



Flexural Behavior of Partially Prestensioned Continuous Concrete Beams

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ABSTRACT

This paper describes flexural behavior of two spans continuous rectangular concrete beams reinforced with mild steel and partially prestressing strands, to evaluate using different prestressing level and prestressing area in continuous prestressed beams at serviceability and ultimate stages. Six continuous concrete beams with 4550 mm length reinforced with mild steel reinforcement and partially prestressed with two prestressing levels of $(0.7f_{py}$ or $0.55f_{py})$ of and different amount of 12.7 mm diameter seven wire steel strand were used. Test results showed that the partially prestressed reinforced beams with higher prestressing level exhibited the narrowest crack width, smallest deflection and strain in both steel and concrete at ultimate service load, the deflection decreased by (3.60% & 32.49%) and the crack width decreased by (20.0%) and (75.0%) when increasing the prestressing level from $(0.55f_{py})$ to $(0.7f_{py})$ for beams reinforced with one and two strands respectively. Deflection of beams with two strands decreased by (44.81% & 22.2%) compared with beams of one strand at prestressing level of $(0.7f_{py})$ and $(0.55f_{py})$, respectively. At ultimate load, using ACI-Code recommended moment redistribution led to more agreement between theoretical and experimental loads for both ordinary reinforced and partially prestressed beams.

Key words: continuous beams; prestressed concrete; deformation; cracking; redistribution

تصرف الأثناء للعتبات الخرسانية المستمرة والمسبقة الشد

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الخلاصة

يصف هذا البحث التصرف الانحنائي للعتبات الخرسانية المستمرة بفضائين والمسلة جزئياً بحديد الاجهاد المسبق والحديد الاعتيادي لتقييم استخدام اجهادات وكميات حديد شد مسبق مختلفة على تصرف العتبات الخرسانية في المراحل التشغيلية والقصى. تم استخدام ستة عتبات خرسانية مستمرة بطول (4550 ملم) مسلحة بحديد التسليح الاعتيادي وحديد التسليح المسبق الاجهاد ذو قطر 12.7 ملم وبمستويين من الشد المسبق (55% و 70%) من اجهاد الخضوع وبعدد مختلف. أظهرت نتائج الفحص بأن العتبات المسلحة جزئياً مع مستوى اجهاد مسبق أعلى كان لها اقل عرض للشقوق و اقل هطول وانفعال لكل من حديد التسليح والخرسانة في المرحلة التشغيلية القصوى. زيادة مستوى الاجهاد المسبق قلل من انخفاض مقدار الهطول بنسبة (3.6% و 32.49%) وقلل عرض الشق بمقدار (20% و 75%) عند زيادة مقدار الشد من (55%) الى (70%) من اجهاد الخضوع للعتبات المسلحة بظفيرة او ظفيرتين من حديد مسبق الاجهاد على التوالي. استخدام ظفيرتين من الحديد المسبق الاجهاد ادى الى تقليل الهطول بمقدار (44.81% و 22.2%) عند مستويات شد (70% و 55%) على التوالي. النتائج النظرية للحمل الاقصى ابدت توافق اكثر مع النتائج العملية عند استخدام نسب اعادة توزيع العزم الموصى بها من قبل ACI-Code للعتبات المسلحة اعتيادياً والمسلة جزئياً بحديد الشد المسبق.

الكلمات الرئيسية: العتبات المستمرة، الخرسانة المسبقة الاجهاد، التشوه، التشقق، اعادة التوزيع



1. INTRODUCTION

The use of continuous concrete beams over the interior supports leads to increase the flexural rigidity through providing an alternated load path among the beam, this alternate load path lead to reduce the moments and stresses at midspans results in shallower beams that are stiffer compared with simply supported beams of equal span and with leaser deflection, **Amlan K.**, and **Devdas M.**, 2011. With low tensile capacity of concrete, flexural concrete members would be cracked at early loading stages with higher deflection. In order to limit tensile stress, cracks and deflection under service load; partial prestressed reinforcement which consists of mild steel and prestressing steel is used in concrete beams, **Nawy**, 2010. Using such composite reinforcement technique has valuable gains throughout controlling the extent and width of cracks of concrete members which will lead to reduce the deflection of the members.

2. RESEARCH SIGNIFICANCE

In this paper, cracking, deformation and ultimate capacity of two spans continuous concrete beams reinforced with mild steel only and partially prestressed reinforcement will be investigated. The effect of using different prestressing level and prestressing area in negative moment regions of continuous prestressed beams on the flexural behavior of such beams at serviceability and ultimate stages is studied.

3. TEST PROGRAM

Six (200x310mm) cross section continuous concrete beams having length of (4550mm) were tested under two point's monotonic loading until failure. Two beams reinforced with ordinary steel reinforcement and having ultimate failure capacity corresponding to both beams having one or two strands, respectively. Two beams partially reinforced with one (12.7mm) prestressing steel strand with two prestressing levels and ordinary steel reinforcement and two beams partially reinforced with two (12.7mm) of (1860MPa) ultimate tensile strength prestressing steel strands with two prestressing levels and ordinary steel reinforcement. For shear reinforcement, all beams reinforced with (10mm) stirrups spaced at (100mm) through the entire length of the beams. Table 1 shows the reinforcement details for all beams and **Fig. 1** and **Fig. 2** show the cross section and elevation of all tested beams. The designed cylindrical compressive strength of the continuous beams is (35MPa) at (28) days. Table 2 shows the concrete properties of the continuous beams at the time of test. The development of prestressing stress for each strand in concrete beams is shown in Table 3. Normally reinforced beams were designed according to ACI-318 Code to have equal theoretical ultimate capacity corresponding to partially prestressed beams, while the partially prestressed beams were analyzed using strain compatibility method. Concrete beams test was conducted in the Structural Laboratory of the Civil Engineering Department, at the College of Engineering, University of Al-Mustansiriyah.

4. INSTRUMENTATION

Four dial gauges of (0.01mm) accuracy with (50mm) travel length were used to measure the deflection under point load and at mid-span of each side. Different sizes of pre-wired strain gauges of (120 Ω) resistance, made in Japan by TML Company are used in this study. Two (2mm) strain gauges are placed on each strand at middle support. Three (5mm) strain gauges are placed on steel bars at both sides of tension zone under load points and at tension zone at middle support. Four (60mm) strain gauges are placed on concrete surface at compression steel level under point load and at (40mm and 80mm) from the bottom face of the beam at middle support.

Six rows of demec points are placed on concrete surface under point load at the right side of the beam at (50, 100, 150, 200, 250 and 300mm) from the bottom face of concrete beam, as shown in **Fig. 3**. Crack width is measured using special tool made of a set of thin steel plates with specific thickness, concrete surface divided to square cells of (50x50mm) to allow for measuring crack propagation at loading stages. Test setup photo shown in **Fig. 4**. One concentrated load was applied on each of the two spans at distance of (725mm) from the center support.

5. ANALYSIS OF TEST RESULTS

5.1 Load Deflection Relationship

Load deflection ($P-\Delta$) curves of tested beams under left load point are shown in **Fig. 5** to **Fig. 7**. **Fig. 5** shows that the increasing of prestressing level in beam (B2) lead to decrease the deflection by (3.60%) at loading level of ($0.5P_u$) compared with beam (B3), and decreased by (7.31%) compared with beam (B1), at same loading level, the deflection of partially prestressed beam (B3) which has the lowest prestressing level decreases by (3.85%) compared with normally reinforced beam (B1). The used beams numbering is same as what mentioned in the original thesis.

Deflection curves at load point in **Fig. 6** shows that when increasing the prestressing level in the partially prestressed beams having two strands, the deflection of beam (B11) decreases by (32.49%) at loading level of ($0.5P_u$) compared with partially prestressed beam (B12) and by (46.15%) compared with normally reinforced beam (B10), at the same loading level, the deflection of partially prestressed beam (B12) which has the lowest prestressing level decreases by (20.24%) compared with normally reinforced beam (B10).

Fig. 7 shows the deflection at left load point of partially prestressed beams, the figure shows that increasing the number of strands in partially prestressed beam (B11) lead to decrease the deflection at load point by (44.81%) at load level of ($0.5P_u$) compared with partially prestressed beam (B2) having one strand with same prestressing level, and for the same reason, the deflection of beam (B12) decreases by (21.20%) compared with partially prestressed beam (B3).

From the last two figures, it can be seen that the ($P-\Delta$) curves of partially prestressed beam (B11) and (B12) reinforced with two strands having two portions only, without yielding and the flat part after yielding, this may be attributed to that the tension steel reinforcement were not yields due to the shear failure type of these beams compared with beams (B2) and (B3). It is agreed that the shear failure causes change in the deflection curve when occurs, for these two beams, the shear starts to participate in the beams deflection at stages earlier than the final stages, this participation were steadily occurred with load increments and did not happen suddenly. Figures show also that beam (B11) has higher deflection than beam (B12) where at the lower loading level the beam (B12) experienced higher deflection after cracking load of (170kN), this may be attributed to the opposite effect of higher prestressing level applied on the compression zone of the section at load point.

5.2 Strain in Steel Reinforcement and Concrete

5.2.1 Strain in steel reinforcement

Tensile strain in mild steel reinforcement at tension zones of left load point and middle support of tested beams are shown in **Fig. 8** and **Fig.9** while the tensile strain increments in seven wire steel strand at tension zone of middle support are shown in **Fig. 10**.

Fig. 8 shows the mild steel strain at left load point taking the influence of increasing number of strand in beams (B11) and (B12) reinforced with two strands compared with beams (B2) and

(B3) reinforced with one strand and having same prestressing level of corresponding to beams (B11) and (B12) respectively, and compared with equivalent normally reinforced beams (B1) and (B10).

It can be seen from this figure that the (P- ϵ) curves approximately coincide up to the cracking of concrete covers of beams having less reinforcement area. These beams show more noticeable change in the curve slope at cracking and the strain increased in higher rate compared with beams reinforced with higher amount of reinforcement. Beams (B10), (B11) and (B12) show higher beam cracking load compared with beams (B1), (B2) and (B3) at both mid-spans and center support although that beams (B11) and (B12) were expected to have lower cracking load at mid-spans due to the opposite effect of prestressing force, these beams shows also less rate of strain increasing after beam cracking and shows less noticeable change in the curve slope at cracking.

Continuous concrete beams (B10), (B11) and (B12) shows higher ultimate failure load compared with beams (B1) (B2) and (B3) without yielding of flexural steel reinforcement at failure due to the fact that these beams failed in shear.

It can be seen from this figure, that when increasing the amount of reinforcement in normally reinforced beam (B10) the strain decreases by (46.82%) at loading level of ($0.5P_u$), compared with beam (B1).

In partially prestressed beam (B11) reinforced with two strands and having prestressing level of ($0.7f_{py}$), steel strain decreases by (53.36%) at loading level of ($0.5P_u$) compared with beam (B2) reinforced with one strand having same prestressing level.

Steel strain at load point of beam (B12) reinforced with two strands and having prestressing level of ($0.55f_{py}$) decreases by (48.22%) at loading level of ($0.5P_u$) compared with partially prestressed beam (B3) reinforced with one strand having same prestressing level.

When comparing the steel strain at left load point of beam (B11) taking the influence of prestressing level compared with (B12), increasing the prestressing level of (B11) lead to decrease the mild steel strain by (37.19%) at loading level of ($0.5P_u$) compared with beam (B12), while it decreased by (46.61%) compared with normally reinforced beam (B10) having equivalent amount of reinforcement.

Fig. 9 shows the steel strain at center support, the same behavior can be seen compared with load point strain in **Fig. 8** at cracking and ultimate points except that it shows higher strain at entire loading stages compared with load point due to the higher moment applied on the sections at center support, it has to be mentioned that the strain gauge reading of beam (B11) at center support were missed from the figure where the strain gauges failed before the test.

It can be seen from this figure, that when increasing the amount of reinforcement in normally reinforced beam (B10) the strain decreases by (41.8%) at loading level of ($0.5P_u$) compared with beam (B1).

Steel strain at center support of beam (B12) reinforced with two strands and having prestressing level of ($0.55f_{py}$) decreases by (39.2%) at loading level of ($0.5P_u$) compared with partially prestressed beam (B3) reinforced with one strand having same prestressing level.

Steel strain at center support of beam (B12) increases by (40.63%) at loading level of ($0.5P_u$) compared with normally reinforced beam (B10).

Fig. 10 shows the strain increments in center support steel strands of beams (B11) and (B12) having two strands with different prestressing level compared with beams (B2) and (B3) having one strand with same prestressing level corresponding to beams (B11) and (B12), respectively.

The figure shows that the beam cracking load of (B11) and (B12) were much higher than beams (B2) and (B3) and the strain values are much lower at entire loading stages, it can be seen that when concrete cover of beams (B11) and (B12) cracks, at different loading level, strand

strain increased rapidly followed by tension stiffening at load of approximately (250kN) lead to significantly decrease the strand strain until failure compared with partially prestressed beam (B2) and (B3) reinforced with one strand. The figure shows also that the strand strain of beams (B11) and (B12) coincide until beam cracking, then after, unexpectedly, the strain curve of beam (B11) becomes more flatter until load of (250kN) lead to have higher strain compared with beam (B12), figure shows also that strand strain were not reached the yielding in both beams.

Fig.10 shows that when using two strand in beam (B11) lead to decreases the strand strain by (41.41%) at loading level of ($0.5P_u$) compared with partially prestressed beam (B2) having same prestressing level and one strand, and the strain in beam (B12) decreased by (56.69%) at loading level of ($0.5P_u$) compared with partially prestressed beam (B3) having same prestressing level and one strand. It can be seen also that the increasing of prestressing level in beam (B11) compared with beam (B12), both beams reinforced with two strands, the strain increases by (11.25%) at loading level of ($0.5P_u$).

It has to be mentioned that **Fig.10** shows only the strain occurred due to external load, when summing these strain increments values with effective pre-strain, can be noted that the strand tensile strain at ultimate failure loads were (99.64%), (107.28%) of beams (B2) and (B3), respectively, while the strands strain at ultimate were (76.29%), and (63.62%) of beams (B11) and (B12), respectively, from strand ultimate tensile strain, this came from the fact that these beams failed in shear.

5.2.2 Strain in concrete

Fig. 11 shows the concrete strain ($P-\epsilon$) curves at right load point taking the influence of increasing number of strand in beam (B11) reinforced with two strands comparing with beam (B2) reinforced with one strand and having same prestressing level of ($0.7f_{py}$), and beam (B12) reinforced with two strands compared with beam (B3) reinforced with one strand and having same prestressing level of ($0.55f_{py}$), and compared with equivalent normally reinforced beams (B10) and (B1).

It can be seen from this figure that the ($P-\epsilon$) curves slopes have noticeable change occurred at beams cover cracking load then the strain increased in higher rate in beams reinforced with less amount of reinforcement. Beams (B10), (B11) and (B12) shows higher beam cracking load compared with beams (B1), (B2) and (B3, respectively).

Continuous concrete beams (B10), (B11) and (B12) shows higher ultimate failure load compared with beams (B1), (B2) and (B3) without yielding of flexural steel reinforcement at failure due to the fact that these beams failed in shear.

It can be seen from this figure, that when increasing the amount of reinforcement in normally reinforced beam (B10) the concrete strain decreases by (15.61%) at loading level of ($0.5P_u$) compared with beam (B1).

In partially prestressed beams (B11) and (B12) reinforced with two strands and having prestressing level of ($0.7f_{py}$) and ($0.55f_{py}$), respectively, concrete strain decreases by (51.82%) and (41.21%) at loading level of ($0.5P_u$) compared with beams (B2) and (B3), respectively, reinforced with one strand having same corresponding prestressing level.

When increasing prestressing level in beam (B11), concrete strain decreased by (23.21%) compared with beam (B12) and the concrete strain decreases by (6.30%) in beam (B2) compared with beam (B3).

When comparing the strain of beams (B11) and (B12) with normally reinforced beam (B10) having equal amount of reinforcement, concrete strain decreases by (35.09%) and (15.47%), respectively, at loading level of ($0.5P_u$).

5.3 Crack Width, Crack Patterns and Ultimate Loads

Load-cracking width development at center support and right span are shown in **Fig. 12** and **Fig. 13**, respectively. Concrete covers cracking load with corresponding maximum crack width at ultimate were presented in Table 4, the cracking pattern were shown in **Fig. 14**.

Fig. 12 shows beams cracking width with load at center support, the figure shows that at loading level of (350kN), when increasing the number of strand in beams (B11) and (B12), the crack width decreased by (100%) and (92.0%) compared with beams (B2) and (B3), respectively, having one strand. While increasing the amount of reinforcement in normally reinforced beam (B10) lead to decreases the crack width at center support by (77.78%) compared with beam (B1).

At same loading level, increasing the prestressing level in beam (B11) leads to decrease the crack width by (75.0%) compared with beams (B12). At lower loading level of (250kN), increasing the prestressing level in (B2) lead to decrease the crack width by (20%) compared with beam (B3).

Fig. 13 shows the crack width at right load point of the same beams, since the beams spans cracked at lower load level compared with center support, hence, the load of (250kN) will be used again in comparison. The figure shows that using of two strands in beam (B11) lead to decrease the left span crack width by (83.34%) at loading level of (250kN) compared with partially prestressed beam (B2), while increasing the prestressing reinforcement in beam (B12) decreases the crack width at load point by (62.5%) at loading level of (250kN) compared with partially prestressed beam (B3), increasing the amount of mild steel reinforcement lead to decreases the crack width by (83.34%) at loading level of (250kN) compared with partially prestressed beam (B1).

Increasing the prestressing level in beam (B11) lead to decrease the crack width by (33.34%) compared with (B12) at load of (250kN).

Fig. 14 shows the crack pattern of beams (B1), (B2), (B3), (B10), (B11) and (B12) at ultimate load, average crack spacing taken at left span for the flexure cracks of beams (B1), (B2) and (B3) were (72mm, 75mm, 85mm), respectively, and at center support were (73mm, 67mm, 65mm), respectively. Beams (B10), (B11) and (B12) were failed in shear before developing the three hinges mechanism at ultimate. Average crack spacing taken at left span for the flexure cracks were (110mm, 105mm and 110mm) for Beams (B10, B11 and B12), respectively. At center support, the crack spacing were (112mm, 105mm and 95mm) for beams (B10, B11 and B12), respectively.

Normally reinforced beam were designed according to ACI-318 code using ultimate design method while the partially prestressed beams reinforced beams were designed according to strain compatibility method. Table 5 represent experimental and theoretical failure loads, theoretical failure load calculated using elastic analysis of indeterminate continuous beams after using ACI-Code and strain compatibility method to determine ultimate moment capacities for both normally reinforces and partially prestressed beams, respectively. Final ultimate loads were determined using moment redistribution percentage factor (α) in Eq. (1) and Eq. (3) stated in ACI-Code for both normally reinforced and partially prestressed continuous beams respectively. Table 5 shows that all beams failed at ultimate in load higher than calculated load using elastic analysis only, after applying recommended moment redistribution percentage, calculated loads show more agreement and consistency with experimental failure load for all tested beams.

$$\alpha = 1000 * \varepsilon_t \quad (1)$$



Where:

$$\varepsilon_t = 0.003 \left(\frac{d}{c} - 1 \right) \quad (2)$$

$$\alpha \leq 20 \left[1 - \frac{\omega_p + \frac{d}{a_p}(\omega - \omega')}{0.36\beta_1} \right] \% \quad (3)$$

6. CONCLUSIONS

1. Increasing of prestressing level in partially prestressed beams reinforced with one strand leads to decrease the deflection under load point by (3.60%), while the deflection decreased by (7.31%) compared with reference normally reinforced beam. When using two strands the reduction becomes (32.49%) and (46.15%), respectively.
2. Using two strands in partially prestressed beams showed decreases in the deflection at load point by (44.81%) and (22.20%) compared with partially prestressing beams having one strand at prestressing level of $(0.7 f_{py})$ and $(0.55 f_{py})$, respectively.
3. Increasing of prestressing level in partially prestressed beams reinforced with two strands leads to decrease mild steel strain by (37.19%), while the deflection decreased by (46.61%). compared with reference normally reinforced beam.
4. Increasing the amount of prestressing reinforcement showed decrease in mild steel strain at load point by (53.36%) and (48.22%) compared with partially prestressing beams having less amount of prestressing reinforcement at prestressing level of $(0.7 f_{py})$ and $(0.55 f_{py})$, respectively, while at center support, using two strand leads to decrease the mild steel strain by (39.20%) for at prestressing level of $(0.55 f_{py})$.
5. Increasing the prestressing level leads to decrease center support steel strand strain by (11.25%) between beams reinforced with two strands, while the steel strand strain decreases by (41.41%) and (56.69%) for both beams compared with partially prestressing beams having one strand at prestressing level of $(0.7 f_{py})$ and $(0.55 f_{py})$, respectively.
6. Increasing the prestressing level leads to decrease concrete strain by (6.30%) and (23.21%) for beams reinforced with one and two strands, respectively. Using two strands leads to decrease concrete strain by (51.82%) and (41.21%) compared with partially prestressing beams having one strand at prestressing level of $(0.7 f_{py})$ and $(0.55 f_{py})$, respectively.
7. Increasing the prestressing level leads to decrease center support crack width by (20.0%) and (75.0%) for beams reinforced with one and two strands respectively. Using two strands leads to decrease center support crack width by (100.0%) and (92.0%) compared with partially prestressing beams having one strand at prestressing level of $(0.7 f_{py})$ and $(0.55 f_{py})$, respectively.
8. At ultimate load, using ACI-Code recommended moment redistribution factor load to more agreement between theoretically calculated and experimental loads for both normally reinforced and partially prestressed beams.
9. Using higher amount of prestressing reinforcement or increasing the prestressing level to control the deflection and crack width and to increase the ultimate load capacity is more effectively cost saving technique than using higher amount of mild steel reinforcement.



7. REFERENCES

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8. NOMENCLATURE

- c : neutral axis depth
 α : allowable percentage of redistribution of support moment calculated by elastic analysis.
 ω_p : reinforcement index for prestressed reinforcement.
 ω : reinforcement index for tension reinforcement.
 ω' : reinforcement index for compression reinforcement.
 d : effective depths of non-prestressed reinforcement.
 d_p : effective depths of prestressed reinforcement.
 β_1 : equivalent rectangular stress block coefficient.

Table 1. Continuous beams reinforcement details.

Beam Symbol	Over center support			At mid-span	
	$A_{s,2}$ mm ²	$A_{ps,2}$ mm ²	$A_{s,2}$ mm ²	$A_{s,2}$ mm ²	$A_{s,2}$ mm ²
B1	625	0	228	242	228
B2	228	96.6			
B3					
B10	938.8	0			
B11				855.9	
B12	228	199.2			

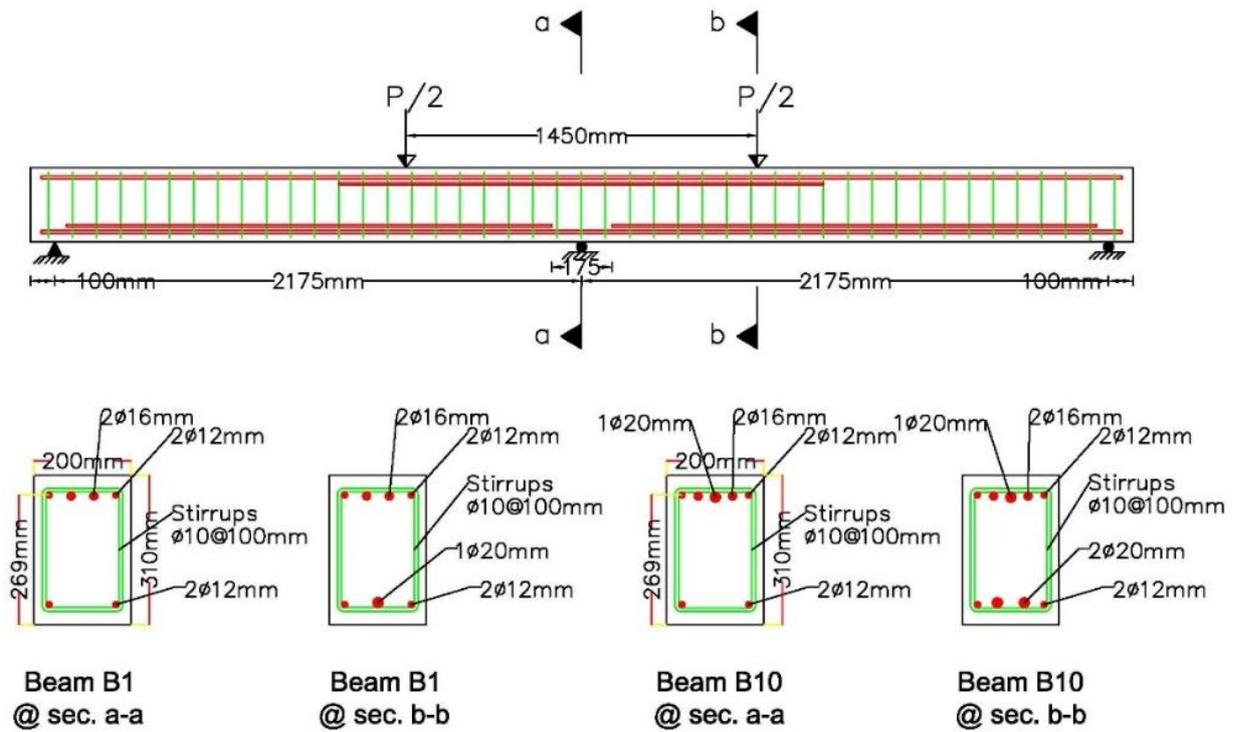


Figure 1. Geometry and reinforcement details of continuous beams (B1 and B10).

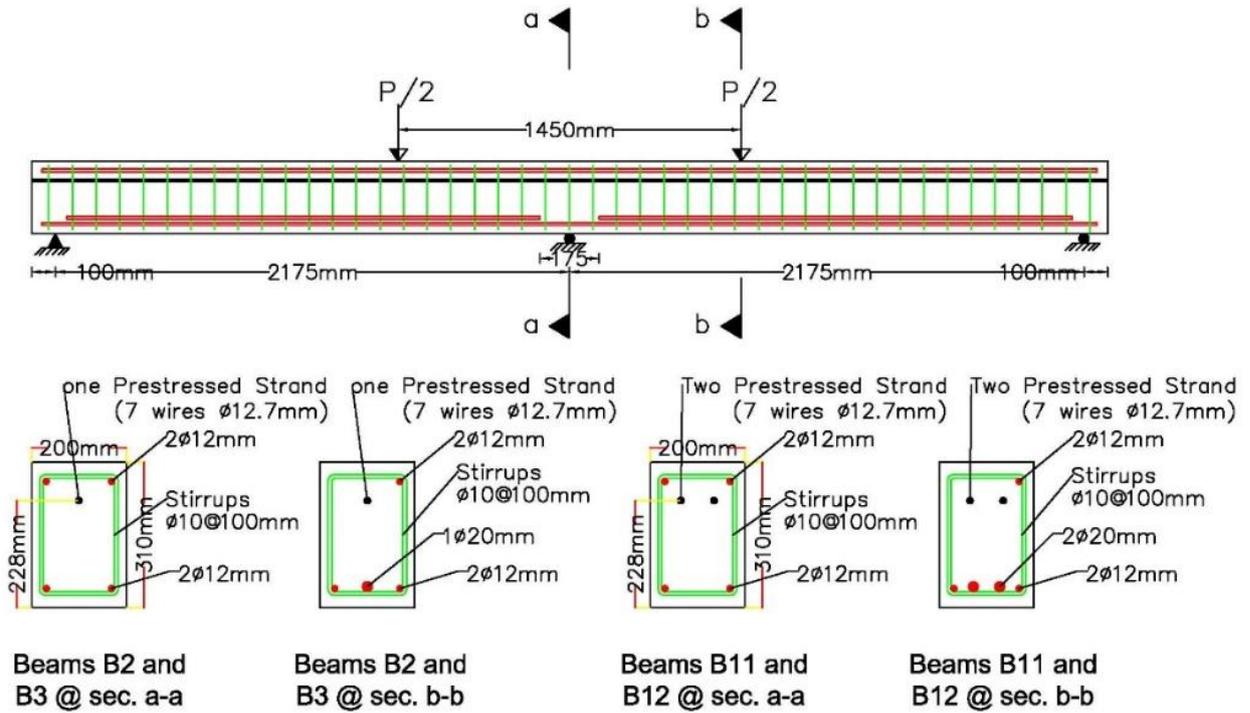


Figure 2. Geometry and reinforcement details of continuous beams (B2, B3, B11 and B12).

Table 2. Concrete properties for tested beams at age of test.

Beam Symbol	Beam properties			
	Compressive Strength. f'_c , (MPa)	Splitting Strength. f_{ct} , (MPa)	Modulus of Rupture. f_r , (MPa)	Modulus of Elasticity. E_c , (MPa)
B1	36.8	3.14	4.01	25373
B2	36.1	3.44	4.27	26319
B3	39.2	3.28	4.13	27599
B10	36.8	3.14	4.01	25373
B11	36.1	3.44	4.27	26319
B12	39.2	3.28	4.13	27599

Table 3. Prestress in beams steel strands.

Beam Symbol	P _j Jacking stress, (MPa)	P _i Before release, (MPa)	P _i After release, (MPa)	P _e Effective at time of test, (MPa)
B2	1162.29	1149.05	1121	1009.05
B3	936.05	929.64	909.29	820.88
B11	1175.67	1158.56	1107.37	979.56
B12	907.61	897.79	868.94	780.34

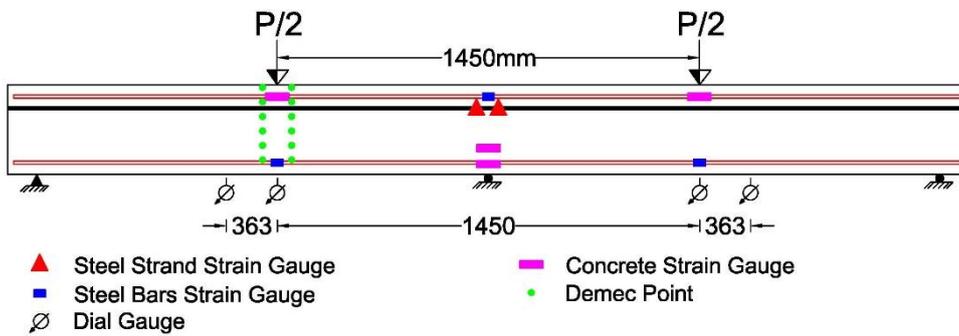
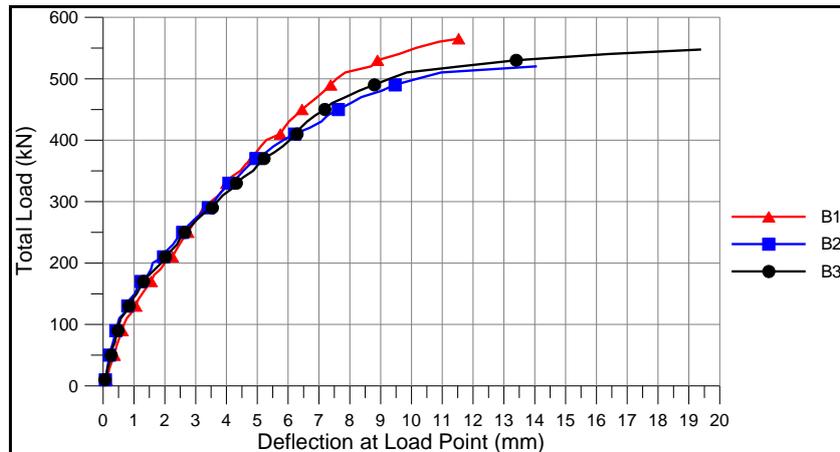


Figure 3. Beams instrumentation and loading details.

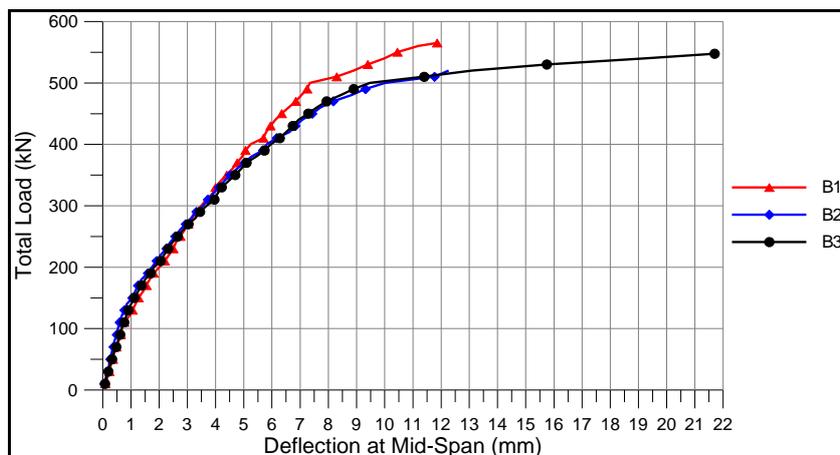


Figure 4. Test setup photo.



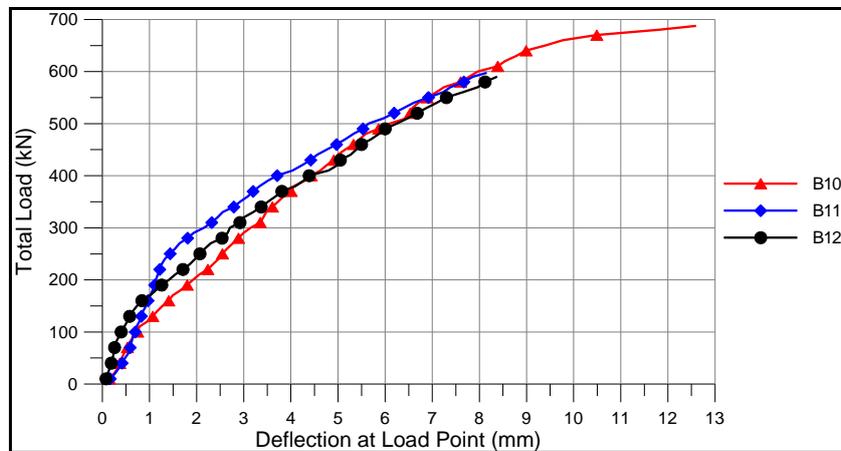
B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$),

Figure 5. Load-deflection curve at load point of beams B1, B2 and B3.



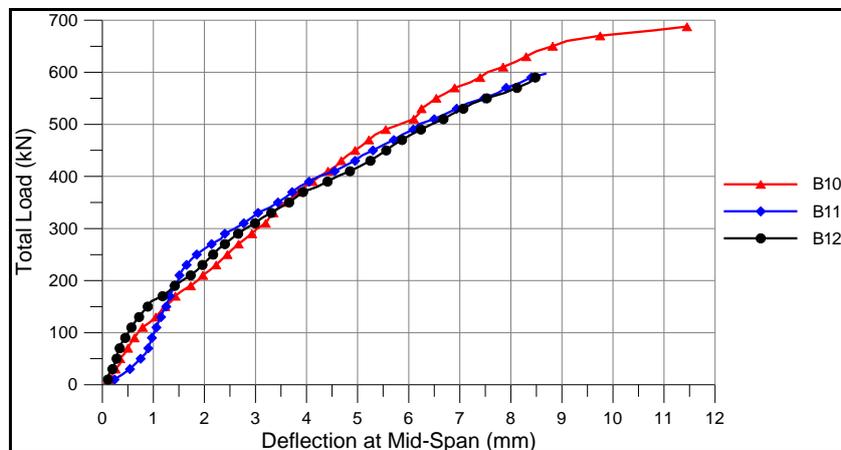
B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$),

Figure 6. Load-deflection curve at mid-span of beams B1, B2 and B3.



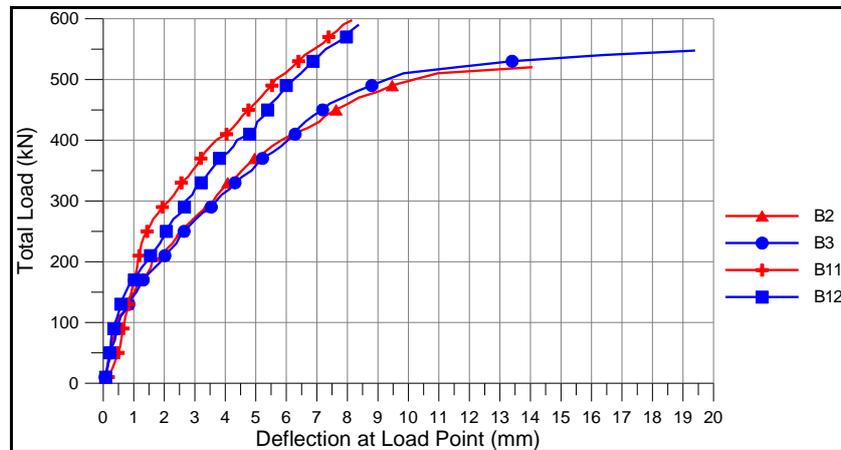
B10: Normally reinforced, **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$),

Figure 6. Load-deflection curve at load point of beams B10, B11 and B12.



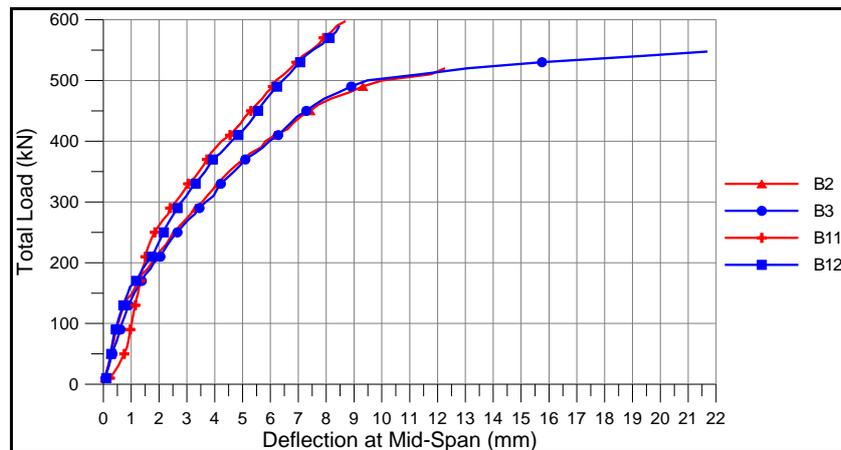
0: Normally reinforced, **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$),

Figure 6. Load-deflection curve at mid-span of beams B10, B11 and B12.



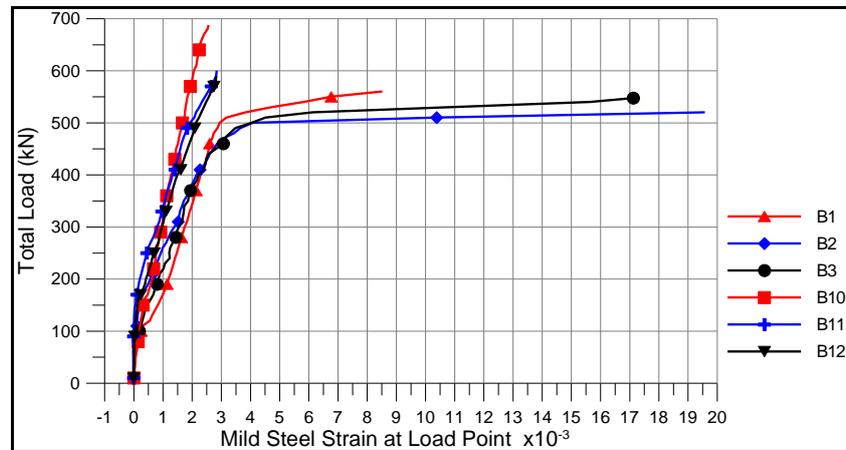
B2: Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 7. Load-deflection curve at Load point of beams B2, B3, B11 and B12.



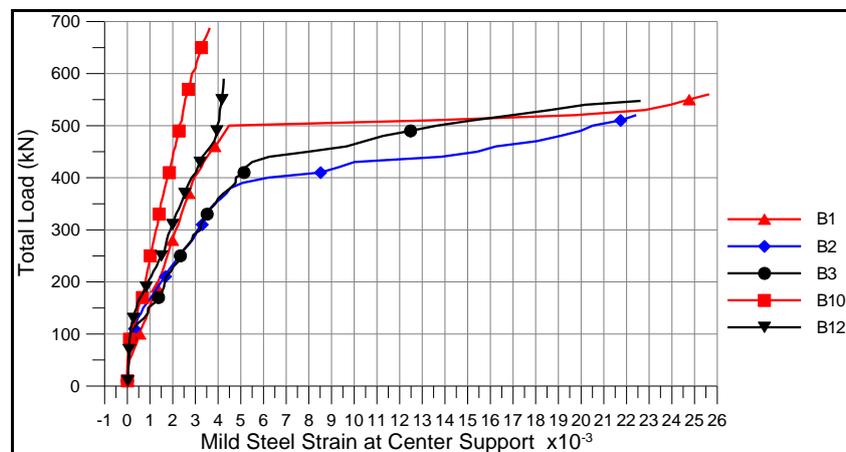
B2: Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 7. Load-deflection curve at mid-span of beams B2, B3, B11 and B12.



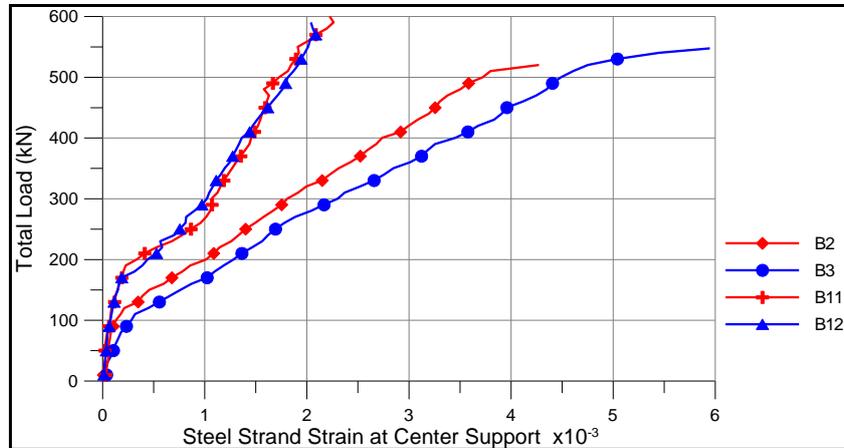
B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B10:** Normally reinforced, **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 8. Load-strain relationship of mild steel at load point of beams B1, B2, B3, B10, B11 and B12.



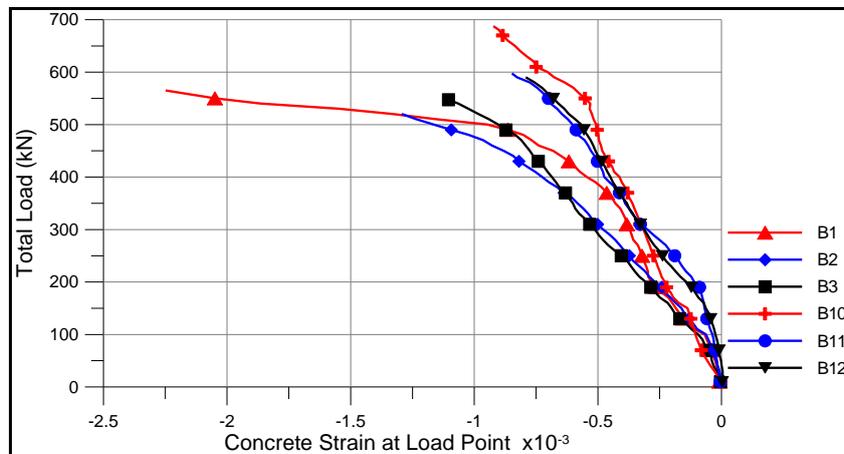
B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B10:** Normally reinforced, **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 9. Load-strain relationship of mild steel at middle support of beams B1, B2, B3, B10 and B12.



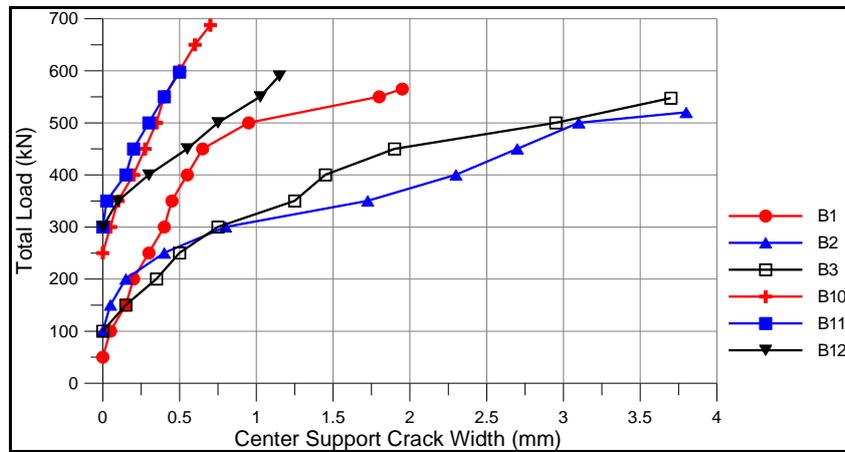
B2: Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 10. Load-strain relationship of steel strand at middle support of beams B2, B3, B11 and B12.



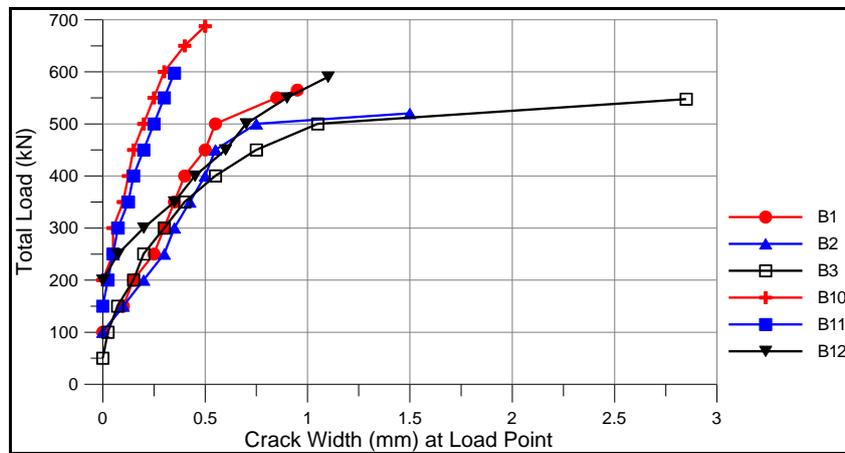
B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B10:** Normally reinforced, **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 11. Load-strain relationship of concrete at load point of beams B1, B2, B3, B10, B11 and B12.



B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B10:** Normally reinforced, **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 12. Cracking width at center support of beams B1, B2, B3, B10, B11 and B12.



B1: Normally reinforced, **B2:** Partially prestressed with one strand ($0.7f_{py}$), **B3:** Partially prestressed with one strand ($0.55f_{py}$), **B10:** Normally reinforced, **B11:** Partially prestressed with two strands ($0.7f_{py}$), **B12:** Partially prestressed with two strands ($0.55f_{py}$)

Figure 13. Cracking width under load point at right span of beams B1, B2, B3, B10, B11 and

B12.

Table 4. Cracking load and maximum crack width at ultimate load.

Beam set	Beam Cracking Load (kN)			Beam Cracking Load/ultimate load % (Pcr/Pu)			Ultimate load, (kN)	Maximum crack width at ultimate (mm)		
	Left Span	Center Support	Right Span	Left Span	Center Support	Right Span		Left Span	Center Support	Right Span
B1	112	58	107	19.82	10.26	18.93	565.0	1.05	1.95	0.95
B2	105	116	102	20.19	22.3	19.61	520.0	1.55	3.8	1.5
B3	94.5	104	107	17.26	18.99	19.54	547.5	3.95	3.7	2.85
B10	170	120	160	24.72	17.45	23.27	687.5	0.5	0.7	0.55
B11	148	196	142	24.76	32.8	23.76	597.5	0.35	0.5	0.45
B12	162	170	162	27.45	28.81	27.45	590.0	1.1	1.65	1.05

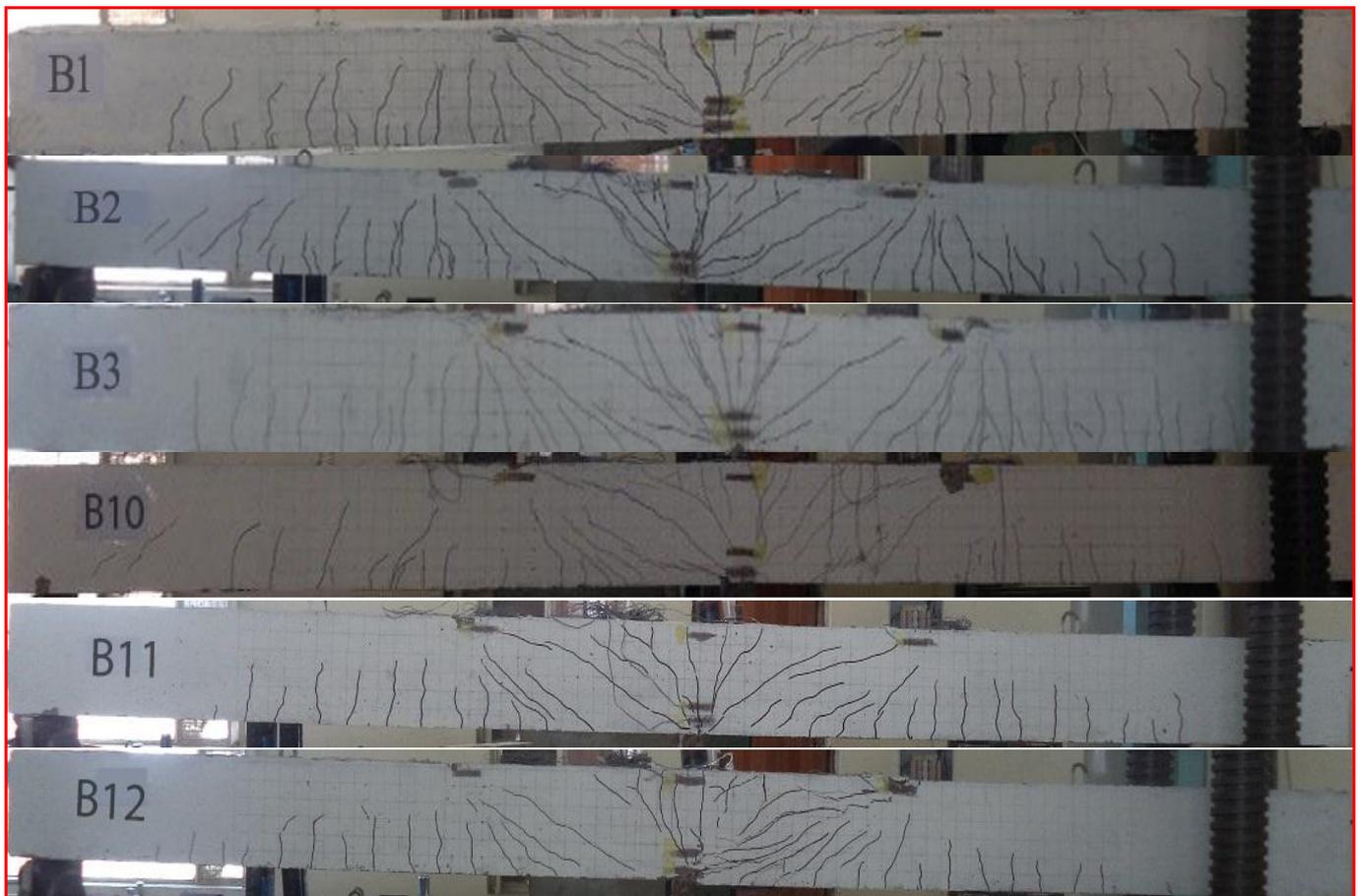


Figure 14. Crack Pattern at ultimate load of tested beams.

**Table 5.** Experimental and theoretical load capacities for tested beams.

Beam Set	Experimental Ultimate Load, P_u, Exp., (kN)	Theoretical Ultimate Load, P_{u1}, Cal., using elastic analysis, (kN)	(P_u, Exp/P_{u1}, Cal.), (%)	Theoretical Ultimate Load, P_{u2}, Cal., using elastic analysis in addition to moment redistribution, (kN)	(P_u, Exp/P_{u2}, Cal.), (%)
B1	565	513.782	109.97	587.64	96.15
B2	520	402.264	129.27	475.49	109.36
B3	547.5	405.348	135.07	481.89	113.61
B10	687.5	676.419	101.64	676.419	101.64
B11	597.5	537.964	111.07	605.32	98.71
B12	590	543.685	108.52	618.63	95.37