laminated plate composite material. It is necessary to know the theories of laminated composite plates, because it is not possible to provide accurate analysis without knowledge of theories. These theories can be classified in to three type's single layer theories, layer-wise theories and continuum based 3D elasticity theories.

Many researchers had studied Vibration analysis of rectangular plates with general elastic boundary supports by classical plate theory (CLPT), and other researchers have studied the natural frequency of composite plates with all boundary conditions. **Pervez, Al-Zebdeh, and Farooq, 2010**

W.L. Li ,2004. used Rayleigh–Ritz method to determine the modal characteristics of a rectangular isotropic plate with general elastic supports alone its edges. Each of the admissible functions here is composed of a trigonometric function and an arbitrary continuous function. He firstly investigated the convergence of his function then he studied many different cases of isotropic plate such as different aspect ratio and different values of elastic restraint constant (k,K). **Y.F. Xing** and **B. Liu ,2009.** solved new exact solutions for free vibrations of thin orthotropic rectangular plates by using a novel separation of variables. The exact normal eigenfunctions and eigenvalue equations for the boundary condition combinations SSCC, SCCC and CCCC are obtained through the mode formulation and boundary conditions. **Henry Khov, Wen L. Li** and **Ronald F. Gibson ,2009.**presented an accurate solution method for the static and dynamic deflections of orthotropic plates with general boundary conditions. The displacement function is expressed as a 2-D Fourier cosine series supplemented with several terms in the form of 1-D series. Thus, a classical solution can be derived by letting the series exactly satisfy the governing differential equation at every field point and all the boundary conditions at every boundary point, respectively**. W.L. Li, X.Zhang, J.Du** and **Z.Liu ,2009.** studied an exact series solution for the transverse vibration of rectangular isotropic plates with general elastic boundary supports. An analytical method is developed for the vibration analysis of rectangular plates with elastically restrained edges. The displacement solution is expressed as a two-dimensional Fourier series supplemented with several one-dimensional Fourier series. Thus, an exact solution can be obtained by letting the series simultaneously satisfy the governing differential equation and the boundary conditions on a point-wise basis. **H.DAL** and **O.K.MORGUL,2011.** studied vibrations of elastically restrained rectangular isotropic plates. Vibrations of plates with boundary conditions were elastic along full edges. Deflections function was expressed as the combination of a Fourier sine series and an auxiliary polynomial. Solution function as employed by **Li**

2002. has been adopted for plates with fully elastic edges. Frequency parameters of plate were calculated for different plate parameters. **H.T.Thai , S.E.Kim ,2012.** obtained Levy-type solution for free vibration analysis of orthotropic plates based on two variable refined plate theory. The theory, which has strong similarity with classical plate theory in many aspects, accounts for a quadratic variation of the transverse shear strains across the thickness, and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. **Kookhyun Kim, B. Kim, T.Choi and D.Cho ,2012.** presented free vibration analysis of rectangular isotropic plate with arbitrary edge constraints using characteristic orthogonal polynomials in assumed mode method. Natural frequencies and their mode shapes of the plate are calculated by solving an eigenvalue problem of a multi-degree-of-freedom system matrix equation derived by using Lagrange's equations of motion. Characteristic orthogonal polynomials having the property of Timoshenko beam functions which satisfies edge constraints corresponding to those of the objective plate are used. **A. Pagani ,2014.** extended free vibration analysis of composite plates by higher-order 1D dynamic stiffness elements based on Carrera Unified Formulation (CUF) and experiments. The principle of virtual displacements is then used to derive the equations of motion and the natural boundary conditions, which are subsequently expressed in the frequency domain by assuming a harmonic solution. After the resulting system of ordinary differential equations of second order with constant coefficients is solved, the frequency dependent DS matrix of the system is derived. Finally the algorithm of Wittrick and Williams is applied to extrapolate the free vibration characteristics of laminated composite plate. **Wan-You Li, W.Li, B. Lv, H. Ouyang, J. Du,H. Zhou, and D. Wang ,2014.** presented a Hybrid Finite Element-Fourier Spectral Method for Vibration Analysis of Structures with Elastic Boundary Conditions. A novel hybrid method, which simultaneously possesses the efficiency of Fourier spectral method (FSM) and the applicability of the finite element method (FEM), is used for the vibration analysis of structures with elastic boundary conditions. The computational domain of general shape is divided into several subdomains firstly, some of which are represented by the FSM while the rest by the FEM. Then, fictitious springs are introduced for connecting these subdomains. Numerical examples of a one-dimensional Euler-Bernoulli beam and a two-dimensional rectangular plate show that the present method has good accuracy and efficiency. Further, one irregular-shaped plate which consists of one rectangular plate and one semi-circular plate also demonstrates the capability of the present method applied to irregular structures. **Firas Hamzah Taya,2014.** presented free vibration and buckling behavior of laminated composite thin plates subjected to in-plane uniform, parabolic, and linear distributed loads is studied using classical laminated plate theory (CLPT). Different functions were used for different boundary conditions applying Ritz method to get homogeneous set of equations and solved as Eigen value problems of buckling load solution for laminated plate. The boundary conditions considered in this study are (SSSS, CCCC, CSCS, SFSF, and CFCF). **G. Jin, T. Ye,** and **S. Shi ,2015.** presented Three-Dimensional Vibration Analysis of Isotropic and

Orthotropic Open Shells and Plates with Arbitrary Boundary Conditions. Vibration characteristics of the shells and plates have been obtained via a unified three-dimensional displacement-based energy formulation represented in the general shell coordinates, in which the displacement in each direction is expanded as a triplicate product of the cosine Fourier series with the addition of certain supplementary terms were introduced to eliminate any possible jumps with the original displacement function and its relevant derivatives at the boundaries. All the expansion coefficients are then treated equally as independent generalized coordinates and determined by the Rayleigh-Ritz procedure.

In present work the function proposed by **W.L. Li ,2004.** is used for laminated symmetric and antisymmetric cross and angle ply composite plate with general elastic supports along its edges.

2. THEORETICAL ANALYSIS:

2.1 Classical Laminated Plate Theory:

The equivalent single layer ESL laminated plate theories are those in which a heterogeneous laminated plate is treated as a statically equivalent single layer having a complex constitutive behavior, reducing the 3-D continuum problem to a 2-D problem. The ESL theories are developed by assuming the form of the displacement field or stress field as a linear combination of unknown functions and the thickness coordinate: **J.N. REDDY,2004.**

$$\psi\_i(x, y, z, t) = \sum_{j=0}^{N} (z)^j \, \psi_i^j(x, y, t) \tag{2.1}$$

where ψ_i is the component of displacement or stress, (x, y) is the in-plane coordinates, z is the thickness coordinate, t is the time, and ψ_i^j are functions to be determined. When ψ_i displacements, then the equations governing are ψ_i^j are determined by the principle of virtual displacements (or its dynamic version when time dependency is to be included)

$$0 = \int_0^T (\delta\Pi + \delta W - \delta E_c)\, dt \tag{2.2}$$

where $\delta\Pi$, δW, and δE_c denote the virtual strain energy, virtual work done by external applied forces, and the virtual kinetic energy, respectively. These quantities are determined in terms of actual stresses and virtual strains, which depend on the assumed displacement functions, and their variations.

The simplest laminated plate theory is the classical laminated plate theory (or CLPT), which is an extension of the Kirchhoff (classical) plate theory to laminated composite plates. It is based on the displacement field

$$u(x,y,z,t) = u_o(x,y,t) - z\frac{\partial w_o}{\partial x}$$

$$v(x,y,z,t) = v_o(x,y,t) - z\frac{\partial w_o}{\partial y}$$

$$w(x,y,z,t) = w_o(x,y,t) \tag{2.3}$$

where (u_o, v_o, w_o) are the displacement components along the (x, y, z) coordinate directions, respectively, of a point on the midplane (i.e., $z = 0$).

The governing differential equation for the free vibration of laminated thin plate is given by: **Henry Khov,2009.**

$$D_{11}\frac{\partial^4 w}{\partial x^4} + D_{22}\frac{\partial^4 w}{\partial y^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} - \rho h \omega^2 w(x,y) \tag{2.4}$$

2.2 Total Mechanical Energy:

The first law of thermodynamics or the principle of conservation of energy serves as the foundation for energy-based methods employed in the analysis of structures, including plates. In the absence of energy dissipation and other non-conservative forces, i.e. if the forces acting on the system are conservative, this principle is reduced to the principle of stationery total energy, **Victor Birman ,2011.**

The total mechanical energy (defined as the sum of its potential and kinetic energies) of a particle being acted on by only conservative forces is constant, **Robert G. Brown ,2007.**

$$E = E_c + \Pi = Constant \tag{2.5}$$

where E: Total mechanical energy of a system
E_c: Total kinetic energy of the system
Π: Total potential energy of the system

In static problems, the principle of stationary total energy reduces to the principle of minimum total potential energy implying that the virtual work of forces acting on the system in equilibrium is equal to zero **Victor Birman,2011.** so that:

$$\Delta E=0 \text{ OR } E= Constant \tag{2.6}$$

Where

$$\Pi = \frac{1}{2}\int_0^b \int_0^a \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] dx\, dy + \frac{1}{2}\int_0^b \left[k_{x0}w^2 + K_{x0}\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]_{x=0} dy + \frac{1}{2}\int_0^b \left[k_{x1}w^2 + K_{x1}\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]_{x=a} dy + \frac{1}{2}\int_0^a \left[k_{y0}w^2 + K_{x0}\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]_{y=0} dx + \frac{1}{2}\int_0^b \left[k_{y1}w^2 + K_{y1}\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]_{y=b} dx \qquad (2.7)$$

and

$$E_c = \frac{1}{2}\omega^2 \iint I_o w_o^2 dx\, dy \qquad (2.8)$$

2.3 Boundary Conditions:

In terms of the flexural displacement, the bending and twisting moments and transverse shearing forces can be expressed as, **Henry Khov ,2009.**

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \qquad (2.9)$$

$$M_y = -D_{22}\frac{\partial^2 w}{\partial y^2} - D_{12}\frac{\partial^2 w}{\partial x^2} \qquad (2.10)$$

$$M_{xy} = -2D_{66}\frac{\partial^2 w}{\partial x \partial y} \qquad (2.11)$$

$$Q_x = -D_{11}\frac{\partial^3 w}{\partial x^3} - (D_{12} + 4D_{66})\frac{\partial^3 w}{\partial y^2 \partial x} \qquad (2.12)$$

$$Q_y = -D_{22}\frac{\partial^3 w}{\partial y^3} - (D_{12} + 4D_{66})\frac{\partial^3 w}{\partial x^2 \partial y} \qquad (2.13)$$

The boundary conditions for an elastically restrained rectangular plate are

$k_{x0}w = Q_x \quad K_{x0}\frac{\partial w}{\partial x} = -M_x$ ....*at x=0* (2.14-15)

$k_{x1}w = -Q_x \quad K_{x1}\frac{\partial w}{\partial x} = M_x$ ....*at x=a* (2.16-17)

$k_{y0}w = Q_y \quad K_{y0}\frac{\partial w}{\partial y} = -M_y$ ....*at y=0* (2.18-19)

$k_{y1}w = -Q_y \quad K_{y1}\frac{\partial w}{\partial y} = M_y$ ....*at y=b* (2.20-21)

where k_{x0} and k_{x1}(k_{y0} and k_{y1}) are the linear spring constants, and $K_{x0}$and K_{x1} (K_{y0} and K_{y1}) are the rotational spring constants at x =0 and a (y = 0 and b), respectively. Eqs. (14)–(21) represent a set of general boundary conditions from which, for example, all the classical homogeneous boundary conditions can be directly obtained by accordingly setting the spring constants equal to an extremely large or small number **Fig 2**

From Eqs. (9–21), the boundary conditions can be finally written as:

$$k_{x0}w = -D_{11}\frac{\partial^3 w}{\partial x^3} - (D_{12} + 4D_{66})\frac{\partial^3 w}{\partial x \partial y^2} \qquad (2.22)$$

$$k_{x1}w = D_{11}\frac{\partial^3 w}{\partial x^3} + (D_{12} + 4D_{66})\frac{\partial^3 w}{\partial x \partial y^2} \quad (2.23)$$

$$K_{x0}\frac{\partial w}{\partial x} = D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2} \quad (2.24)$$

$$K_{x1}\frac{\partial w}{\partial x} = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \quad (2.25)$$

2.4 Admissible functions

Admissible functions play a critical role in the Rayleigh–Ritz method. For plate problems, the products of the beam functions are often chosen as the admissible functions and the displacement function can be accordingly expressed as, **W.L. Li ,2004.**

$$w(x, y) = \sum_{m,n=1} A_{mn} X_m(x) Y_n(y) \quad (2.26)$$

where $X_m(x)$ and $Y_n(y)$ are the characteristic functions for beams that have the same boundary conditions in the x- and y-direction, respectively.

Although beam functions can be generally obtained as a linear combination of trigonometric and hyperbolic functions, they include some unknown parameters that have to be determined from the boundary conditions. Consequently, each boundary condition basically leads to a different set of beam functions. In real applications, this is clearly inconvenient, not to mention the tediousness of determining the characteristic functions for a generally supported beam. In order to avoid this difficulty, an improved Fourier series method have been proposed for beams with arbitrary supports at both ends in which the characteristic functions are sought in the form of, **W.L. Li,2000.**

$w(x) = \sum_{m=0}^{\infty} a_m \cos\lambda_{am} x + p(x) \quad (\lambda_{am} = m\pi/a), \quad 0 \le x \le a \quad (2.27)$

The function $p(x)$ in Eq. (27) represents an arbitrary continuous function that, regardless of boundary conditions, is always chosen to satisfy the following equations:

$p^{```}(0) = w^{```}(0) = \alpha_0$, $p^{```}(a) = w^{```}(a) = \alpha_1$ (2.28-29)

$p^{`}(0) = w^{`}(0) = \beta_0$, $p^{`}(a) = w^{`}(a) = \beta_1$ (2.30-31)

As explained in Ref. **W.L. Li ,2000.** the function $p(x)$ is here introduced to take care of the potential discontinuities of the (original) displacement function and its derivatives at the end points. Accordingly, the Fourier series now simply represents a residual displacement function, $\acute{W}(x) = W(x) - p(x)$; that is periodic continuous and has at least three continuous derivatives over the entire x-axis. Mathematically, it is already known that the smoother a periodic function is, the faster its Fourier expansion converges. Therefore, the addition of the function $p(x)$ will have two immediate benefits: (1) the Fourier series expansion is now applicable to any boundary conditions, and (2) the Fourier series solution can be drastically improved regarding its accuracy convergence.

So far, $p(x)$ has only been understood as a continuous function that satisfies Eqs. (28)–(31), its form is not a concern with respect to the convergence of the series solution. Thus, the function $p(x)$ can be selected in any desired form. As a demonstration, suppose that $p(x)$ is a polynomial function

$$p(x) = \sum_{n=0}^{4} c_n p_n\left(\frac{x}{a}\right) \qquad (2.32)$$

where c_n is the expansion coefficient and $p_n(x)$ is the Legendre function of order n. It is obvious that the function $p(x)$ needs to be at least a fourth order polynomial to simultaneously satisfy Eqs. (28)–(31). Substituting Eq. (32) into Eqs. (28)–(31) results in

$$c_3 p_3^{```}(0) + c_4 p_4^{```}(0) = a^3 \alpha_0 \qquad (2.33)$$

$$c_3 p_3^{```}(1) + c_4 p_4^{```}(1) = a^3 \alpha_1 \qquad (2.34)$$

$$c_1 p_1^{`}(0) + c_2 p_2^{`}(0) + c_3 p_3^{`}(0) + c_4 p_4^{`}(0) = a\beta_0 \qquad (2.35)$$

$$c_1 p_1^{`}(1) + c_2 p_2^{`}(1) + c_3 p_3^{`}(1) + c_4 p_4^{`}(1) = a\beta_1 \qquad (2.36)$$

From the above equations, the coefficients, c_n (n = 1, 2, 3, 4), are directly obtainable in terms of the boundary constants, α_0, α_1 , β_0 and β_1. Since the constant c_0 does not actually appear in Eqs. (33)–(36), it can be an arbitrary number theoretically. For instance, c_0 is here selected to satisfy

$$\int_0^a p(x)\, dx = 0 \qquad (2.37)$$

The final expression for the function $p(x)$ can be written as

$$p(x) = \zeta_a(x)\bar{\alpha} \qquad (2.38)$$

Where $$\bar{\alpha} = \{\alpha_0\ \alpha_1\ \beta_0\ \beta_0\} \qquad (2.39)$$

and $$\zeta_a(x)^T = \begin{Bmatrix} -(15x^4-60x^3+60a^2x^2-8a^4)/360a \\ (15x^4-30a^2x^2+7a^4)/360a \\ (6ax-2a^2-3x^2)/6a \\ (3x^2-a^2)/6a \end{Bmatrix} \qquad (2.40)$$

The results in Eqs. (38)–(40) were previously derived from a more straightforward but less general approach, **W.L. Li,2004 .**

In order to determine the unknown boundary constants, α_0, α_1, β_0 and β_1, substitution of Eqs. (27) and (38) into the boundary conditions Eqs. (22)–(25) results in

$$\bar{\alpha} = \sum_{m=0}^{\infty} H_a^{-1} Q_{am} a_m \quad (2.41)$$

Where

$$H_a = \begin{bmatrix} 1 + \frac{8k_{x0}a^3}{360D_{11}} & \frac{7k_{x0}a^3}{360D_{11}} & \frac{-k_{x0}a}{3D_{11}} & \frac{-k_{x0}a}{6D_{11}} \\ \frac{7k_{x1}a^3}{360D_{11}} & 1 + \frac{8k_{x1}a^3}{360D_{11}} & \frac{-k_{x1}a}{6D_{11}} & \frac{-k_{x1}a}{3D_{11}} \\ \frac{a}{3} & \frac{a}{6} & \frac{K_{x0}}{D_{11}} + \frac{1}{a} & \frac{-1}{a} \\ \frac{a}{6} & \frac{a}{3} & \frac{-1}{a} & \frac{K_{x1}}{D_{11}} + \frac{1}{a} \end{bmatrix} \quad (2.42)$$

and

$$Q_{am} = \left\{ (-1)\frac{k_{x0}}{D_{11}} \quad (-1)^m \frac{k_{x1}}{D_{11}} \quad -\lambda_{am}{}^2 \quad (-1)^m \lambda_{am}{}^2 \right\}^T \quad (2.43)$$

It should be mentioned that the matrix Ha will become singular for a completely free beam. However, this problem can be overcome to a certain extent by artificially attaching one or more springs with very small stiffness to the ends of a beam. It has been shown in **W.L. Li ,2002.** that although the matrix may be ill-conditioned in such a treatment, the natural frequencies can still be accurately calculated for a completely free beam.

Nevertheless, the characteristic functions are well known for this particular case and can be readily used as the admissible functions in the Rayleigh–Ritz method.

Making use of Eqs. (38) and (41), Eq. (27) can be rewritten as

$$w(x) = \sum_{m=0}^{\infty} a_m \varphi_m^a(x) \quad (2.44)$$

Where $$\varphi_m^a(x) = cos\lambda_{am}x + \zeta_a(x) H_a^{-1} Q_{am} \quad (2.45)$$

Mathematically, Eq. (44) indicates that each of the beam functions can be viewed as a function in the functional space spanned by the basic functions $\{\varphi_m^a(x)$; m = 0, 1, 2,.......}. Thus, Eq. (26) can be accordingly rewritten as

$$w(x,y) = \sum_{m,n=0}^{\infty} A_m \varphi_m^a(x) \varphi_n^b(y) \quad (2.46)$$

Where $$\varphi_n^b(y) = cos\lambda_{bn}y + \zeta_b(y) H_b^{-1} Q_{bn} \quad (2.47)$$

The expressions for $\zeta_b(y)$, H_b and Q_{bn} can be, respectively, obtained from Eqs. (40), (42) and (43) by simply replacing the x-related parameters by the y-related.

2.5 Determination of Natural Frequency:

Consider an orthotropic laminated plate, the material directions of which coincide with the plate directions. This plate is subjected to free vibration along the edges x = 0-a and y = 0-b.

Where the transverse displacement (w) is substituted in the total mechanical energy as mentioned in section 2.4. To calculate the natural frequency ω. Performing the required mathematical processes (differentiations and then integrations) of eqs. (2.7) and (2.8) and then putting the mechanical energy in the following equation:

$$\frac{\partial E}{\partial A_{mn}} = 0 \qquad (2.48)$$

eq. 2.48 will lead to a set of linear homogeneous algebraic equations as follow:

$$f(A_{mn}, \omega) = 0 \quad for\ vibration \qquad (2.49)$$

Solving the last equation as an Eigen-value problem to get the following form:

$$\begin{bmatrix} a_{1,1} & \cdots & a_{1,(m*n)} \\ \vdots & \ddots & \vdots \\ a_{(m*n),1} & \cdots & a_{(m*n),(m*n)} \end{bmatrix} \begin{Bmatrix} A_{11} \\ \vdots \\ A_{mn} \end{Bmatrix} = 0 \qquad (2.50)$$

where a_{ij} are the coefficients of the nonzero unknowns A_{mn}. Finding the determinant of the first term of eq. (2.50) and equating it to zero will lead to get the natural frequency ω. When M and N are more than 1, the natural frequency ω are determined by solving Eigen value problem. For different arbitrary boundary conditions and M & N are greater than 1, the solution becomes more difficult and needs computer programming to determine the natural frequency ω. In this work, Matlab R2013a is used to solve the Eigen-value problem.

3-RESULT AND CONCLUSIONS

3-1-Results

The natural frequency of isotropic and composite laminated plate with elastic boundary condition is analyzed and solved using MATLAP version13 programming. To examine the validly of the derived equations and performance of computer programming for vibration analysis of composite laminated plate, numerical results of isotropic plate are compared with those obtained by **W.L. Li ,2004.** and **Hüseyin DAL ,2011.** as shown in **Tables (1,2)** which give very close results for different boundary conditions. While present results for laminated composite plate with different boundary conditions give good agreement when compared with **Henry (2009)** and those obtained by numerical program **ANSYS** as shown in **Table 3.** and figures (3) and (4), also present work results are compared with those obtained by **Reddy** for ssss cross ply plate with different orthotropy ratio as shown in **table(4).**

Also cross ply scheme (symmetric and non-symmetric) with different layer number was studied as shown in **Table 5.** where the frequency parameter for symmetric ply is more than that for non-symmetric because the stiffness of the rare is larger than symmetric ply as proved by many researchers, while the other presented scheme is angle ply scheme (symmetric and non-symmetric) with different layer number as shown in **Table6.** where the frequency parameter for symmetric angle ply have no great change than that for non-symmetric because the stiffness of the two type are close to each other as proved by many researchers Also we study isotropic scheme in **Tables (7,8)** which give very close results for different boundary conditions and aspect ratio with **Wen le 2004**, it can be noted that when the aspect ratio increases the frequency parameter increases for same boundary condition .

While the other presented scheme is laminated cross ply scheme (symmetric and non-symmetric) with different boundary conditions and aspect ratio as shown in **Table 9.** where the frequency parameter decreases when the aspect ratio increases, and the largest frequency parameter for clamped boundary at all edges of the symmetric cross-ply plate. The next presented scheme is laminated angle ply $[45\ \ -45]_2$ as shown in **Table10.** where the frequency parameter decreases when the aspect ratio increases. Also the next presented scheme is laminated angle ply $[30\ \ -30]_2$ as shown in **Table11.** where the frequency parameter decreases when the aspect ratio increases. It can be noted that in **Tables (10,11)** the largest frequency parameter for SSCC boundary conditions for square plate .

The thickness ratio schemes of laminated plate with different boundary conditions are changed, once symmetric cross ply and $[45\ \ -45]_s$ schemes are studied with different boundary conditions and thickness ratio as shown in **Table12.** where the frequency parameter decreases when the thickness ratio increase (reduces the thickness) for same boundary condition and the largest frequency parameter for clamped boundary at all edges because it has largest stiffness. The next presented scheme is angle ply $[30\ \ -30]_s$ as shown in **Tables 13.** Where the frequency parameter decreases when the thickness ratio increase (reduces the thickness) for same boundary condition and the largest frequency parameter for clamped boundary at all edges because it has largest stiffness.

Rotational restraint along edges of laminated and isotropic plate with different boundary conditions are changed, consider plates are elastically restrained along edges. The first one involves a simply supported square isotropic plate with a uniform elastic restraint against rotation along each edge, that is, $K_{x0}a/D$=$K_{x1}a/D = K_{y0}a/D = K_{y1}a/D = Ka/D$, in **Table 14**. the first six frequency parameters are shown for a few different stiffness values. Because of the symmetries about the x- and y-axis, the second and third frequency parameters are identical. The fifth and sixth frequency parameters are also the same for $Ka/D = 0$. However, they become slightly different for other stiffness values. The frequency parameter increases when the stiffness values increases, when it zero the behavior of the frequency parameter like a simply supported along all edge , but when the stiffness values equal to infinity the behavior of the frequency parameter like a clamped along all edge. **Table 15** shows the frequency parameter of a simply supported square laminated plate $[0\ 90]_2$with a uniform elastic restraint against rotation along each edge, that is, $K_{x0}a/D_{22}$ =$K_{x1}a/D_{22} = K_{y0}a/D_{22} = K_{y1}a/D_{22} = Ka/D_{22}$, the first four frequency parameters are shown for a few different stiffness values. Because of the symmetries about the x- and y-axis, the second and third frequency parameters are identical.

The frequency parameter increases when the stiffness values increases, when it is zero the behavior of the frequency parameter is like a simply supported along all edge, but when the stiffness values is equal to infinity the behavior of the frequency parameter like a clamped along all edge.The orthotropy ratio schemes of laminated plate with different boundary conditions are changed, **Fig.5** shows the frequency parameter of non-symmetric cross ply with different boundary conditions and orthotropy ratio , where the frequency parameter increases when the orthotropy ratio increases, the largest frequency parameter for CFCF boundary conditions and then less in SSSS and smallest in SFSF boundary conditions. **Fig6** shows the frequency parameter of angle ply$[60\ \ -60]_2$, where the frequency parameter increases when the orthotropy ratio increases, the largest frequency parameter for SSSS boundary conditions and then less in CFCF and smallest in SFSF boundary conditions .

3.2. Conclusions

This study presented investigations for free vibration of a composite laminated plate. Some assumptions are made to solve the vibration problems and determine the results desired for this paper.

The results are determined mainly by analytic method and compared with numerically found results, and with obtained by other researchers; the comparison showed high agreement between them.

The vibration results lead to the following conclusions:

1- The number of half wavelengths affects the natural frequency, where the increasing aspect ratio requires larger number (M & N) to get more accurate results where the error is found 4.71% for cccc isotropic plate a/b=2.5 at ($m = n = 3$).

2- The boundary conditions affect the fundamental natural frequency. Clamped edges conditions offer high stiffness, results in high natural frequency. Clamped boundaries make the plate holds larger frequency than simply supported boundaries, where the fundamental natural frequency for SSSS cross ply, is less by 47.3% than fundamental natural frequency of CCCC cross ply, while for angle ply, is less by 58.21%,and the fundamental natural frequency for SSSS isotropic plate, is less by 55.28% than of CCCC plate.

3- The aspect ratio is inversely proportional to the frequency parameter $\Omega = \omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$ of the orthotropic plate and frequency parameter $\Omega = \omega b^2/\pi^2\sqrt{(\rho h\ /\ D)}$ of the isotropic plate.

4- Rotational restraint along edges of laminated and isotropic plate affects the natural frequency, where the value of (Ka/D) and (Ka/D_{22}) is zero the behavior of the natural frequency like a (SSSS) boundary conditions, but when the value is infinity the behavior of the natural frequency like a (CCCC) boundary conditions.

5- The orthotropy ratio is directly proportional with the frequency parameters, $\Omega = \omega a^2/h\sqrt{(\rho\ /\ E_2)}$ of the orthotropic plate.

6- The increasing of the lamination angle is inversely proportional with frequency parameters, $\Omega = \omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$ of the orthotropic plate.

7- The thickness ratio is inversely proportional with the natural frequency of the orthotropic plate.

NOMENCLATURE

Symbol	Discretion	Units
a	Length of a plate	m
b	width of a plate	m
h	Plate thickness	m
A	vector of the expansion or Rayleigh–Ritz coefficients	
A_{mn}	expansion or Rayleigh–Ritz coefficients	
a_m	expansion or Rayleigh–Ritz coefficient	
D_{ij}	flexural rigidity	-
E_1 , E_2, E_3	Elastic modulus components	Gpa
$G12, G23, G13$	Shear modulus components	Gpa
M,N	numbers of expansion terms used in x- and y-direction, respectively	
M_x , M_y , M_{xy}	Moment result per unit length	N.m/m
Q_x , Q_y	Transverse shear force result	N
K_{x0} , K_{x1}	rotational stiffnesses at x = 0 and a, respectively	Rad.N/m
K_{y0} , K_{y1}	rotational stiffnesses at y = 0 and b, respectively	Rad.N/m.
k_{x0} , k_{x1}	translational stiffnesses at x = 0 and a, respectively	N/m
k_{y0} , k_{y1}	translational stiffnesses at y = 0 and b, respectively	N/m
$P(x)$	a simple polynomial function	
x,y,z	Cartesian coordinate system	m
E, E_c	Total mechanical and kinetic energies of a system	N.m
Π	Total potential energy of the system	N.m
U	Strain energy of deformation	N.m
V_c	the elastic potential energy	N.m
$\epsilon_x, \epsilon_y, \epsilon_z$	Strain components	
γ_{xz} , γ_{yz}	Transverse shear strain	
ν_{12}	Poisson's ratio components	-
σ_{xx} , σ_{yy} , σ_{xy} , σ_{yz}	Stress components	Gpa

σ_{xz}		
u, v, w	Displacements in x, y, z directions	m
u_o, v_o, w_o	Displacements of the reference surface in the x, y, z directions	m
$W(X)$	flexural displacement of a beam	m
$w(x,y)$	flexural displacement of a plate	m
$X_m(x)$, $Y_n(y)$	beam characteristic function	
α_1 , α_0	$=w^{```}(a), w^{```}(0)$	
β_1, β_0	$=w^{`}(a), w^{`}(0)$	
λ_{am}	$\frac{m\pi}{a}$	-
λ_{bn}	$\frac{n\pi}{b}$	-
ρ	Density of material	Kg/m^3
$\varphi_m^a(x)$	admissible functions in x-direction	-
$\varphi_n^b(y)$	admissible functions in y-direction	
ω	Natural frequency	Cycle/sec

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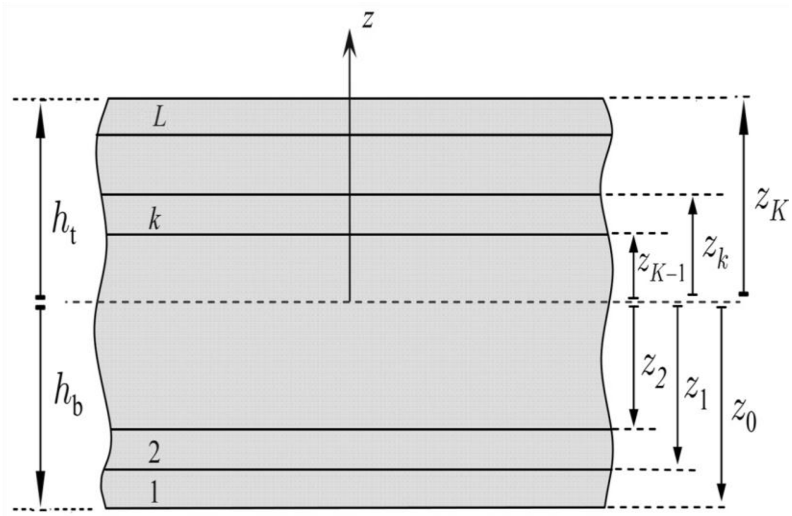

Figure 1 Distances from the reference plane.

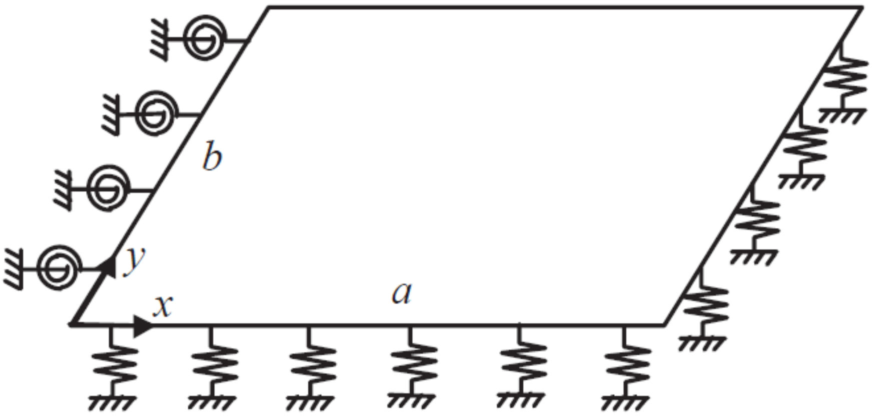

Figure 2 A rectangular plate elastically restrained along edges.

Table 1. Frequency parameters, Ω= ω$a^2\sqrt(\rho h\ /\ D)$, for all clamped boundary conditions (cccc) plate with ($\frac{a}{b}=2.5$). For $m=n=1{,}2{,}3$.

M=N	mod 1	mod2	mod 3	mod 4	mod 5
1	147.43	172.60	395.47	419.39	--------
2	147.87	172.54	220.33	394.94	419.39
3	146.87 $(147.8)^a$ 0.62%	172.52 $(173.8)^a$ 0.73%	220.33 $(221.4)^a$ 0.48%	293.49 $(291.7)^a$ -0.61%	387.85 $(384.4)^a$ -0.89%

a : **Wen le 2004 .**

Tabel 2. Frequency parameters, Ω= ω$a^2\sqrt(\rho h\ /\ D)$, for squar plates of different boundary conditions

B C S		mod 1	mod2	mod 3	mod 4	mod 5	mod 6
ssss	present	19.67	49.14	49.14	78.04	98.37	98.38
	H. DAL	19.72	49.32	49.32	78.88	98.56	98.56
	Diff %	0.25	0.36	0.36	1.06	0.19	0.19
	Wen le 2004	19.74	49.35	49.35	78.96	98.70	98.70
	Diff %	0.35	0.42	0.42	1.16	0.33	o.33
cscs	present	28.71	54.18	68.73	92.2	101.36	128.55
	H.DAL	28.91	54.64	69.37	93.91	102.04	128.58
	Diff %	0.69	0.84	0.92	1.82	0.66	0.02
	Wen le 2002	28.95	54.74	69.32	94.61	102.23	129.09
	Diff %	0.82	1.02	0.85	2.54	0.85	0.41
Ssfs	present	11.82	27.92	41.55	59.22	63.28	91.09
	H. DAL	11.58	27.68	41.11	59.20	62.93	90.42
	Diff %	-2.07	-0.86	-1.07	-0.03	-0.55	-0.74
	Wen le 2002	11.68	27.79	41.23	59.24	62.37	90.51
	Diff %	-1.19	-0.46	-0.77	0.03	-1.45	-0.64

Table 3. Natural frequencies (Hz) of graphite–epoxy plates consisting of 12 plies oriented at 0 under different boundary conditions (E1=127.9Gpa , E2 =10.27Gpa , G12=7.312Gpa , υ12=0.22)

Mod	ssss				ccfc			
	Ansys	Henry 2009	Present	Diff %	Ansys	Henry 2009	Present	Diff %
1	108.10	108.7	108.82	-0.11	70.63	70.96	71.01	-0.07
2	170.09	171.4	171.91	-0.29	166.96	167.5	169.35	-1.10
3	292.84	294.8	297.55	-0.93	215.69	219.7	219.88	-0.08
4	384.97	388.4	390.52	-0.54	294.77	298.1	298.8	-0.23
5	429.83	435.0	437.27	-0.52	314.51	314.8	323.27	-2.69

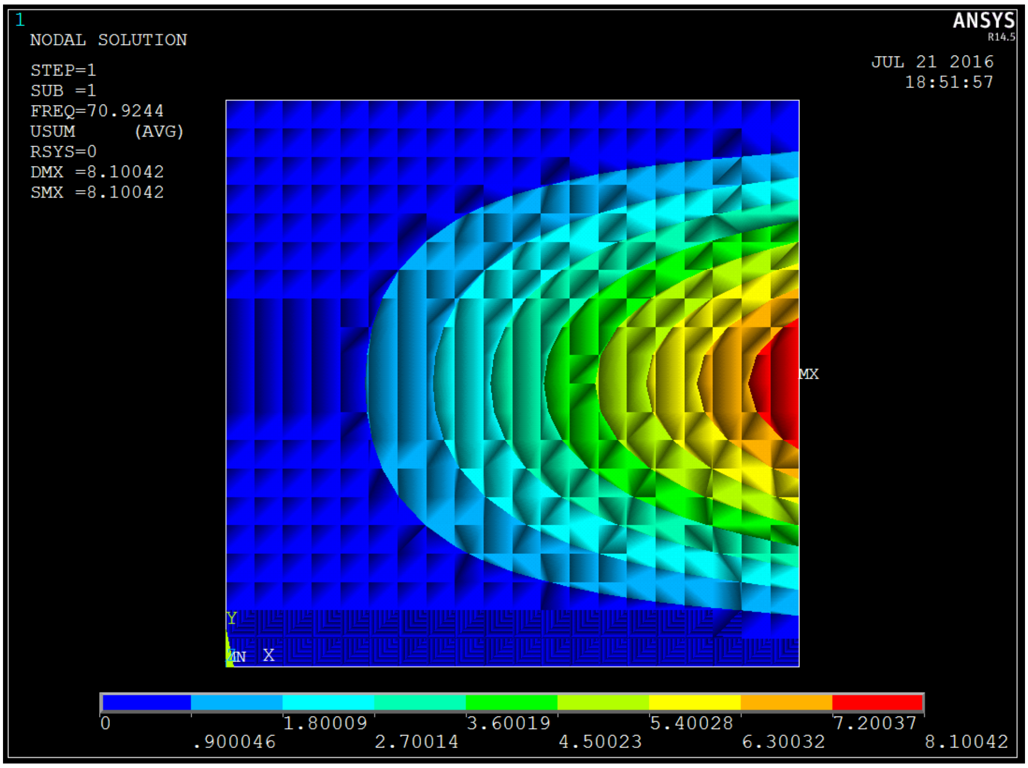

Figure 3 . First Mode shape for free vibration of a CCFC orthotropic square plate.

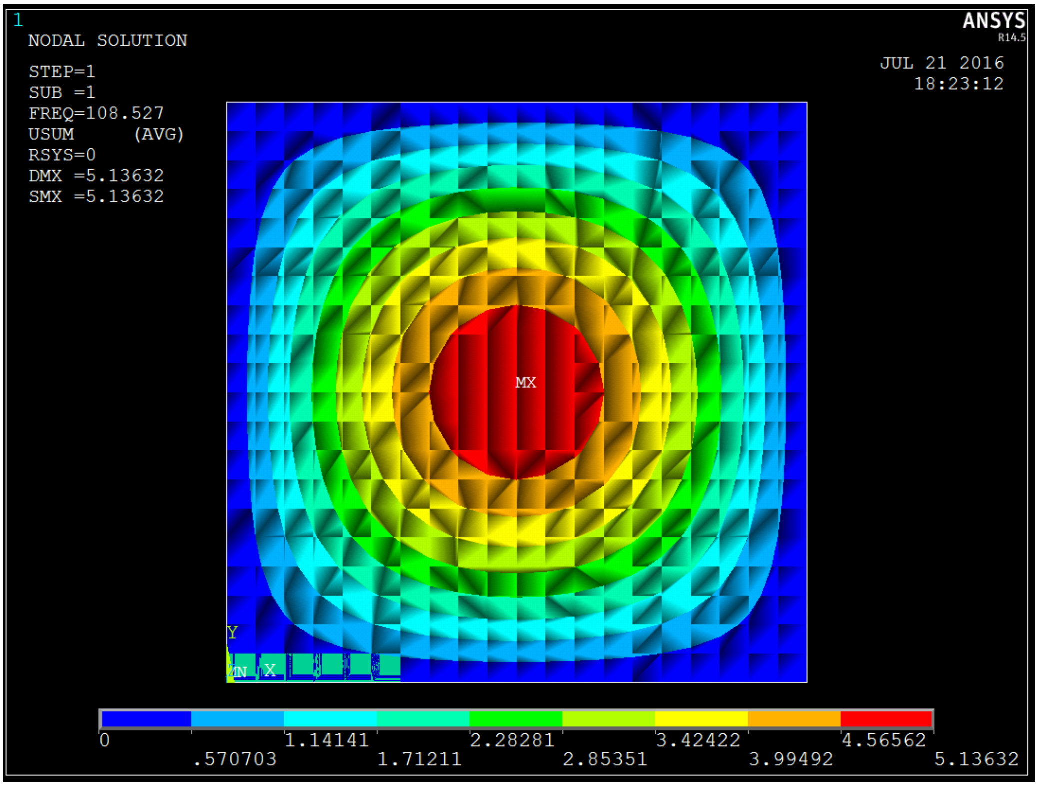

Figure 4 . First Mode shape for free vibration of a SSSS orthotropic square plate.

Table 4. Frequency parameters, Ω= $\omega a^2/h\sqrt{(\rho / E_2)}$, for s-s-s-s squar plates and (a/h=10, G12=0.6E2, υ12=0.25).

E_1 / E_2	[0 90 90 0]		
	Present	Reddy	Diff %
3	7.53	7.47	-0.80
10	10.65	10.56	-0.85
20	13.95	13.83	-0.86
30	16.61	16.47	-0.85
40	18.90	18.73	-0.90

Table 5. Frequency parameters, Ω= $\omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$, of a square plate of various laminations and boundary conditions. (E1/E2=10, G12=0.6E2, υ12=0.25)

Angle Ply Orientations		ssss	cccc	cscs	cfcf
[0 90]s	present	2.56	5.37	4.93	4.65
	Firas	2.55	5.35	4.91	4.63
	Diff %	-0.39	-0.37	-0.40	-0.43
	Ansys	2.55	5.31	4.91	4.63
[0 90 0 90]	present	1.58	3.33	2.606	2.27
	Firas	1.58	3.33	2.606	2.26
	Diff %	0	0	0	-0.44
	Ansys	1.60	3.39	2.58	2.31
[0 90 0]s	present	2.04	4.28	3.73	3.45
	Ansys	2.03	4.31	3.72	3.43
$[0\ 90]_3$	present	1.59	3.33	2.61	2.27
	Ansys	1.58	3.24	2.55	2.19

Table 6. frequency parameters, Ω= $\omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$, of a square plate of various laminations and boundary conditions. (E1/E2=10, G12=0.6E2, υ12=0.25).

Angle Ply Orientations		ssss	cccc	cscs	cfcf
[45 -45]s	present	2.52	4.09	3.38	2.27
	Firas	2.51	4.12	3.39	2.26
	Diff %	-0.39	0.78	0.29	-0.44
	Ansys	2.72	4.49	3.15	1.98
[45 -45 45 -45]	present	2.52	4.09	3.38	2.27
	Firas	2.51	4.12	3.39	2.26
	Diff %	-0.39	0.78	0.29	-0.44
	Ansys	2.31	4.01	3.24	2.25
[30 -30]s	present	3.40	5.84	5.30	4.32
	Ansys	3.17	5.76	5.14	4.12
[30 -30 30 -30]	present	3.40	5.84	5.30	4.32
	Ansys	3.30	5.62	5.04	4.09
[45 -45 45]s	present	2.52	4.09	3.38	2.27
	Ansys	2.43	4.08	3.31	2.16
$[45\ -45]_3$	present	2.52	4.09	3.38	2.27
	Ansys	2.48	3.98	3.30	2.14

[30 -30 30]s	present	3.40	5.84	5.30	4.32
	Ansys	3.31	5.84	5.23	4.19
$[30\ -30]_3$	present	3.40	5.84	5.30	4.32
	Ansys	3.36	5.79	5.18	4.13

Tabel 7. Frequency parameters, Ω= $\omega a^2\sqrt{(\rho h\ /\ D)}$, for all clamped boundary conditions(cccc) plates with different aspect ratios.

a/b		mod 1	mod2	mod 3	mod 4	mod 5	mod 6
1	present	35.58	72.44	72.44	104.48	130.18	131.44
	Wen.le	35.99	73.4	73.4	108.2	131.6	132.2
	Diff %	1.13	1.30	1.30	3.43	1.07	0.57
1.5	present	60.31	93.05	148.28	149.88	176.38	225.82
	Wen.le	60.76	93.84	148.8	149.7	179.6	226.8
	Diff %	0.74	0.84	0.34	-0.12	1.79	0.43
2	present	97.67	126.24	178.75	251.68	256.46	278.91
	Wen.le	98.31	127.3	179.1	253.3	255.9	284.3
	Diff %	0.65	0.83	0.19	0.63	-0.21	1.89
2.5	present	146.87	172.52	220.33	293.49	387.85	413.81
	Wen.le	147.8	173.8	221.4	291.7	384.4	394.3
	Diff %	0.62	0.73	0.48	-0.61	-0.89	-4.94

Table 8. Frequency parameters, Ω= $\omega a^2\sqrt{(\rho h\ /\ D)}$, for CSSF plates with different aspect ratios .

a/b	Reference	mod 1	mod2	mod 3	mod 4	mod 5	mod 6
1	present work	16.88	31.18	51.67	65.19	67.27	100.31
	Wen.le	16.87	31.14	51.64	64.03	67.64	101.2
	Diff %	-0.05	-1.27	-0.05	-1.81	0.54	0.87
1.5	present work	18.68	50.62	54.14	89.09	110.64	130.16
	Wen.le	18.54	50.43	53.72	88.78	108.2	126.10
	Diff %	-0.75	-0.37	-0.78	-0.34	-2.25	-3.21
2	present work	20.89	56.83	77.54	113.13	117.81	176.94
	Wen.le	20.65	56.54	77.33	111.3	117.3	176.0
	Diff %	-1.16	-0.51	-0.27	-1.64	-0.43	-0.53
2.5	present work	23.41	60.46	112.14	117.19	153.89	196.47
	Wen.le	23.07	59.97	111.9	115.1	153.1	189.6
	Diff %	-1.47	-0.81	-0.21	-1.18	-0.51	-3.62

Table 9. show Frequency parameters, Ω= $\omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$, of effect of aspect ratio and boundary conditions. (E1/E2=10, G12=0.6E2, υ12=0.25).

a/b		[0 90 0 90]			[0 90]s		
		ssss	cccc	cffc	ssss	cccc	cffc
1	present	1.58	3.33	0.583	2.56	5.37	0.94
	Firas	1.58	3.33	0.592	2.55	5.35	0.95
	Diff %	0	0	1.52	-0.39	-0.37	1.05
1.5	present	1.19	2.55	0.435	1.57	3.24	0.579
2	present	1.09	2.38	0.393	1.27	2.67	0.467
2.5	present	1.052	2.32	0.377	1.16	2.48	0.421

Table 10. Frequency parameters, Ω= $\omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$, for (45/-45/45/-45) plates of different aspect ratios and boundary conditions ,(E1/E2=10, G12=0.6E2, υ12=0.25).

	Type of boundary conditions				
a/b	ssss	sscc	ssff	ccff	ccfc
1	2.52	3.239	0.496	0.854	2.581
1.5	1.77	2.326	0.329	0.596	2.405
2	1.467	1.998	0.246	0.494	2.347
2.5	1.313	1.844	0.196	0.444	2.321

Table 11. Frequency parameters, Ω= $\omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$, for (30/-30/30/-30) plates of different aspect ratios and boundary conditions,(E1/E2=10, G12=0.6E2, υ12=0.25).

	Type of boundary conditions				
a/b	ssss	sscc	ssff	ccff	ccfc
1	3.404	4.502	0.633	1.166	2.818
1.5	2.194	2.862	0.422	0.754	2.499
2	1.723	2.291	0.316	0.589	2.396
2.5	1.486	2.027	0.252	0.507	2.351

Table 12. Natural Frequency ω(rad/sec) , of effect of thickness ratio for squar plate ($E1/E2=10$, $G12=0.6E2$, $\upsilon 12=0.25$) .

	[0 90 90 0]			[45 -45 -45 45]		
a/t	cccc	ssss	cscs	cccc	ssss	cscs
20	1.11	0.532	1.025	1.09	0.67	0.90
40	0.55	0.266	0.513	0.545	0.335	0.45
100	0.223	0.106	0.205	0.218	0.134	0.18

Table 13. Natural Frequency ω(rad/sec), effect of thickness ratio for square plate ($E1/E2=10$, $G12=0.6E2$, $\upsilon 12=0.25$) (30/-30/-30/30).

a/t	Type of boundary conditions				
	cccc	ssss	cscs	cfcf	sfsf
20	1.097	0.638	0.995	0.811	0.356
40	0.548	0.319	0.497	0.405	0.178
100	0.219	0.127	0.199	0.162	0.071

Table 14. Frequency parameters, $\Omega = \omega a^2 \sqrt{(\rho h / D)}$, for SSSS square plate with different uniform rotational restraint along edges.

K a/D	Reference	mod 1	mod2	mod 3	mod 4	mod 5	mod 6
0	present work	19.67	49.14	49.14	78.04	98.38	98.38
	Wen.le 2004	19.74	49.35	49.35	78.96	98.70	98.70
	Diff %	0.35	0.42	0.42	1.16	0.32	0.32
10	present work	28.50	60.52	60.52	90.97	112.82	113.07
	Wen.le 2004	28.50	60.22	60.22	90.81	111.20	111.40
	Diff %	0	-0.49	-0.49	-0.17	-1.45	-1.49
20	present work	31.08	64.68	64.68	95.97	118.83	119.25
	Wen.le 2004	31.08	64.31	64.31	95.82	116.80	117.20
	Diff %	0	-0.57	-0.57	-0.15	-1.73	-1.74
100	present work	34.67	71.27	71.27	104.61	129.69	130.36
	Wen.le 2004	34.67	70.78	70.78	104.50	127	127.60
	Diff %	0	-0.69	-0.69	-0.10	-2.11	-2.16
∞	present work	35.58	72.44	72.44	104.48	130.18	131.44
	Wen.le 2004	35.99	73.40	73.40	108.20	131.60	132.20
	Diff %	1.13	1.30	1.30	3.431	1.07	0.57

Table 15. Frequency parameters, Ω= $\omega b^2/\pi^2\sqrt{(\rho h\ /\ D22)}$,for SSSS square plate [0 90 0 90] with uniform rotational restraint along edges.

$K\,a/\,D22$	mod 1	mod2	mod 3	mod 4
0	1.59	4.39	4.39	6.39
10	2.60	5.61	5.61	7.80
20	2.86	6.04	6.04	8.34
100	3.21	6.69	6.69	9.19
∞	3.33	6.95	6.95	9.54

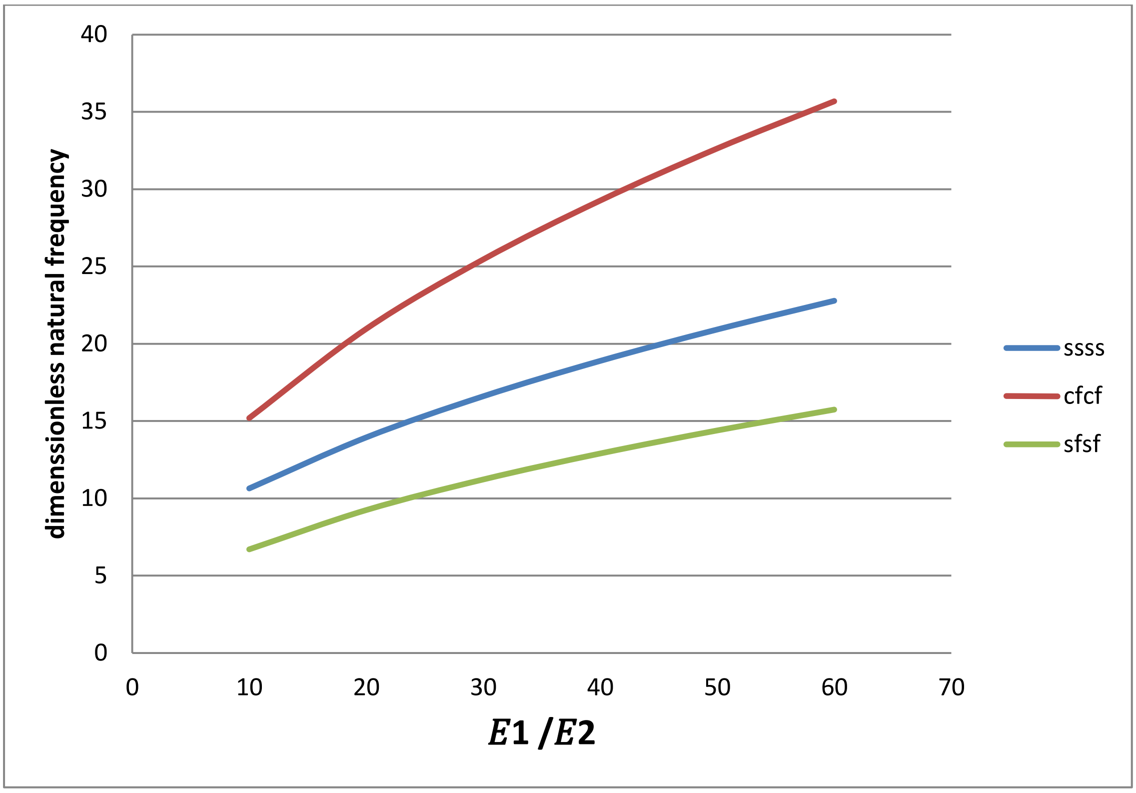

Figure 5. Frequency parameters, Ω= $\omega a^2/h\sqrt{(\rho\ /\ E_2)}$, for square plates with different orthotropic ratio, (a/h=20, G12=0.6E2, υ12=0.25) (0/90/0/90).

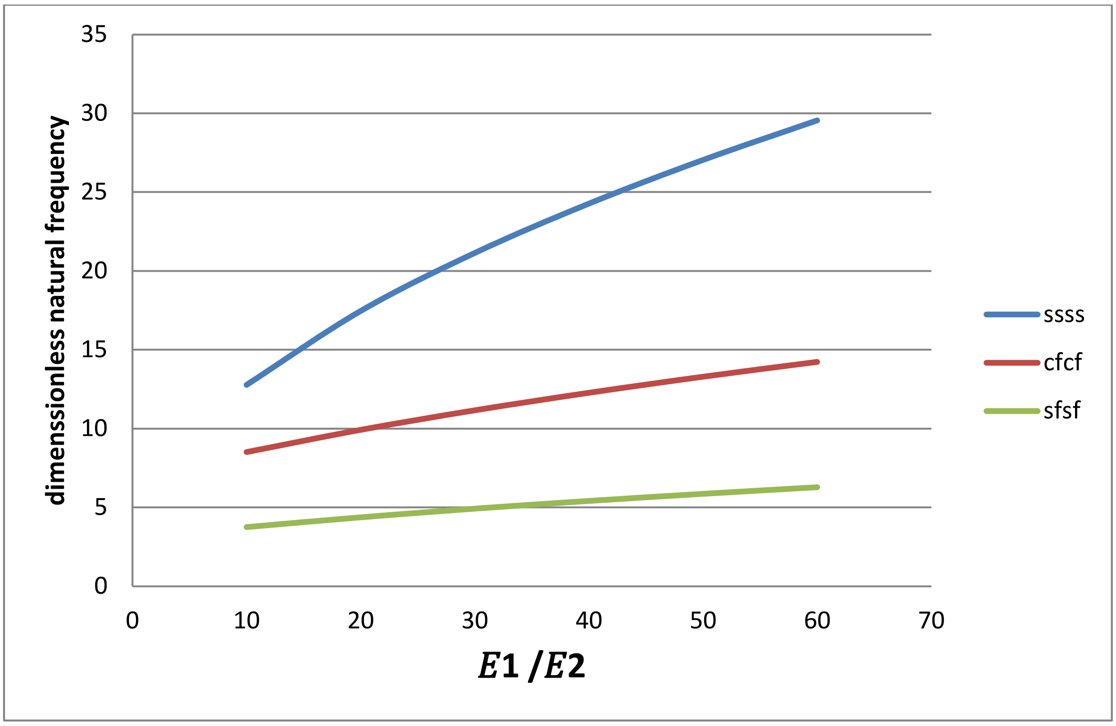

Figure 6. Frequency parameters, $\Omega = \omega a^2/h\sqrt{(\rho / E_2)}$, for square plates with different orthotropic ratio, (a/h=20, G12=0.6E2, υ12=0.25) (60/-60/60/-60).